Optimal Monetary Policy and the Sources of Local-Currency Price Stability[1]

Giancarlo Corsetti[a]

European University Institute, University of Rome III and CEPR

Luca Dedola[b]

European Central Bank and CEPR

Sylvain Leduc[c]

Federal Reserve Board

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[a]Address: Via dei Roccettini 9, San Domenico di Fiesole 50016, Italy; email: Giancarlo.Corsetti@eui.eu.

[b]Address: Postfach 16 013 19, D-60066 Frankfurt am Main, Germany; email: luca.dedola@ecb.int.

[c]Address: 20th and C Streets, N.W., Stop 43, Washington, DC 20551, USA; email: Sylvain.Leduc@.frb.gov.

Abstract

We analyze the policy trade-offs generated by local currency price stability of imports in economies where upstream producers strategically interact with downstream firms selling the final goods to consumers. We study the effects of staggered price setting at the downstream level on the optimal price (and markup) chosen by upstream producers and show that downstream price movements affect the desired markup of upstream producers, magnifying their price response to shocks. We revisit the international dimensions of optimal monetary policy, unveiling an argument in favor of consumer price stability as the main prescription for monetary policy. Since stable consumer prices feed back into a low volatility of markups among upstream producers, this contains inefficient deviations from the law of one price at the border. However, efficient stabilization of different CPI components will not generally result into perfect stabilization of headline inflation. National policies optimally respond to the same shocks in a similar way, thus containing volatility of the terms of trade, but not necessarily of the real exchange rate. The latter will be more volatile, among other things, the larger the home bias in expenditure and the content of local inputs in consumer goods.

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1 Introduction

The high degree of stability of import prices in local currency, documented both at the border and at consumer level, vis-à-vis large movements in exchange rates, raises issues at the core of the design of national monetary policies in a globalizing world economy.[1] On the one hand, a low elasticity of import prices with respect to the exchange rate can result from the presence of costs incurred locally before the imported goods reach the consumers, such as distribution costs or assembling costs, i.e. costs of combining imported intermediated inputs with domestic inputs. By the same token, it may result from optimal markup adjustment by monopolistic firms, which maximize profits through price discrimination across national markets ("pricing to market"). These are real sources of local-currency price stability of imports, which influence pricing even in the absence of nominal rigidities. Although these factors may result in inefficiencies — like deviations from the law of one price due to pricing to market — they also shield price and wage dynamics from currency volatility, thus helping central banks maintain a low and stable headline inflation. On the other hand, stable import prices and low exchange rate pass-through can also stem from nominal frictions impeding desired markup adjustment, thus interfering with equilibrium movements in relative prices. When local-currency-price stability of imports is due to price stickiness, it creates policy trade-offs between competing objectives, e.g. between stabilizing the prices of domestically produced goods as opposed to the (relative) price of imported goods, which raise the importance of international considerations in the conduct of monetary policy.

In this paper we reconsider these policy trade-offs in economies where stable import prices in local currency result from both nominal rigidities and endogenous destination-specific markup adjustment. We specify a two country model where each economy produces an array of country-specific, differentiated traded goods. In each country, we model local downstream firms as using one intermediate traded good, and possibly local inputs, to produce nontradable final goods. In other words, each intermediate good is produced by an upstream monopolist and sold to a continuum of monopolistic downstream firms, active in each country, from which local consumers can directly buy further differentiated final varieties. Thus, because both upstream

[1] See, e.g., Engel and Rogers, [1996]; Goldberg and Knetter, [1997]; Campa and Goldberg, [2005] and Frankel et al. [2004].

and downstream firms have monopoly power, final prices reflect double marginalization. We posit that markets are segmentated across national borders, so that intermediate producers price-discriminate between domestic and foreign local downstream producers as a group, although not among individual local producers (charging different prices within the same country).

As in standard monetary models, we assume that firms set prices in local currency, adjusting them infrequently according to the Calvo mechanism.[2] Different from the previous literature, however, we explicitly model strategic interactions among upstream and downstream firms: upstream firms exercise their monopoly power by taking into account country-specific differences in the properties of the demand for their products. Relative to the literature already modelling vertical interactions between exporters and local firms (e.g. Bacchetta and Van Wincoop [2005], Corsetti and Dedola [2005], Devereux and Engel [2007], Monacelli [2005]), an important novel contribution of this paper consists of analyzing the effects of staggered price setting at the downstream level on the optimal price (and markup) chosen by upstream producers.

Specifically, our analysis establishes three key characteristics of the perceived demand elasticity by upstream producers when nominal rigidities constrain price decisions by downstream firms. *First*, this elasticity is a decreasing function of the rate of change of final prices in each industry: the higher this rate (thus the higher the price dispersion among final producers selling an industry product), the higher the intermediate producer's equilibrium markup. *Second*, the perceived demand elasticity is market-specific, depending on differences in industry-specific inflation rates across the domestic and the export market. Sticky prices at consumer level create an incentive for upstream firms to price discriminate across borders, which leads to equilibrium deviations from the law of one price, independently of the degree of nominal rigidities in the upstream firms' own prices. *Third*, if either local inputs in downstream production are good substitute of intermediate imported goods, or their share in the downstream firms' costs is low, the demand elasticity is decreasing in the price charged by upstream producers. In other words, downstream nominal rigidities magnify the price response to shocks by upstream monopolists who optimally reset their price in any given period. This generates strategic substitutability among upstream producers: a rise in marginal costs will lead to an increase in their desired markups.

[2]See the literature review in section 2 below.

These results have at least two notable implications for policy modelling and design. *First*, by shedding light on the link between optimal price adjustment at the dock and domestic inflation rates, our results suggest a specific reason why, in line with the observations by Taylor [2000], lower CPI inflation volatility and price dispersion may result in a lower degree of exchange-rate pass-through: stable inflation reduces at the margin the producers' incentives to price discriminate across countries, decreasing the sensitivity of their 'desired markup' to cost changes.

Second, by showing that downstream price rigidities result into strategic substitutability among upstream producers, our results emphasize that adding several layers of nominal rigidities do not necessarily results into more price inertia. Strategic interactions among vertically integrated firms with sticky prices may create incentive for large price adjustment, feeding back into inflation volatility.

In addition, our model specification implies an important dimension of heterogeneity across firms which has a bearing on optimal monetary policy. In contrast to standard models, the marginal costs of our downstream firms are generally not symmetric, not even when the economy is completely closed to foreign trade and there are no markup shocks. Thus, monetary authorities are not able to achieve complete stabilization of final prices.[3]

The mechanism underlying these results is different from that emphasized by the previous literature focusing on vertical interactions between upstream and downstream firms, but that stresses real determinants of the local currency price stability of imports. In previous work of ours, we assume that local firms produce consumer goods by combining intermediate tradable goods with local inputs (Corsetti and Dedola [2005], Corsetti, Dedola, and Leduc [2007]). In this framework, provided that the tradable goods and the local inputs are poor substitutes in production, the presence of local inputs tends to mute the response of upstream prices to shocks (corresponding to a case of strategic complementarity), and makes the exchange-rate pass-through incomplete, even in the absence of nominal rigidities. Building an example of an economy encompassing both channels, we analyze conditions under which the properties of the demand elasticity faced by upstream producers are dominated by the effect of local inputs in production, as opposed to the effect of downstream nominal rigidities.

[3] In standard models with cost-push and markup shocks, monetary authorities can achieve complete price stability, but face trade-offs that motivate deviations from it — see for instance the discussion in Woodford [2003]. In our model, instead, stability of all prices is unfeasible to start with.

We characterize the optimal cooperative monetary policy under commitment. In order to reduce inefficiencies due to price stickiness, monetary policy does mitigate fluctuations in the major components of consumer price inflation. However, it falls short of stabilizing completely either the CPI, or the price of domestic intermediate goods.[4]

Optimal monetary policies address different trade-offs, specific to both the international and the domestic dimensions of the economy. First, as in Corsetti and Pesenti [2005], nominal rigidities in local currency at upstream level lead benevolent monetary authorities to attach a positive weight to stabilizing the consumer price of imports, and thus deviate from perfect stabilization of the final prices of domestic goods. Second, downstream technology shocks prevent perfect stabilization of all consumer prices, because vertical interactions with upstream firms, which may or may not adjust their prices, induce heterogeneity of marginal costs at retail level. This effect is compounded in an open economy setting, because of the response of the intermediate price of imports to exchange rate fluctuations. Third, the elasticity of the producer's demand curve falls with the industry's dispersion of final goods prices, motivating policy emphasis on final price stabilization.

None of these trade-offs, however, entail specific prescriptions regarding the volatility of the real exchange rate. In the literature, optimal monetary policy in models with nominal rigidities in local currency is sometimes associated with a limited degree of real exchange rate volatility, relative to the terms of trade — see e.g. Devereux and Engel [2007]. In contrast, we find that implementing the optimal policy in our economy with nominal rigidities leads the real exchange rate to be more volatile, and the terms of trade to be less volatile, than in the same economy under flexible prices. This is because of the combined effects of nominal rigidities, and the presence of nontradable components in final goods. We take these findings as a caution against strong policy prescriptions on the need to curb the volatility of the real exchange rate. The point is that, while there are good reasons to expect optimal policies to contain the volatility of the terms of trade, these reasons cannot be mechanically extended to the real exchange rate, whose volatility is bound to depend on a number of structural features of the economy.

This paper is organized as follows. In the next section, we will briefly survey the literature

[4] In our model economy the isomorphism between optimal monetary policy in closed and open economies characterized by Clarida, Galí and Gertler [2002] obviously does not hold, as policymakers face several trade-offs which make perfect stabilization of domestic inflation suboptimal.

on pricing to market and monetary policy, with the goal of clarifying our contributions to it. Section 3 specifies the model, while Section 4 provides analytical results on the link between price stickiness and price discrimination arising from vertical interactions. Section 5 describes our calibration of the model. Section 6 presents the equilibrium dynamics of prices in response to shocks, while Section 7 discusses the allocation under the optimal policy, relative to alternative policy rules and the case of flexible prices. Section 8 concludes. An appendix provides analytical details on the model and the derivation of the main results.

2 Local currency price stability and efficient monetary stabilization

In this section we briefly reconstruct the main development of recent debates on the local currency price stability of imports, and their implications for the international transmission and the optimal design of monetary policy, with the goal of clarifying our contribution to the literature. A core issue underlying these debates is whether monetary policy should react to international variables, such as the exchange rate or the terms of trade, beyond the influence that these variables have on the domestic output gap (for example, via external demand) and on the domestic good prices — so that it would have a specific 'international dimension.' As discussed below, models stressing the stability of import prices in local currency have provided one possible answer to this question stressing the implications of nominal rigidities for monetary transmission and stabilization policy.

2.1 Nominal rigidities and the international dimensions of optimal monetary policy

At the heart of the international dimension of monetary policy lies the role of the exchange rate in the international transmission mechanism. Consistent with traditional open macroeconomic models, the seminal contribution to the New Open Economy Macroeconomics (henceforth NOEM) by Obstfeld and Rogoff [1995] embraces the view that exchange rate movements play the stabilizing role of adjusting international relative prices in response to shocks, when frictions prevent or slow down price adjustment in the local currency. The idea is that nominal depreciation transpires into real depreciation, making domestic goods cheaper in the world markets, hence redirecting world demand towards them: exchange rate movements therefore have 'expenditure switching effects'. Accordingly, NOEM contributions after Obstfeld and Rogoff [1995] draw on the Mundell–Fleming and Keynesian tradition, and posit that firms preset the price for their products in domestic currency, implying that export prices are sticky in the currency of the producers — this is why such hypothesis is commonly dubbed 'producer currency pricing' (henceforth PCP). Under this hypothesis, nominal import prices in local currency move one-to-one with the exchange rate and pass-through is perfect. In the baseline model with preset prices, to the extent that the demand elasticities are identical across countries, there is no incentive for producers to charge different prices in different markets: in equilibrium there would be no deviations from the law of one price even if national markets were segmented.

In model economies with PCP, optimal monetary policy rules tend to be 'inward-looking' and 'iso-morphic' to the rules derived in closed-economy models: welfare-maximizing central banks pursue the stabilization of domestic producers marginal costs and markups — hence they aim at stabilizing the GDP deflator —, while letting the consumer price index (CPI) fluctuate with efficient movements in the relative price of imports. There is no need for monetary policies to react to international variables — a result that in the baseline NOEM model after Corsetti and Pesenti [2001] goes through under different assumptions regarding nominal rigidities, including staggered price setting and partial adjustment (see, for example, Clarida, Galí and Gertler [2002], or Benigno and Benigno [2003] for a generalization of the baseline model).

The high elasticity of import prices to the exchange rate underlying the contributions after Obstfeld and Rogoff [1995], however, is clearly at odds with a large body of empirical studies showing that the exchange rate pass-through on import prices is far from complete in the short run, and deviations from the law of one price are large and persistent (see, for example, Engel and Rogers [1996]; Goldberg and Knetter [1997]; Campa and Goldberg [2005]). Based on this evidence, several contributions have engaged in a thorough critique of the received wisdom on the expenditure switching effects of the exchange rate. Specifically, Betts and Devereux [2000] and Devereux and Engel [2003], among others, posit that firms preset export prices in the currency of the market where they sell their goods. This assumption, commonly dubbed 'local currency pricing' (henceforth LCP), attributes local currency price stability of imports entirely to nominal frictions. The far-reaching implications of LCP for the role of the exchange rate in the international transmission mechanism have been widely discussed by the literature (see e.g.

Engel [2003]).

To the extent that import prices are sticky in the local currency, a Home depreciation does not affect the price of Home goods in the world markets; hence, it has no expenditure switching effects. Instead, it raises the ex-post markups on Home exports: at given marginal costs, revenues in domestic currency from selling goods abroad rise. In contrast with the received wisdom, nominal depreciation strengthens a country's terms of trade: if export prices are preset during the period, the Home terms of trade improves when the Home currency weakens.

As opposed to earlier literature, models assuming LCP unveil a clear-cut argument in favour of policies with an 'international dimension.' One way to present the argument is as follows. To the extent that exporters' revenues and markups are exposed to exchange rate uncertainty, firms' optimal pricing strategies internalize the monetary policy of the importing country. In the benchmark model by Corsetti and Pesenti [2005], for instance, foreign firms optimally preset the price of their goods in the Home market one period ahead, by charging the equilibrium markup over expected marginal costs evaluated in Home currency. The preset price of Home imports then depends on the joint distribution of Home monetary policy and Foreign productivity shocks: in the model, it is increasing in the variance of nominal marginal costs.

The reason why the isomorphism between closed-economy and open-economy monetary rules breaks down is apparent. Suppose that the Home monetary authorities ignore the influence of their decisions on the price of Home imports. Incomplete stabilization of Foreign firms marginal costs and markup in local currency will translate into inefficiently high local prices of their product (relative to their level in the flexible-price allocation). On the other hand, if Home monetary authorities wanted to stabilize Foreign firms' marginal costs, they could only do so at the cost of raising costs and markup uncertainty for Home producers, resulting in inefficient Home good prices. It follows that, to maximize Home welfare, Home policymakers should optimally trade-off the stabilization of marginal costs of all producers (domestic and foreign) selling in the Home markets. The optimal response to Foreign shocks by domestic policymakers depends, among other factors, on the degree of openness of the economy, as indexed by the overall share of imports in the CPI (see Corsetti and Pesenti [2005], and Sutherland [2005], for a discussion of intermediate degrees of pass-through).

In section 7 of this paper we will show that these basic principles of the 'international dimensions of optimal monetary policies' go through in models assuming LCP and staggered price adjustment. Namely, in our model monetary authorities will optimally attempt to stabilize the CPI, although CPI stabilization will not be complete because of the asymmetry in shocks hitting different economies and different sectors of the same national economy, creating the need for relative price adjustment. At an optimum, welfare-maximizing policymakers will thus trade-off inefficient misalignment of import prices, with inefficient relative price dispersion among domestic and foreign goods (see also Smets and Wouters, [2002], and Monacelli [2005]).

2.2 Interactions of nominal and real determinants of local currency price stability of imports

While most of the discussion in the NOEM literature has focused on incomplete pass-through as an implication of nominal rigidities, a low pass-through, in itself, is not necessarily incompatible with expenditure switching effects — a point stressed by Obstfeld [2000] among others. In this respect, Obstfeld and Rogoff [2000] points out that, in the data (and consistent with the received wisdom), nominal depreciation does tend to be associated with deteriorating terms of trade. This piece of evidence clearly sets an empirical hurdle for LCP models: specifications which assume a very high degree of price stickiness in local currency cannot pass this test (see Corsetti, Dedola, and Leduc [2005], for a quantitative assessment). Interestingly, estimates of LCP models attributing incomplete pass-through exclusively to nominal rigidities in local currency, tend to predict that the degree of price stickiness is implausibly higher for imports than for domestic goods — a result suggesting model misspecification (see, for example, Lubik and Schorfheide [2006]).

The key issue is the extent to which the evidence of local currency price stability of imports can be explained by nominal rigidities. In the literature, it is well understood that the low elasticity of import prices at the retail level with respect to the exchange rate is in large part due to the incidence of distribution (see Burstein, Eichenbaum, and Rebelo [2006] for a recent reconsideration of this point). Namely suppose that import prices at the dock move one-to-one with the exchange rate, but the distribution margin account for 50 percent of the retail price, mostly covering local costs. A one percent depreciation of the currency will then affect the final price of the imported good only by 1/2 percent.

In addition, several macro and micro contributions have emphasized that import prices at the dock do not move one-to-one with the exchange rate because of optimal destination-specific markup adjustment by monopolistic firms. Instances of these studies include Dornbusch [1987], stressing market structure, as well as previous work by two of us (Corsetti and Dedola [2005]), where upstream monopolists sell their tradable goods to downstream firms, which combine them with local inputs before reaching the consumer. The latter contribution establishes that, to the extent that the tradable goods and the local inputs are not good substitutes in the downstream firms' production, the demand elasticity faced by upstream monopolists will be (a) market specific, causing optimal price discrimination across markets, and (b) increasing in the monopolists price, thus leading to incomplete exchange rate pass-through independently of nominal rigidities. Based on this principle, that paper then generalizes the model with distribution by Burstein, Neves and Rebelo [2003], as to encompass local currency price stability due to endogenous movements of markups implied by the presence of distribution services intensive in local inputs. The same principle nonetheless can be applied to models where intermediate imported inputs are assembled using local inputs — a case analyzed by Bacchetta and Van Wincoop 2003. Whether one has in mind markets with high distribution margins (such as the market for cups of coffee at Starbucks in the US), or markets for goods with a relatively high incidence of imported parts (such as the market for cars 'made in the US'), incomplete exchange rate pass through can be traced back to some degree of complementarity between imported goods and local input/services.

Analyses of the relative importance of these different sources of import price stability (especially local costs) are provided by several market-specific studies — such as Goldberg and Verboven [2001], Goldberg and Hellerstein [2007], and Hellerstein [2006]. The main result emerging from these partial-equilibrium contributions is that real factors can explain a large extent of local currency price stability of imports. Most interestingly, similar conclusions can be reached using quantitative, general equilibrium models, as suggested by the numerical exercises in Corsetti and Dedola [2005].

Yet, quantitative studies incorporating these factors also corroborate the idea that a realistic degree of nominal rigidities can improve substantially the performance of the model. In Corsetti, Dedola, and Leduc [2005], we show that a model assuming LCP together with vertical interaction between producers and distributors can pass the empirical hurdle set by Obstfeld and Rogoff [2000], provided that the average frequency of price adjustment is consistent with the evidence by Bils and Klenow (2004).

Research is therefore increasingly focused on the interaction between real and monetary determinants of low exchange rate pass-through and deviations from the law of one price. A first early instance of research focused on such interaction is provided by contributions which emphasize the need to treat the currency denomination of exports as an endogenous choice by profit maximizing firms. Bacchetta and Van Wincoop [2005], Devereux, Engel and Storgaard [2004], and Friberg [1998] have developed models where firms can choose whether to price export in domestic or in foreign currency, knowing that price updates will be subject to frictions. A number of factors — from the market share of exporters to the incidence of distribution, and the availability of hedging instruments — potentially play a crucial role in this choice (see Engel [2006], for a synthesis).

Although most of these models are developed assuming an arbitrary monetary policy, the role of optimal stabilization policy in the choice between LCP and PCP is addressed by Corsetti and Pesenti [2001]. The main idea is that expansionary monetary shocks unrelated to fundamental shocks (e.g. productivity) raise nominal wages and marginal costs while depreciating the currency. Consider a firm located in a country with noisy monetary policy, i.e. hit by frequent monetary shocks unrelated to fundamentals. For such firm, pricing its exports in foreign currency (that is, choosing LCP) is attractive in the following sense: it ensures that revenues from exports in domestic currency will move in parallel with nominal marginal costs, with stabilizing effects on the markup. This is because any expansionary monetary shock depreciating the Home currency would simultaneously raise wages and the domestic currency revenue from unit sales abroad (at an unchanged local price). This observation may help explain why exporters from countries with relatively unstable domestic monetary policies (e.g. some developing countries) prefer to price their exports to developed countries in the importers' currency. The same argument, however, suggests that LCP is not necessarily optimal for exporters producing in countries where monetary policy systematically stabilizes marginal costs (see Goldberg and Tille [2005], for empirical evidence). For firms located in these countries, real factors arguably become more relevant in the choice.

A second instance of the new directions taken by the literature consists of studies taking the LCP choice as given, and combining it with different determinants of pricing to market and incomplete pass-through. This is the approach we take in this paper. In contrast from previous contributions, where price stickiness is not linked to price discrimination (see e.g. Monacelli [2005]), or where nominal rigidities and price discrimination coexist without feeding into each other (see e.g. Corsetti, Dedola, and Leduc [2005]), we specify a model building on the intuitive idea that the frequency of price changes by local downstream firms selling products to consumers, is bound to affect the elasticity of demand perceived by upstream producers of intermediate (tradable inputs or) goods. The novel result of our study is that, looking at the interactions between nominal and real determinants of price discrimination in an otherwise standard monetary model, nominal rigidities at the retail level do not necessarily lower the equilibrium reaction of final prices to exchange rate movements, thus increasing price inertia. As mentioned in the Introduction, downstream price rigidities tend to generate strategic substitutability among upstream producers and an overall larger sensitivity of all prices to exchange rate changes.

3 The model economy

The world economy consists of two countries of equal size, H and F. Each country specializes in one type of tradable good, produced in a number of differentiated industries defined over a continuum of unit mass. Tradable goods are indexed by $h \in [0,1]$ in the Home country and $f \in [0,1]$ in the Foreign country. In each industry, the firm producing the tradable good h (or f) is a monopolistic supplier of one good, using labor as the only input to production. These firms set prices in local currency units and in a staggered fashion as in Calvo [1983].

A distinctive feature of our setting is that we model a downstream sector in each country. Specifically, we assume that each producer's good h is sold to consumers in many varieties by a continuum of local firms indexed by $r_h \in [0,1]$. These firms buy the h tradable goods, and turn them into consumer goods — which are not traded across borders — with random productivity. We will distinguish between two cases: one in which local firms use domestic labor as an input; the other in which they do not. Similar to upstream producers, also downstream operate under monopolistic competition and set prices in a staggered fashion as in Calvo [1983].

By the logic of the Calvo adjustment, local downstream firms buying goods from upstream producers charge different prices to final users, with a constant fraction of them reoptimizing prices in each period. In principle, one could assume that upstream firms exercise their monopoly power by charging individual prices which are specific to each downstream firm. However, we find it more realistic and convincing to assume that upstream producers are not able to price-discriminate across individual local firms, but only across groups of them — namely, across domestic and foreign local firms. So, we assume that upstream producers exercise their monopoly power and set prices by taking into account the total demand for their product in each market, at Home and in the Foreign country.

In what follows, we describe our set up focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy — variables referred to Foreign firms and households are marked with an asterisk.

3.1 The Household's Problem

3.1.1 Preferences

The representative Home agent maximizes the expected value of her lifetime utility, given by the following standard functional form:

$$V_0 = E \sum_{t=0}^{\infty} \beta^t U \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\sigma}}{1-\sigma} + \kappa \frac{(1-L_t)^{1-\upsilon}}{1-\upsilon}, \right], \tag{1}$$

where instantaneous utility U is a function of a consumption index, C_t, leisure, $(1-L_t)$, and real money balances $\frac{M_{t+1}}{P_t}$. Households consume both domestically produced and imported goods. We define $C_t(h, r_h)$ as the Home agent's consumption as of time t of the variety r_h of the Home good h, produced and distributed by the firm r_h; similarly, $C_t(f, r_f)$ is the Home agent's consumption of the variety r_f of the good f, produced and distributed by firm r_f. For each good h (or f), we assume that one final good variety r_h (r_f) is an imperfect substitute for all other final good varieties, with constant elasticity of substitution $\eta > 1$:

$$C_t(h) \equiv \left[\int_0^1 C_t(h, r_h)^{\frac{\eta-1}{\eta}} dr \right]^{\frac{\eta}{\eta-1}}, \qquad C_t(f) \equiv \left[\int_0^1 C_t(f, r_f)^{\frac{\eta-1}{\eta}} dr \right]^{\frac{\eta}{\eta-1}},$$

$C_t(h)$ is the consumption of (all varieties of) the Home good h, by the Home agent, at time t; similarly, $C_t(f)$ is the same agent's consumption of the Foreign good f. We then assume that the good produced by the h industry is an imperfect substitute for all other goods produced by the Home industries, with the same constant elasticity of substitution $\eta > 1$ as between final good varieties. Aggregate consumption of Home and Foreign goods by the Home agent is thus defined as:

$$C_{\mathrm{H},t} \equiv \left[\int_0^1 C_t(h)^{\frac{\eta-1}{\eta}} dh \right]^{\frac{\eta}{\eta-1}}, \qquad C_{\mathrm{F},t} \equiv \left[\int_0^1 C_t(f)^{\frac{\eta-1}{\eta}} df \right]^{\frac{\eta}{\eta-1}},$$

The full consumption basket, C_t, in each country, aggregates Home and Foreign goods according to the following standard CES function:

$$C_t \equiv \left[a_{\mathrm{H}}^{1-\phi} C_{\mathrm{H},t}{}^{\phi} + a_{\mathrm{F}}^{1-\phi} C_{\mathrm{F},t}{}^{\phi} \right]^{\frac{1}{\phi}}, \qquad \phi < 1, \tag{2}$$

where a_{H} and a_{F} are the weights on the consumption of Home and Foreign traded goods, respectively and $\dfrac{1}{1-\phi}$ is the constant elasticity of substitution between $C_{\mathrm{H},t}$ and $C_{\mathrm{F},t}$.

3.1.2 Budget constraints and asset markets

For simplicity, we posit that domestic and international asset markets are complete and that only domestic residents hold the Home currency, M_{t+1}. Households derive income from working, $W_t L_t$, from domestic firms ' profits and from previously accumulated units of currency, as well as from the proceeds from holding state-contingent assets, B_t. They pay non-distortionary (lump-sum) net taxes $\mathbb{T}$, denominated in Home currency. Households use their disposable income to consume and invest in state-contingent assets. The individual flow budget constraint for the representative agent j in the Home country is therefore:

$$\begin{aligned} \mathbb{P}_{\mathrm{H},t} C_{\mathrm{H},t} + \mathbb{P}_{\mathrm{F},t} C_{\mathrm{F},t} + \int_s p_{bt,t+1} B_{t+1} + M_{t+1} \leq W_t L_t + M_t + B_t + \int_0^1 \Pi(h) dh \\ + \int_0^1 \int_0^1 \Pi(h, r_h) dh dr_h + \int_0^1 \int_0^1 \Pi(f, r_f) df dr_f + \mathbb{T}_t \end{aligned} \tag{3}$$

where $\Pi(.)$ denotes the agent's share of profits from all firms h and r in the economy. The price indexes are as follows: $\mathbb{P}_{\mathrm{H},t}$ denotes the consumer price of the aggregate Home traded good; $\mathbb{P}_{\mathrm{F},t}$ denotes the consumer price of aggregate Home imports. We will also denote the overall consumer price index (CPI) by P_t. All these indices are defined below.

The household's problem consists of maximizing lifetime utility, defined by (1), subject to the constraint (3).

3.2 Production structure and technology

International price discrimination is a key feature of the international economy captured by our model. In what follows we show that, even if Home and Foreign consumers have identical constant-elasticity preferences for consumption, vertical interactions between upstream and downstream firms causes differences in the elasticity of demand for the h (f) product at wholesale level across national markets. Upstream firms will thus want to charge different prices at

Home and in the Foreign country. We will focus our analysis on Home firms — optimal pricing by Foreign firms can be easily derived from it. To distinguish between upstream and downstream firms, we will denote variables referred to the former with an upper bar.

We begin by specifying the technology used by upstream firms producing Home tradables. These firms employ domestic labor to produce a differentiated product h according to the following linear production function:

$$\overline{Y}(h) = \overline{Z} \cdot \overline{L}(h),$$

where $\overline{L}(h)$ is the demand for labor by the producer of the good h and $\overline{Z}$ is a technology shock common to all upstream producers in the Home country, which follows a statistical process to be specified below. The letter h will be indifferently referred to an upstream producer selling to downstream firms r_h, or the corresponding 'industry.'

In each industry h, downstream firms r_h combine the traded input, bought from upstream producers, with some local nontraded input. For analytical convenience, in most of our analysis we do not model the local nontraded input explicitly, but posit that the production function of firms r_h is linear in the traded input only

$$Y(h, r_h) = ZX(h, r_h) \tag{4}$$

where $X(h, r_h)$ is the demand for tradable good h by firm r_h, Z is a random technology component that affects the amount of traded input required to produce the variety r_h and distribute it to consumers. This random shock is country-specific, and hits symmetrically all national downstream firms.

The use of the local input is consequential for our results, to the extent that it is a poor substitute with X. This case has been made in previous work of ours (see Corsetti and Dedola 2005 and in Corsetti, Dedola and Leduc 2005), in which we have assumed that downstream firms in an industry h combine the tradable good h with a local input according to a fixed-proportion production function, such as

$$Y(h, r_h) = Min\left[X(h, r_h), ZL(h, r_h)\right] \tag{5}$$

Here, $L(h, r_h)$ is the demand for labor by the downstream firm r_h, and the random technology component Z now affects the amount of labor required to produce the variety r_h and distribute it to consumers.

In our previous contributions, we have shown that the above specification can generate endogenous movements in upstream firms' markups and cross-border price discrimination independently of nominal price rigidities. In this paper, we make a different, but complementary point. Namely, we show that vertical interactions among upstream and downstream firms can lead to price discrimination *exclusively* as a consequence of nominal rigidities. To focus sharply on the mechanism underlying this new result, throughout our analysis we specify the production function of downstream firms as in (4), abstracting from the local nontraded input. For the sake of comparison, however, in the next sections we will also show analytical results for the production function (5).

3.3 The problem of downstream firms

Both upstream and downstream firms are subject to nominal rigidities à la Calvo. Hence, at any time t downstream firms will buy either from a producer h which updates its price in the same period, or from a producer still charging an old price. Conversely, in each period, upstream producers updating their price will need to consider that only a fraction of downstream firms buying their products will also re-optimize in the period. In characterizing optimal pricing decisions, it is instructive to go over these cases one by one. Let θ be the probability that a downstream firm within the industry h keeps its price fixed — in each period a firm r_h sets a new price with probability $(1-\theta)$. The corresponding probabilities for the upstream producers will be denoted by $\overline{\theta}$ and $\left(1-\overline{\theta}\right)$.

Consider first the optimization problem of the downstream firms r_h which can reset their product prices in the current period t. The representative firm r_h chooses $P_t(h, r_h)$ to maximize the expected discounted sum of profits:

$$\pi(h) = E_t \left\{ \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k \left(P_t(h, r_h) C_{t+k}(h, r_h) - MC_{t+k}(h) C_{t+k}(h, r_h) \right) \right\},$$

where $p_{bt,t+k}$ is the firm's stochastic nominal discount factor between t and $t+k$. This firm faces the following final demand:

$$C_t(h, r_h) = \left(\frac{P_t(h, r_h)}{\mathbb{P}_t(h)} \right)^{-\eta} \left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}} \right)^{-\eta} C_{\mathrm{H},t},$$

where $\mathbb{P}_t(h)$ is the price index of the good (or industry) h, and $\mathbb{P}_{\mathrm{H},t}$ is the price index of all Home goods. The optimal price charged to consumers can then be written in the following standard form:

$$P_t^o(h, r_h) = \frac{\eta}{\eta - 1} \frac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k MC_{t+k}(h) C_{t+k}(h, r_h)}{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k C_{t+k}(h, r_h)}. \tag{6}$$

where, depending on whether we consider (4) or (5), the firm's marginal cost, $MC_t(h)$, will be given by either of the following expressions:

$$\begin{aligned} MC_t(h) &= \frac{\overline{P}_t(h)}{Z_t} \\ MC_t(h) &= \overline{P}_t(h) + \frac{W_t}{Z_t} \end{aligned}$$

where $\overline{P}_t(h)$ is the price of good h charged by the producer in the industry.

Now, if the downstream firm operates in an industry in which the upstream producer does not re-optimize its product price during the period, the price $\overline{P}_t(h)$ in the above expression will coincide with the price charged in the previous period, i.e. $\overline{P}_t(h) = \overline{P}_{t-1}(h)$. Conversely, if the downstream firm r_h operates in an industry h in which the upstream firm has also reset the price of its product during the same period, the marginal cost will be depending on the new, optimized price, discussed in the following section. This has the noteworthy implication that downstream firms in different industries will be facing different marginal costs even in the face of common productivity shocks Z_t — a key feature of our model which will be important in determining the characteristics of the optimal monetary policy.

3.4 Price indexes and market clearing

3.4.1 Price indexes

Before getting to the analytical core of our contribution and delving into our numerical experiments, we conclude the presentation of the model by formally defining the price indexes repeatedly used in the analysis so far, and writing down the market clearing conditions in the goods market. In an industry in which the producer updates its price, the price index of the good h at consumer level is given by:[5]

$$\mathbb{P}_t(h) = \left[\int_0^1 P_t(h, r_h)^{1-\eta} dr_h\right]^{\frac{1}{1-\eta}} = \left[(1-\theta)P_t^o(h)^{1-\eta} + \theta\mathbb{P}_{t-1}(h)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Denoting with a tilde the prices in an industry in which the producer doesn't update its price, the price index is:

$$\widetilde{\mathbb{P}}_t(h) = \left[\int_0^1 P_t(h, r_h)^{1-\eta} dr_h\right]^{\frac{1}{1-\eta}} = \left[(1-\theta)\widetilde{P}_t^o(h)^{1-\eta} + \theta\widetilde{\mathbb{P}}_{t-1}(h)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

The price index of Home tradables consumed at home thus becomes:

$$\begin{aligned}
\mathbb{P}_{\mathrm{H},t} &= \left[\int_0^1 \mathbb{P}_t(h)^{1-\eta} dh\right]^{\frac{1}{1-\eta}} \quad => \\
\mathbb{P}_{\mathrm{H},t}^{1-\eta} &= \left[(1-\overline{\theta})\mathbb{P}_t(h)^{1-\eta} + \overline{\theta}\widetilde{\mathbb{P}}_t(h)^{1-\eta}\right] \\
&= (1-\theta)\left[(1-\overline{\theta})P_t^o(h)^{1-\eta} + \overline{\theta}\widetilde{P}_t^o(h)^{1-\eta}\right] + \theta\left[(1-\overline{\theta})\mathbb{P}_{t-1}(h)^{1-\eta} + \overline{\theta}\widetilde{\mathbb{P}}_{t-1}(h)^{1-\eta}\right] \\
&= (1-\theta)\left[(1-\overline{\theta})P_t^o(h)^{1-\eta} + \overline{\theta}\widetilde{P}_t^o(h)^{1-\eta}\right] + \theta\mathbb{P}_{\mathrm{H},t-1}^{1-\eta}
\end{aligned}$$

The price index associated with the consumption basket, C_t, is:

$$P_t = \left[a_{\mathrm{H}}\mathbb{P}_{\mathrm{H},t}^{\frac{\phi}{\phi-1}} + a_{\mathrm{F}}\mathbb{P}_{\mathrm{F},t}^{\frac{\phi}{\phi-1}}\right]^{\frac{\phi-1}{\phi}}.$$

Let $\mathcal{E}_t$ denote the Home nominal exchange rate, expressed in units of Home currency per unit of Foreign currency. The real exchange rate is costumarily defined as the ratio of CPIs' expressed in the same currency, i.e. $\frac{\mathcal{E}_t P_t^*}{P_t}$. The terms of trade are instead defined as the relative price of domestic imports in terms of exports, namely $\frac{\mathbb{P}_{\mathrm{F},t}}{\mathcal{E}_t\mathbb{P}_{\mathrm{H},t}^*}$.

3.4.2 Equilibrium in the goods market

To characterize the equilibrium conditions in the goods market, we equate supply to demand at each firm level. Integrating over all downstream firms in a given industry we get:

$$\overline{Y}_t(h) = a_{\mathrm{H}}\left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}}\right)^{-\eta}\left(\frac{\mathbb{P}_{\mathrm{H},t}}{P_t}\right)^{\frac{1}{\phi-1}}\frac{C_t}{Z_t}S_t(h) + (1-a_{\mathrm{H}})\left(\frac{\mathbb{P}_t^*(h)}{\mathbb{P}_{\mathrm{H},t}^*}\right)^{-\eta}\left(\frac{\mathbb{P}_{\mathrm{H},t}^*}{P_t^*}\right)^{\frac{1}{\phi-1}}\frac{C_t^*}{Z_t^*}S_t^*(f),$$

[5]We drop the index r_h in our notation of the optimal final prices, since in any given industry firms that can update their price will choose the same optimal one.

where $S_t(h)$ denotes industry h's relative price dispersion at the consumer level:

$$S_t(h) \equiv \int \left(\frac{P_t(h, r_h)}{\mathbb{P}_t(h)} \right)^{-\eta} dr_{h.}.$$

Integrating over all industries, aggregate output is:

$$\overline{Y}_t = a_{\mathrm{H}} \left(\frac{\mathbb{P}_{\mathrm{H},t}}{P_t} \right)^{\frac{1}{\phi-1}} \frac{C_t}{Z_t} \overline{S}_t + (1 - a_{\mathrm{H}}) \left(\frac{\mathbb{P}^*_{\mathrm{H},t}}{P^*_t} \right)^{\frac{1}{\phi-1}} \frac{C^*_t}{Z^*_t} \overline{S}^*_t$$

where the price dispersion term, $\overline{S}_t$, is defined as:

$$\overline{S}_t = \int \left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}} \right)^{-\eta} S_t(h) dh,$$

Observe that $\overline{S}_t$ captures the relative price dispersion within and across industries. Since $\overline{S}_t$ and $S_t(h)$ are bounded below by 1, price dispersion implies a real resource cost.

4 Modelling the sources of local currency price stability: Price discrimination and nominal rigidities

In this section we fully characterize pricing to market by upstream firms, as a function of final prices. A crucial feature of our model is that the demand price elasticity perceived by upstream producers is time varying as a function of downstream price inflation. Since results differ depending on the specification of the downstream firms' production function, we will characterize the optimal producer price $\overline{P}^o_t(h)$, and discuss its main properties, looking first at the case of downstream linear (Cobb-Douglas) production, then at the case of downstream Leontief production.

4.1 The problem of the upstream firms

Consistent with the logic of the Calvo model, we posit that, when upstream producers update their prices, they do so simultaneously in the Home and in the Foreign market, in the respective currencies. The maximization problem is then as follows:

$$Max_{\bar{p}(h),\bar{p}^*(h)} \;\; E_t \left\{ \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k \left(\left[\overline{P}_t(h) D_{t+k}(h) + \mathcal{E}_t \overline{P}^*_t(h) D^*_{t+k}(h) \right] - \overline{MC}_{t+k}(h) \left[D_{t+k}(h) + D^*_{t+k}(h) \right] \right) \right\} \tag{7}$$

where the marginal cost of the producer is given by:

$$\overline{MC}(h) = \frac{W_t}{\overline{Z}_t},$$

and, depending on the production function downstream, the Home and Foreign demands for the firm's variety are given by:

$$\begin{aligned} D_t(h) &= \frac{1}{Z_t}\int \left(\frac{P_t(h,r_h)}{\mathbb{P}_t(h)}\right)^{-\eta}\left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}}\right)^{-\eta} C_{\mathrm{H},t}dr_h \\ D_t^*(h) &= \frac{1}{Z_t^*}\int \left(\frac{P_t^*(h,r_h^*)}{\mathbb{P}_t^*(h)}\right)^{-\eta}\left(\frac{\mathbb{P}_t^*(h)}{\mathbb{P}_{\mathrm{H},t}^*}\right)^{-\eta} C_{\mathrm{H},t}^*dr_h^* \end{aligned}$$

in our linear production specification, or

$$\begin{aligned} D_t(h) &= \int \left(\frac{P_t(h,r_h)}{\mathbb{P}_t(h)}\right)^{-\eta}\left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}}\right)^{-\eta} C_{\mathrm{H},t}dr_h \\ D_t^*(h) &= \int \left(\frac{P_t^*(h,r_h^*)}{\mathbb{P}_t^*(h)}\right)^{-\eta}\left(\frac{\mathbb{P}_t^*(h)}{\mathbb{P}_{\mathrm{H},t}^*}\right)^{-\eta} C_{\mathrm{H},t}^*dr_h^* \end{aligned}$$

for the case of Leontief production function. In these expressions, $\mathbb{P}_t^*(h)$ and $\mathbb{P}_{\mathrm{H},t}^*$ denote the price index of industry h and of Home goods, respectively, in the Foreign country, expressed in Foreign currency. In comparing the two sets of demands above, recall that in the linear production case (4) the firm's productivity affect the quantity of tradable good h needed to satisfy a given final demand for each variety r_h: hence the demand for the monopolist's product is scaled by productivity.

For each industry h, we can write the relative price dispersion at the consumer level as:

$$S_t(h) = (1-\theta)\left(\frac{P_t(h)}{\mathbb{P}_t(h)}\right)^{-\eta} + \theta\pi_t^{\eta}(h)S_{t-1},$$

where $\pi_t^{\eta}(h) = \dfrac{\mathbb{P}_t(h)}{\mathbb{P}_{t-1}(h)}$.[6] Using this result, we can rewrite the demand faced by each upstream producer as a function of price dispersion. In other words, the producer's demand curve depends on the price dispersion at the consumer level, induced by infrequent price adjustment by downstream firms.[7]

[6] See appendix for details.

[7] In the Leontief case, for instance, the demand curve can be written as:

$$D_{t+k}(h) = \left[(1-\theta)P_{t+k}^{-\eta}(h) + \theta\mathbb{P}_{t+k-1}^{-\eta}(h)S_{t+k-1}(h)\right]\mathbb{P}_{\mathrm{H},t+k}^{\eta}C_{\mathrm{H},t+k}.$$

By the first order condition of the producer's problem, the optimal price $\overline{P}_t^o(h)$ in domestic currency charged to domestic downstream firms is:

$$\overline{P}_t^o(h) = \frac{E_t \sum_{k=0}^{\infty} \overline{\theta}^k p_{bt,t+k} \varepsilon_{t+k}(h) D_{t+k}(h) \overline{MC}_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \overline{\theta}^k p_{bt,t+k} \left(\varepsilon_{t+k}(h) - 1\right) D_{t+k}(h)}; \tag{8}$$

while the price (in foreign currency) charged to downstream firms in the foreign country is:

$$\overline{P_t^*}^o(h) = \frac{E_t \sum_{k=0}^{\infty} \overline{\theta}^k p_{bt,t+k} \varepsilon_{t+k}^*(h) D_{t+k}^*(h) \frac{\overline{MC}_{t+k}(h)}{\mathcal{E}_{t+k}}}{E_t \sum_{k=0}^{\infty} \overline{\theta}^k p_{bt,t+k} \left(\varepsilon_{t+k}^*(h) - 1\right) D_{t+k}^*(h)}.$$

where the elasticities

$$\begin{aligned} \varepsilon_{t+k}(h) &= -\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} \frac{\overline{P}_t(h)}{D_{t+k}(h)} \\ \varepsilon_{t+k}^*(h) &= -\frac{\partial D_{t+k}^*(h)}{\partial \overline{P^*}_t(h)} \frac{\overline{P^*}_t(h)}{D_{t+k}^*(h)} \end{aligned}$$

summarize how the price set by the producer as of t, will affect the choice of downstream firms that will have a chance to change their prices in the current period and in the future.

Now, it is well understood that, when $\overline{P}_t(h)$ and $\overline{P}_t^*(h)$ are sticky in local currency, exchange rate movements translate into systematic violation of the law of one price. However, comparing the expressions for the optimal prices above, it is apparent that the law of one price is bound to be systematically violated even when the firm has a chance to reset its prices, reflecting differences in the two market elasticities $\varepsilon(h)$ and $\varepsilon^*(h)$. In this respect, it is worth emphasizing that in our economy deviations from the law of one price across markets are not an exclusive implication of nominal rigidities in local currency. They also depend on the way vertical interactions among upstream and downstream monopolists affect optimal pricing by producers, as shown below.

4.2 Demand price elasticities, price variability and strategic interactions

We now characterize the elasticities in (9) for our specification in which the downstream firms' production function is linear in the traded good, as in (4), and discuss its main properties and implications for pricing. The derivative of the producer's demand with respect to its own price is:

$$\begin{aligned}\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} &= \sum_{s=0}^{k} \frac{\partial C_{t+k}(h, r_h)}{\partial P_{t+k-s}(h, r_h)} \frac{\partial P_{t+k-s}(h, r_h)}{\partial \overline{P}_t(h)} \\ &= -\eta\,(1-\theta) \sum_{s=0}^{k} \theta^s \left(\frac{P^o_{t+k-s}(h)}{\mathbb{P}_{\mathrm{H},t+k}} \right)^{-\eta} \frac{C_{\mathrm{H},t+k}}{Z_{t+k}} \frac{1}{P^o_{t+k-s}(h)} \frac{\partial P^o_{t+k-s}(h)}{\partial \overline{P}_t(h)}.\end{aligned}$$

where the partial derivative $\frac{\partial P^o_{t+k}(h)}{\partial \overline{P}_t(h)}$ captures the extent to which current and future optimal pricing decision by firms r_h are affected by the current producer pricing decision — here $P^o_{t+k}(h)$ denotes the optimal price set by the downstream firms which will reoptimize in each period $t+k$, while facing the traded input price $\overline{P}_t(h)$.

In the appendix, we show that this derivative is simply equal to the ratio of the two prices themselves; e.g. at time t we have $\frac{\partial P^o_t(h)}{\partial \overline{P}_t(h)} = \frac{P^o_t(h)}{\overline{P}_t(h)}$. Using this fact, the impact on current and future demand of a price change by the producer can be simplified as follows:[8]

$$\begin{aligned}\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} &= -\eta\,(1-\theta) \frac{C_{\mathrm{H},t+k}}{Z_{t+k}} \frac{\mathbb{P}^\eta_{\mathrm{H},t+k}}{\overline{P}_t(h)} \sum_{s=0}^{k} \theta^s \left(P^o_{t+k-s}(h)\right)^{-\eta} \\ &= -\eta\,(1-\theta) \frac{C_{\mathrm{H},t+k}}{Z_{t+k}} \frac{\mathbb{P}^\eta_{\mathrm{H},t+k}}{\mathbb{P}^\eta_{t+k}(h)} \frac{1}{\overline{P}_t(h)} \sum_{s=0}^{k} \theta^s \left(\frac{P^o_{t+k-s}(h)}{\mathbb{P}_{t+k}(h)} \right)^{-\eta} \\ &= -\eta \frac{C_{\mathrm{H},t+k}}{Z_{t+k}} \frac{\mathbb{P}^\eta_{\mathrm{H},t+k}}{\mathbb{P}^\eta_{t+k}(h)} \frac{1}{\overline{P}_t(h)} \left(S_{t+k}(h) - \theta^{k+1} \frac{\mathbb{P}^\eta_{t+k}(h)}{\mathbb{P}^\eta_{t-1}(h)} S_{t-1}(h) \right).\end{aligned}$$

The sum $\sum_{s=0}^{k} \theta^s \left(\frac{P^o_{t+k-s}(h)}{\mathbb{P}_{t+k}(h)} \right)^{-\eta}$ in the second line of this expression reflects the fact that, when setting the optimal price as of t, upstream monopolists internalize its effects on final demand in each future period between t and $t+k$. Observe that in the last line in the above expession, this sum has been substituted out using the definition of $S_{t+k}(h)$ in the appendix.

Using again the definition of $S_{t+k}(h)$, the price elasticity of demand at each point in time as

[8] We note here that this result is due to the fact that for prices P_{t+k} reset optimally as of $t+k$, $\frac{\partial MC_{t+k+s}(h)}{\partial \overline{P}_t(h)} = \frac{1}{Z_{t+k+s}} = \frac{MC_{t+k+s}(h)}{\overline{P}_t(h)}$, $\forall s > 0$, and

$$\frac{\partial C_{t+k+s}(h, r_h)}{\partial \overline{P}_t(h)} = -\eta \frac{C_{t+k+s}(h, r_h)}{P_{t+k}(h)} \frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)}.$$

See appendix for details.

perceived by the producer, $\varepsilon_{t+k}(h)$ becomes:

$$\begin{aligned}\varepsilon_{t+k}(h) &= -\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} \frac{\overline{P}_t(h)}{D_{t+k}(h)} \qquad (9)\\ &= \eta \frac{C_{\mathrm{H},t+k}}{Z_{t+k}} \frac{\mathbb{P}^{\eta}_{\mathrm{H},t+k}}{\mathbb{P}^{\eta}_{t+k}(h)} \left(S_{t+k}(h) - \theta^{k+1} \frac{\mathbb{P}^{\eta}_{t+k}(h)}{\mathbb{P}^{\eta}_{t-1}(h)} S_{t-1}(h) \right) \frac{1}{D_{t+k}(h)}\\ &= \eta \left(1 - \frac{\theta^{k+1} \left(\frac{\mathbb{P}_{t+k}(h)}{\mathbb{P}_{t-1}(h)} \right)^{\eta} S_{t-1}(h)}{S_{t+k}(h)} \right)\end{aligned}$$

This demand elasticity is a function of the producer price $\overline{P}_t(h)$ only indirectly, through the impact of $P^o_{t+k}(h)$ on the final price level $\mathbb{P}_{t+k}(h)$: absent downstream nominal rigidities ($\theta = 0$), the price elasticity of the producer would be constant and proportional to that perceived by the downstream firm, η — the final price charged would simply be $\frac{\eta}{\eta-1}\frac{\overline{P}_t(h)}{Z_t}$.

In (9), the implications for the demand elasticity of nominal rigidities at the downstream level are captured by the negative term inside the brackets. An important and novel result is that the demand price elasticity perceived by upstream firms under sticky prices is time-varying and, up to first order[9] a decreasing function of the cumulated rate of inflation at the consumer level, $\frac{\mathbb{P}_{t+k}(h)}{\mathbb{P}_{t-1}(h)}$. Namely, with positive inflation, such elasticity will be lower than with flexible prices (in which case it is constant and equal to η). Any change in the consumer prices within a specific h industry — either in response to productivity shocks hitting downstream firms, or in response to price changes by the upstream firms — modifies the elasticity of the demand faced by upstream producers in the same industry. A notable implication is that differences in national inflation rates will induce differences in demand price elasticities for a product, creating an incentive for producers to price-to-market across borders.

To provide an intuitive account of these results, observe that, from the vantage point of an upstream producer of tradables h, the marginal revenue from a price change reflects the fact that only some downstream firms update their prices in any given period. Specifically, the upstream monopolist does not know which individual firm r_h will be updating its price in the period, but knows that a fraction $1-\theta$ of them will do so, while a fraction θ will keep their price unchanged. Because of the latter, the upstream producers will optimally respond to shocks to own marginal costs by charging a price which is higher than she/he would ideally charge if all downstream

[9] We show below that up to first order price dispersion $S_t(h)$ is equal to 1 around a zero inflation steady state.

producers set new prices.

This leads to *strategic substitutability* among producers: namely, a rise in their marginal costs will lead to an increase in the desired markups by producers. Strategic substitutability in our model is important because it implies that producer prices will be more reactive to shocks to their demand conditions and to marginal costs: when vertical interactions among firms with sticky prices are considered, it may not be necessarily the case that several layers of nominal rigidities bring about more inertia in prices.

It is worth stressing that, if monetary policy stabilized consumer prices completely, removing any within-industry price dispersion for each good h, such policy would make the producer's demand elasticity and thus its desired markup constant. To wit: in this case, the producer's demand elasticity would be given by $\eta\left(1-\theta^{k+1}\right)$. Through price stabilization, monetary authorities would therefore eliminate the incentive to price discriminate. However, observe that the elimination of consumer price variability (and consumer price dispersion) would not make the producer's markups independent of downstream price rigidities. The steady-state markup of upstream firms would still be a function of θ, and equal to $\dfrac{\eta}{\eta-\dfrac{1-\theta\beta\overline{\theta}}{1-\theta}}$, implying that the steady state elasticity is lower than η, and equal to

$$\varepsilon_{ss}=\eta\frac{1-\theta}{1-\theta\beta\overline{\theta}}<\eta.$$

4.3 Inflation variability, optimal markups and exchange rate pass-through

To characterize further firms' equilibrium behavior, we log-linearize expression (8) around a zero inflation steady state. Using standard procedures, the optimal price charged by updating upstream producers can be approximated as follows:

$$\widehat{P_t^o}(h)=\left(1-\beta\overline{\theta}\right)\left[\widehat{MC}_t(h)-\frac{\widehat{\varepsilon}_t(h)}{\varepsilon_{ss}-1}\right]+\beta\overline{\theta}E_t\widehat{P_{t+1}^o}(h),$$

where in turn the elasticity as of t is approximately given by

$$\widehat{\varepsilon}_t(h)=-\theta\eta\widehat{\pi}_t\left(h\right),$$

with $\widehat{\pi}_t\left(h\right)$ denoting downstream inflation deviations from steady state in sector h. Since downstream inflation changes depend on the final price set by firms adjusting during the period:

$$\widehat{\pi}_t\left(h\right)=\left(1-\theta\right)\widehat{P_t^o}(h),$$

it is clear that the elasticity $\widehat{\varepsilon}_t(h)$ will be ultimately a decreasing function of the upstream price. Using the difference equation for optimally reset downstream prices,

$$\widehat{P^o_t}(h) = (1-\beta\theta)\,\widehat{MC}_t(h) + \beta\theta E_t\widehat{P^o_{t+1}}(h),$$

together with the fact that $\widehat{MC}_t(h) = \widehat{\overline{P}^o_t}(h) - \widehat{Z}_t$, we can characterize downstream inflation in each industry h as

$$\widehat{\pi}_t\left(h\right) = \left(1-\theta\right)\left(1-\beta\theta\right)\left[\widehat{\overline{P}^o_t}(h) - \widehat{Z}_t\right] + \theta\beta E_t\widehat{\pi}_{t+1}\left(h\right),$$

and thus derive a dynamic expression for the the optimal pricing by upstream firms:

$$\begin{aligned}
\widehat{\overline{P}^o_t}(h) &= \left(1-\beta\overline{\theta}\right)\left(\widehat{\overline{MC}}_t(h) + \frac{\eta\theta}{\varepsilon_{ss}-1}\left[\left(1-\theta\right)\left(1-\beta\theta\right)\left(\widehat{\overline{P}^o_t}(h) - \widehat{Z}_t\right) + \theta\beta E_t\widehat{\pi}_{t+1}\left(h\right)\right]\right) \\
&\quad + \beta\overline{\theta}E_t\widehat{\overline{P}^o_{t+1}}(h) \\
&= \frac{\left(1-\beta\overline{\theta}\right)\left(\widehat{\overline{MC}}_t(h) + \frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\left(1-\beta\overline{\theta}\right)\theta\left[-\left(1-\theta\right)\left(1-\beta\theta\right)\widehat{Z}_t + \theta\beta E_t\widehat{\pi}_{t+1}\left(h\right)\right]\right)}{1-\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\theta\right)\left(1-\beta\overline{\theta}\right)^2} \\
&\quad + \frac{\beta\overline{\theta}E_t\widehat{\overline{P}^o_{t+1}}(h)}{1-\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\theta\right)\left(1-\beta\overline{\theta}\right)^2}
\end{aligned}$$

The term $1-\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\theta\right)\left(1-\beta\overline{\theta}\right)^2$ in the denominator of the above expression is lower than 1.[10] This means that, as already discussed above, the time varying elasticity due to downstream nominal rigidities will transpire into a larger response of the optimal price to changes in marginal costs, relative to the the case in which the upstream price elasticity is constant.

Now, the price charged to foreign downstream firms by domestic upstream producers will be:

$$\begin{aligned}
\widehat{\overline{P^*}^o_t}(h) &= \frac{\left(1-\beta\overline{\theta}\right)\left(\widehat{\overline{MC}}_t(h) - \widehat{\mathcal{E}}_t + \frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\overline{\theta}\right)\left[-\left(1-\beta\theta\right)\left(1-\theta\right)\widehat{Z^*}_t + \theta\beta E_t\widehat{\pi^*}_{t+1}\left(h\right)\right]\right)}{1-\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\theta\right)\left(1-\beta\overline{\theta}\right)^2} \\
&\quad + \frac{\beta\overline{\theta}E_t\widehat{\overline{P^*}^o_{t+1}}(h)}{1-\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\theta\right)\left(1-\beta\overline{\theta}\right)^2}.
\end{aligned}$$

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[10] It is also possible to show that this term will be positive as long as the upstream markup is not too large. A sufficient condition is that

$$\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1} < 4\beta \Longleftrightarrow \eta > \frac{4\beta}{4\beta-1}\frac{1-\theta\beta\overline{\theta}}{1-\theta}.$$

The coefficient multiplying exchange rate deviations $\widehat{\mathcal{E}}_t$ in this expression, which we write for convenience below

$$\frac{1-\beta\overline{\theta}}{1-\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}\theta\left(1-\beta\theta\right)\left(1-\beta\overline{\theta}\right)^2}$$

measures the structural exchange rate pass-through, as defined in e.g. Corsetti, Dedola, and Leduc [2005]. This coefficient highlights the two mechanisms determining how exchange rate movements are passed through into local prices according to our analysis. On the one hand, upstream nominal rigidities ($\overline{\theta} > 0$) tend to lower short-run pass-through irrespective of vertical interactions. But, as we noted above, downstream nominal rigidities ($\theta > 0$) lower the denominator in the above expression below 1 — because of strategic substitutability. Thus, the response of the optimal price to exchange rate changes will be stronger when the elasticity is time varying due to downstream nominal rigidities, relative to the case of a constant elasticity. For instance, if upstream prices were fully flexible — corresponding to $\overline{\theta} = 0$ — the structural pass-through coefficient would be larger than 1 per effect of the vertical interactions with downstream sticky price firms. However, for any given value of θ, a sufficiently large $\overline{\theta}$ will generally reduce ERPT below 100 percent in the short run — unless the upstream steady state markup is unreasonably large.[11]

4.4 Price rigidities versus local costs (the Leontief technology case)

As already mentioned, in previous work (see Corsetti, Dedola, and Leduc [2005]) we have analyzed a different model specification, assuming that the production function of the downstream firm includes a local input, which is a poor substitute for the traded intermediate goods. We showed that the demand price elasticity faced by upstream producers is also market specific in

[11] Precisely, it can be shown that a sufficient condition for the ERPT coefficient to be less than one is that:

$$\overline{\theta} \geq \frac{1}{\beta} - \frac{\varepsilon_{ss}-1}{\varepsilon_{ss}}\left(\sqrt{1+\frac{2}{\beta}\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1}}-1\right).$$

If it is also assumed that the markup is not too large, e.g. $\frac{\varepsilon_{ss}}{\varepsilon_{ss}-1} \leq 2\beta$, which is reasonable for β close to 1, then a sufficient condition for incomplete ERPT in the short run is:

$$\overline{\theta} \geq \frac{2-\sqrt{3}}{\beta} = \frac{0.268}{\beta}.$$

Finally, observe that the ERPT coefficient is clearly a nonmonotonic function of the degree of downstream price rigidity, θ.

this case (independently of nominal rigidities). The properties of this model are however quite different from the ones discussed so far. In the rest of this subsection, we analyze these differences within a single analytical framework. Our main conclusion is that the presence of local inputs (which are weak substitutes for intermediate goods) in downstream production leads to an attenuation of the main effect of price stickiness on upstream producers' optimal markups, without necessarily overturning it.

When the technology of the downstream firms is as in (5), the derivative of the producer demand with respect to its price becomes:

$$\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} = -\eta\,(1-\theta)\sum_{s=0}^{k}\theta^s\left(\frac{P^o_{t+k-s}(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta}\frac{C_{\mathrm{H},t+k}}{P^o_{t+k-s}(h)}\frac{\partial P^o_{t+k-s}(h)}{\partial \overline{P}_t(h)},$$

This is similar to the expression derived for the linear case, except that the right-hand side is not scaled by downstream firms' productivity. After some simplifications (detailed in the appendix), the derivative of the final price to the producer price can be shown to be a constant depending on η. Evaluating this derivative at time t we can write:[12]

$$\frac{\partial P^o_t(h)}{\partial \overline{P}_t(h)} = \frac{\eta}{\eta-1}.$$

Intuitively, the effect of an increase in the upstream producer price (and thus of the marginal cost) on the price optimally charged by downstream firms in the same period will be proportional to the markup charged by the the latter, $\frac{\eta}{\eta-1}$ — a clear instance of double marginalization.

[12]The formula in the text can be derived using the same reasoning as in the appendix, using with the fact that $\frac{\partial MC_{t+k}(h)}{\partial \overline{P}_t(h)} = 1$, and

$$\begin{aligned}\frac{\partial C_{t+k}(h,r_h)}{\partial \overline{P}_t(h)} &= -\eta\left(\frac{P_t(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta} C_{\mathrm{H},t+k}\frac{1}{P_t(h)}\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} \\ &= -\eta\frac{C_{t+k}(h,r_h)}{P_t(h)}\frac{\partial P_t(h)}{\partial \overline{P}_t(h)}.\end{aligned}$$

The derivative of current and future demands with respect to the wholesale price becomes:[13]

$$\begin{aligned}\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} &= -\frac{\eta^2}{\eta-1}(1-\theta)\sum_{s=0}^{k}\theta^s\left(\frac{P^o_{t+k-s}(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta}\frac{C_{\mathrm{H},t+k}}{P^o_{t+k-s}(h)} \\ &= -\frac{\eta^2}{\eta-1}C_{\mathrm{H},t+k}\frac{\mathbb{P}^{\eta}_{\mathrm{H},t+k}}{\mathbb{P}^{\eta+1}_{t+k}(h)}(1-\theta)\sum_{s=0}^{k}\theta^s\left(\frac{P^o_{t+k-s}(h)}{\mathbb{P}_{t+k}(h)}\right)^{-\eta-1} \\ &= -\frac{\eta^2}{\eta-1}C_{\mathrm{H},t+k}\frac{\mathbb{P}^{\eta}_{\mathrm{H},t+k}}{\mathbb{P}^{\eta+1}_{t+k}(h)}\left[\widetilde{S}_{t+k}(h)-\theta^{k+1}\left(\frac{\mathbb{P}_{t+k}(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta+1}\widetilde{S}_{t-1}(h)\right],\end{aligned}$$

where we have defined

$$\widetilde{S}_{t+k}(h)\equiv\int\left(\frac{P^o_{t+k}(h,r_h)}{\mathbb{P}_{t+k}(h)}\right)^{-\eta-1}dr=(1-\theta)\mathbb{P}^{\eta+1}_{t+k}(h)\sum_{j=0}^{\infty}\theta^j P^o_{t+k-j}(h)^{-\eta-1}.$$

. The price elasticity of demand at each point in time as perceived by the upstream producer is then given by:

$$\varepsilon_{t+k}(h)=\frac{\eta^2}{\eta-1}\frac{\overline{P}_t(h)}{\mathbb{P}_{t+k}(h)}\left[\widetilde{S}_{t+k}(h)-\theta^{k+1}\left(\frac{\mathbb{P}_{t+k}(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta+1}\widetilde{S}_{t-1}(h)\right], \tag{10}$$

where again, as shown in the appendix, it is true that

$$\widetilde{S}_t(h)=(1-\theta)\left(\frac{P_t(h)}{\mathbb{P}_t(h)}\right)^{-\eta-1}+\theta\pi_t^{\eta+1}(h)\widetilde{S}_{t-1}(h).$$

In contrast to the linear production case, the demand elasticity is now a function of the wholesale price $\overline{P}_t(h)$ not only indirectly, through the final price index $\mathbb{P}_{t+k}(h)$, but also directly. Specifically, this elasticity reflects three effects.

The first arises from the double marginalization due to the presence of two vertically integrated monopolists and is captured by the term $\frac{\eta}{\eta-1}$ in (10): absent nominal rigidities ($\theta=0$) and the nontraded input among downstream firms, the price elasticity of the producer would be constant and equal to that perceived by these firms, η — the price charged to consumers by all firms would simply be $\frac{\eta}{\eta-1}\overline{P}_t(h)$.

The second effect, arising from nominal rigidities, is captured by the term in brackets in (10) and has been discussed extensively already in the previous subsection — it links the demand

[13]The following is due to the fact that for retail prices P_{t+k} reset as of $t+k$, it will still be true that $\frac{\partial MC_{t+k+s}(h)}{\partial \overline{P}_t(h)}=1$ and

$$\frac{\partial C_{t+k+s}(h,r_h)}{\partial \overline{P}_t(h)}=-\eta\frac{C_{t+k+s}(h,r_h)}{P_{t+k}(h)}\frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)}.$$

See also the appendix.

elasticity to downstream price inflation and price dispersion among final producers. Its presence tends to make the demand elasticity an *increasing* function of $\overline{P}_t(h)$.

The third and last effect arises from the assumption that downstream firms combine the traded and labor inputs in fixed proportion and is captured by the term $\dfrac{\overline{P}_t(h)}{\mathbb{P}_{t+k}(h)}$: absent downstream nominal rigidities this ratio would be equal to $\dfrac{\overline{P}_t(h)}{\frac{\eta}{\eta-1}\left(\overline{P}_t(h)+\frac{W_t}{Z_t}\right)}$, as in Corsetti and Dedola [2005]. However, in contrast to our results above, this last effect tends to make the demand elasticity *decreasing* in $\overline{P}_t(h)$.

Summing up: our analysis above suggest that price stickiness and local inputs which are complement to intermediate tradables in final good production affect producers' markups in different ways: the former makes the producers' demand elasticity decreasing, the latter increasing, in the producer price. Under what conditions would one effect prevail over the other?

Taking a log-linear approximation to the upstream price and the elasticity, we find as before that

$$\widehat{\overline{P}_t^o}(h) = \left(1-\beta\overline{\theta}\right)\left[\widehat{\overline{MC}}_t(h) - \frac{\widehat{\varepsilon}_t(h)}{\varepsilon_{ss}-1}\right] + \beta\overline{\theta}E_t\widehat{\overline{P}_{t+1}^o}(h),$$

where the elasticity as of t is now given by:

$$\widehat{\varepsilon}_t(h) = \widehat{\overline{P}_t^o}(h) - (1+\theta\eta)\,\widehat{P_t^o}(h).$$

and

$$\varepsilon_{ss} = \eta\delta\frac{1-\theta}{1-\theta\beta\overline{\theta}}.$$

Relative to our previous analysis, this steady state elasticity depends on δ, which is defined as the steady state share of the upstream product in the downstream firms' costs, with $0 < 1-\delta < 1$. Since downstream marginal cost can be approximated as

$$\widehat{MC}_t(h) = \delta\widehat{\overline{P}_t^o}(h) + (1-\delta)\left(\widehat{W}_t - \widehat{Z}_t\right),$$

the expression for the optimal upstream price becomes:

$$\begin{aligned}
\widehat{\overline{P}_t^o}(h) &= \left(1-\beta\overline{\theta}\right)\left[\widehat{\overline{MC}}_t(h) - \frac{\widehat{\varepsilon}_t(h)}{\varepsilon_{ss}-1}\right] + \beta\overline{\theta}E_t\widehat{\overline{P}_{t+1}^o}(h) \\
&= \left(1-\beta\overline{\theta}\right)\left[\widehat{\overline{MC}}_t(h) - \frac{\widehat{\overline{P}_t^o}(h) - (1+\theta\eta)\left((1-\beta\theta)\left(\delta\widehat{\overline{P}_t^o}(h) + (1-\delta)\left(\widehat{W}_t - \widehat{Z}_t\right)\right) + \beta\theta E_t\widehat{P_{t+1}^o}(h)\right)}{\varepsilon_{ss}-1}\right] \\
&\quad + \beta\overline{\theta}E_t\widehat{\overline{P}_{t+1}^o}(h),
\end{aligned}$$

or

$$\widehat{\overline{P}^{o}_{t}}(h) = \frac{\left(1-\beta\overline{\theta}\right)\left[\widehat{\overline{MC}}_{t}(h) - \frac{(1+\theta\eta)\left((1-\beta\theta)(1-\delta)\left(\widehat{W}_{t}-\widehat{Z}_{t}\right)+\beta\theta E_{t}\widehat{P^{o}_{t+1}}(h)\right)}{\varepsilon_{ss}-1}\right]}{1+\frac{1-\delta(1+\theta\eta)\left(1-\beta\overline{\theta}\right)(1-\beta\theta)}{\varepsilon_{ss}-1}} + \frac{\beta\overline{\theta}E_{t}\widehat{\overline{P}^{o}_{t+1}}(h)}{1+\frac{1-\delta(1+\theta\eta)\left(1-\beta\overline{\theta}\right)(1-\beta\theta)}{\varepsilon_{ss}-1}}.$$

The denominator of the coefficient multiplying marginal costs can now have either sign, i.e. the time varying elasticity can either magnify or mute the response of the optimal upstream price to marginal costs. This means that we can have either strategic substitutability (the denominator is negative, as was in the previous subsection) or strategic complementarity (positive). A sufficient condition for strategic complementarity is:

$$\delta < \frac{1}{1+\theta\eta}.$$

In other words, the share of local inputs in downstream firms should be sufficiently high. Observe that the above inequality is more likely to hold when η is low (markups are high), or θ is low, so that downstream prices are not too sticky.

5 Calibration

This section describes the benchmark calibration for our numerical experiments, which we assume symmetric across countries. We used dynare++ to solve for the optimal monetary policy and to simulate our different economies. In each exercise, we report statistics averaged over 500 simulations of 100 periods each.

5.1 Preferences and production

We posit that the period-by-period utility function has the form already shown by (1), that we reproduce here for convenience:

$$U\left[C_{t}, \frac{M_{t+1}}{P_{t}}, L_{t}\right] = \frac{C_{t}^{1-\sigma}}{1-\sigma} + \chi\frac{\left(\frac{M_{t+1}}{P_{t}}\right)^{1-\sigma}}{1-\sigma} + \kappa\frac{(1-L_{t})^{1-\upsilon}}{1-\upsilon}, \tag{11}$$

We set κ so that in steady state, one third of the time endowment is spent working. In our benchmark calibration, we assume υ equal to σ (risk aversion), which we in turn set to 2. Since the utility function is separable in consumption and real money balances, money demand is determined residually and does not play any role in our results. We therefore set χ arbitrarily to 0.1.

We set the constant elasticity of substitution across brands, η, so that the markup of downstream firms in steady state is 15 percent. Following Backus, Kehoe, and Kydland [1992], we chose ϕ so that the trade elasticity is 1.5. As regards the weights of domestic and foreign tradables in the consumption basket, $a_{\rm H}$ and $a_{\rm F}$ (normalized $a_{\rm H} + a_{\rm F} = 1$) are set such that imports are 10 percent of aggregate output in steady state, roughly in line with the average ratio for the U.S. in the last 30 years. We pick the steady state value of Z to ensure that the price of traded goods accounts for 50 percent of the final price in steady state. This value corresponds to the empirical estimates by Burstein, Neves and Rebelo [2003], for the distribution margin only. In our specification, downstream firms can do more than distributing goods to final users, suggesting that the value we select is on the conservative side.

As benchmark, we set the probability that upstream and downstream firms update their prices to 0.5. This overall frequency of price adjustment is in line with the evidence in Bils and Klenow [2004] and Nakamura and Steinsson [2007], if sales are treated as price changes.

5.2 Productivity shocks

Let the vector $\mathbf{Z} \equiv \{Z, \overline{Z}, Z^*, \overline{Z^*}\}$ represent the sectoral technology shocks in the domestic and foreign economies. We assume that sectoral disturbances to technology follow a trend-stationary AR(1) process

$$\mathbf{Z}^{'} = \boldsymbol{\lambda}\mathbf{Z} + \mathbf{u}, \tag{12}$$

whereas $\mathbf{u}$ has variance-covariance matrix $V(\mathbf{u})$, and $\boldsymbol{\lambda}$ is a $4x4$ matrix of coefficients describing the autocorrelation properties of the shocks, that are the same for both sectoral shocks. Since we assume a symmetric economic structure across countries, we also impose symmetry on the autocorrelation and variance-covariance matrices of the above process. Because of lack of sectoral data on productivity, we posit that sectoral shocks follow a rather conventional process. First, in line with most of the international business cycle literature — e.g., BKK — we assume that these shocks are very persistent, and set their autocorrelation to 0.95. Second, the standard deviation of the innovations is set to 0.007. For simplicity, we set the shock correlation and the spillovers across countries and sectors to zero.

5.3 Monetary policy

To characterize the optimal monetary policy, we let the planner choose the growth rates of money in the Home and Foreign economies, to maximize the world welfare subject to the first-order conditions for households and firms and the economy-wide resource constraints. We assume that the planner places equal weights on Home and Foreign welfare, so that world welfare is given by the following expression:

$$Welfare = \frac{V_0 + V_0^*}{2},$$

where V_0 and V_0^* do not take into account utility accruing from real balances in (11). We follow an approach similar to that in Khan, King, and Wolman [2003] and consider an optimal policy that has been in place for a long enough time that initial conditions do not matter. When solving our economies, we assume the presence of fiscal subsidies, financed via lump-sum taxation, to ensure that all prices would equal marginal costs if prices were fully flexible.

In describing our results, we also compare the optimal policy to other well-known policy rules. We first consider a Taylor-type rule that sets the short-term nominal interest rate as a function of the deviations of CPI inflation and real GDP from steady state values:

$$R_t = \rho R_{t-1} + \chi(1-\rho)E(\pi_t - \pi^{ss}) + \gamma(1-\rho)(y_t - y^{ss}). \tag{13}$$

We conventionally parameterize the policy rule using the estimates in Lubik and Schorfheide [2004]: $\rho = 0.84$, $\chi = 2.19$, $\gamma = 0.3$. We also consider inflation targeting rules in which the central bank stabilizes either the inflation rate at the final or intermediate level, which we label CPI and GDP inflation targeting, respectively.[14]

[14] Our price index of Home intermediates is a CES function of the price of Home intermediates in the Home market and the price of Home intermediates sold abroad (expressed in Home currency). We set the weights over those prices to be the same as in the CPI.

6 The response of producers and consumer prices to shocks

In this section, we use our quantitative framework to discuss key properties of our model regarding the behavior of prices and markups in response to productivity shocks. Figures 1 and 2 show the impulse responses of prices, markups and inflation — all in percentage deviations from their steady state values — to productivity shocks, distinguishing between the intermediate and final production sectors. Throughout these exercises we assume that central banks in the two countries set monetary policy to implement a strict CPI inflation targeting. Similar results can however be obtained assuming that central banks implement the optimal policy, discussed in the next section.

6.1 Technology shocks to upstream firms

Figure 1 focuses on the effects of an unexpected and persistent productivity increase in the Home tradable goods sector. Consistent with strict inflation targeting, the monetary authorities react to the shock by expanding the country's monetary stance in line with productivity, causing a depreciation of the nominal exchange rate — given CPI inflation targeting, the nominal and real exchange rate move together (see the graph in the lower right corner of Figure 1).

As shown by Figure 1, upstream producers that update their prices lower them both in the domestic and the foreign market (see the first chart on the upper left corner of the figure). The fall in the home good price is however larger abroad than in the domestic market, in violation of the law of one price. In the graph, a positive deviation from the law of one price means that domestic prices are larger than foreign prices.

The behavior of prices is mirrored by the response of the desired and actual markups of the upstream Home producers, shown by the fourth and fifth graphs of Figure 1. As discussed in Section 4, downstream nominal rigidities lead to pricing substitutability at the level of upstream producers. As a result, the *desired markup* by these producers fall with their prices in either market, but relatively more in the Foreign one.[15] Nonetheless, since prices are sticky in local currency, the nominal depreciation of the Home exchange rate raises export revenues in the exporters' own currency: the *average markup* in the country actually rises.

[15] Recall that we also show that this result is attenuated when the production function include local labor input, with a low degree of substitutability with the intermediate tradable goods.

The impact of the same shock on *consumer prices* clearly differs, depending on whether the upstream monopolist in a particular industry h updates its price, or leaves it unchanged. In the former case (shown in the second graph of Figure 1), downstream firms face a drop in their marginal costs. Hence, those firms which can reset prices will lower them, both domestically and abroad. As we have seen previously, the intermediate good price falls more in the Foreign country. Thus, Foreign downstream firms decrease their price by more than the domestic ones, so that deviations from the law of one price have the same sign at both consumer prices' and producer prices' level.

Interestingly, our results show that consumer prices fall also in industries in which the upstream producers *do not* update their prices during the period — albeit by a smaller amount than in the other case (see the third graph of Figure 1). This is so for two reasons. First, although marginal costs of downstream firms in these industry do not fall in the period, these firms nonetheless take into account that the productivity shock is persistent: they thus anticipate that their marginal costs are likely to decrease in the future. Second, a lower price helps these firms respond to increasing competition by firms operating in the other industries, where the price of intermediate product have already gone down.

In these industries, the deviations from the law of one price are larger, but of the opposite sign, relative to the industries in which the upstream price is updated. This is because, for a constant upstream price, consumer prices decrease on impact by more at Home than abroad. To wit: in the first period, the sign of the deviations from the law of one price is positive in the second graph, negative in the third graph of Figure 1.

6.2 Technology shocks to downstream firms

Figure 2 displays the responses to an unexpected persistent increase in the productivity of Home downstream firms. As in the previous case, under the assumed strict CPI inflation targeting the Home monetary authorities react with an expansion, which leads to nominal and real depreciation of the Home currency (see last graph of Figure 2).

Recall that downstream technology shocks are also country-specific: they lower the marginal costs of downstream firms at Home, but do not affect the costs of downstream firms in the industry located in the Foreign country. So, in all industries in which the upstream producers do not update their current price within the period, domestic downstream firms updating their prices will optimally lower them, while downstream firms abroad will keep their prices virtually unchanged. This is at the root of the deviations from the law of one price shown in the third graph of Figure 2, which are further magnified by the fact that monetary authorities react to the shock by engineering Home currency depreciation.

More complex is the case of industries in which upstream producers change their prices (second graph in the Figure), since the overall effects of the shock will depend on a number of general equilibrium effects. Key to understanding these effects is the fact that higher productivity by downstream firms causes an increase in their output, and thus in real domestic consumption. In our model specification, the increase in downstream output does not affect the labor market and thus the real wage directly — under the linear production function specified above, a higher downstream output has no direct impact on the demand for labor, since these firms are assumed not to employ any labor input. However, it does so indirectly: higher domestic consumption is associated with a positive income effect, which reduces labor supply and ultimately translates into a downward shift in hours worked. Given that at the same time the demand for intermediate products is increasing, the labor market tightens, causing a rise in real wages. Facing higher labor costs, upstream firms which can reoptimize their prices raise them, thus increasing the marginal costs of downstream firms. Somewhat surprisingly, as shown in the second graph of Figure 2, the feedback effect on consumer prices is positive.

This transmission mechanism was discussed early on by Friedman, in his celebrated critical analysis of cost-push inflation (see e.g. Nelson [2007] and references within). In the industries where upstream producers adjust their prices, they raise them in response to higher costs in the form of higher nominal wages. Yet, one key factor raising wages is the demand expansion engineered by monetary policy makers in response to productivity improvement at retail level. Changes in prices which appear to be motivated by costs consideration, are actually the result of a demand stimulus, working its way up through the vertical links between downstream and upstream producers, and ultimately raising the price of scarse production inputs supplied in competitive markets.

Observe that domestic upstream producers slightly lower their wholesale prices in foreign currency. Nonetheless, because of currency depreciation, these prices in Home currency are higher than the ones charged in the domestic market, again in violation of the law of one price. Consistently, the desired markup of Home producers increases in the Home market, while falls abroad — in line with the change in prices. The average markup nonetheless falls everywhere in the economy, per effect of nominal rigidities.

7 International dimensions of optimal stabilization policy

This section is devoted to the analysis of stabilization policies under the assumption of cooperation between the Home and Foreign monetary authorities and full commitment. In order to shed light on how policy works in our model, we find it useful to discuss the problem of stabilizing economies hit by shocks to upstream or downstream shocks in isolation, and then proceed to present results for our complete baseline calibration. Thus, results are shown in 3 tables. For a set of macrovariables, Table 1 and 2 report volatilities conditional on shocks to upstream and downstream productivity, respectively; Table 3 report results when both shocks are considered. In each table, the first column shows result for the flexible price benchmark, in which monetary policy targets a zero rate of CPI inflation at all times; the other columns refer to economies with price rigidities under different policy regimes. Tables 1 and 2 only show results under the optimal cooperative policy, including a case in which there is no home bias in consumption expenditure (i.e. $a_{\mathrm{H}} = a_{\mathrm{F}} = 1/2$). Table 3, instead, includes the alternative monetary policies specified in Section 5 — CPI inflation targeting, GDP inflation targeting, and the Taylor-type rule. As we assume subsidies that exactly offset steady-state markups, under the optimal policy long-run inflation is zero. To facilitate comparison across experiments, we also posit that steady state inflation is nil when solving the model under the alternative policies.

7.1 Upstream shocks only

Starting from the simplest case, consider first the problem of stabilizing technology shocks to upstream production only. As an important benchmark, we first establish that, if our Home country were a closed economy, monetary authorities would be able to stabilize completely upstream marginal costs, and therefore upstream prices, preventing any dispersion in the prices charged by adjusting and not-adjusting firms. Monetary authorities can do so by matching any change in upstream marginal costs driven by productivity, with a change in the monetary stance in the opposite direction, which ultimately moves nominal wages in tandem with productivity. The specific reason why such a policy would stabilize all sticky prices (at both producer and consumer level) is that, in our specification, fluctuations in nominal wages are not consequential for downstream firms, by virtue of our assumption that these firms employ no labor resources in producing final goods. So, downstream marginal costs only change with the intermediate goods' prices, or with downstream productivity: without shocks to the latter, once upstream prices are constant in equilibrium, so are downstream prices. Similarly to the standard closed-economy monetary model, the policy just described would replicate the allocation under flexible prices — this policy is optimal in our environment since we assume that steady-state monopolistic distortions in production are corrected with fiscal instruments.[16]

The optimality of complete price stabilization, however, does not carry over to an open economy setting, as shown in Table 1. With an optimal monetary policy in place (second column of the table), the variability of the CPI is close to, but not zero — domestic and imported goods prices are actually much more variable than the CPI. Observe that prices and markups in both countries fluctuate much less for domestic goods than for imported goods. This corresponds to the fact that monetary policymakers concentrate their efforts to reduce the volatility of markups of domestic producers selling in the domestic markets. The reason has already been laid out in Section 2.1, but is worth reconsidering here in the framework of our model with staggered price setting.

By mirroring the logic of Corsetti and Pesenti [2005], assume an equilibrium where there is no price dispersion in either domestic market for domestically produced goods: the monetary authority completely stabilizes the marginal costs of *upstream* firms, once again matching any increase in productivity with an appropriate expansion in the Home monetary stance. While domestic goods prices remain constant and identical to each other, at both intermediate and final

[16]It is worth stressing that, had we assumed that downstream firms use labor, complete stability of upstream prices would be incompatible with complete stability of downstream prices. Even if prices of intermediate goods were held constant, movements in nominal wages in response to endogenous monetary policy changes would additionally affect marginal costs of final good producers, creating an immediate incentive for these to reset prices when possible, thus generating price variability at the retail level. As a result, relative to our baseline model specification, introducing a labor input in final good production implies that welfare maximizing monetary authorities would face a trade-off between stabilization of upstream marginal costs, and downstream price dispersion. They would therefore tend to react by less to upstream productivity shocks, with the objective of containing price dispersion at consumer level. As should become clear in the rest of the text, abstracting from labor inputs in downstream production is helpful in focusing most sharply on the policy trade-offs arising specifically from vertical interactions between downstream and upstream firms.

level, any monetary decision affecting the nominal exchange rate would create both a misalignment of the relative price between domestic and foreign goods, and import price dispersion at consumer level (since a fraction $\overline{\theta}$ of Foreign producers would react to, e.g., Home depreciation, by inefficiently raising the price they charge to Home downstream firms).

At the margin, depending on the degree of openness of the economy, it would be optimal to move away from such outcome. Specifically, it is efficient to stabilize the marginal costs of domestic intermediate producers by less, as to stabilize somewhat marginal costs and thus reduce the incentive to move prices in the import sector. Monetary authorities can raise welfare by trading-off lower import price dispersion and misalignment at the consumer level, against some price dispersion in the Home markets for domestic goods.

We observe here that optimal Foreign monetary policy would mimic Home monetary policy in response to Home shocks, for essentially the same reason. For a given Foreign monetary policy, a Home currency depreciation generates price variability in local currency of Foreign imports from Home, as Home exporters updating their price will lower them. An expansion allows the Foreign monetary authorities to contain the variability of import prices, as well as their misalignment relative to the domestic goods prices, at the cost of some price dispersion in the domestic market for domestic goods. This is exactly what underlies our numerical results in Table 1.

As is well understood in the literature, with LCP, endogenous changes in monetary stance across countries tend to be positively correlated. In the limiting case in which there is no home bias in consumption (the case reported in the third column in Table 1), domestic and foreign goods in the Home and the Foreign consumer price indexes have exactly the same weights. This implies that, in response to disturbances to upstream productivity, national monetary policy stances react to the same weighted average of shocks, becoming perfectly correlated in the optimum. As a result, the nominal exchange rate does not respond to shocks (in the third column of Table 1, the volatility of the real exchange rate is 0), even if shocks are country-specific and uncorrelated — a finding discussed at length by the literature surveyed in Section 2. What induces optimal exchange rate variability under cooperation is home bias in consumption, which obviously raises the importance of stabilizing the marginal costs of domestic producers relative to those of the importers (the case shown in the second column in Table 1). In this respect, our results generalize the point discussed by Corsetti [2006] to an environment with staggered price adjustment.

By comparing the first and the second column of Table 1, it is apparent that the positive co-movements in optimal national monetary policies induced by LCP distortions curb the *volatility of the terms of trade*, relative to the case of flexible prices. With LCP, nominal exchange rate movements do not help correct international relative prices. The only way in which a nominal expansion cum exchange rate depreciation can reduce the price of domestic goods sold abroad is via price adjustment in foreign currency, but by the Calvo mechanism only a subset of firms can reduce their prices. For all the other firms, the terms of trade actually move in the direction of an appreciation. In other words, while exchange rate movements induced by a monetary expansion have nothing to do with expenditure switching effects, a fall in the relative price of imports can only materialize at the cost of import price dispersion. This is why, depending on the relative weight of domestic and imported goods in the CPI, optimal stabilization policy tends to contain international relative prices and thus terms of trade variability.

However, observe that in our results the volatility of the real exchange rate, like that of consumption and hours worked, is higher with nominal rigidities (under the optimal policy), than with flexible prices — the opposite of our results on the terms of trade. We will return on this important point below.

7.2 Downstream shocks only

Shocks hitting final good producers substantially modify the monetary policy problem, in at least two respects. First, in our baseline specification without labor input in downstream production, monetary authorities would never be able to achieve complete stability of final prices, not even in a closed-economy environment. In other words, these shocks create policy trade-offs among competing objective independently of openness. The problem is that complete price stability at consumer level requires monetary policy to respond to technology shocks downstream. Since the resulting fluctuations in wages (see Section 6.2) induce (inefficient) price dispersion among upstream firms, it follows that final producers will face different costs of their intermediate input, depending on which industry they operate in. In this sense, vertical interactions in our model bring about an important dimension of heterogeneity across firms which should be appropriately emphasized. Differently from standard sticky price models, the marginal costs of our downstream firms are generally not symmetric, not even when the economy is completely closed to foreign trade, and there are no markup shocks, due e.g. to stochastic preferences.

Second, since final producers differentiate locally the products they bring to consumers, downstream shocks add an important element of nontradability to consumer goods. Hence, even when consumer expenditure is not biased towards domestic goods, consumption baskets would still be effectively different across countries. When the expenditure weights a_{H} and a_{F} are identical — a case of no home bias in terms of upstream products — monetary authorities would efficiently provide the same degree of stabilization across all categories of domestic and imported goods. Yet, in contrast to the case of upstream disturbances only, the optimal monetary stance will be sufficiently different across countries as to induce nominal and real exchange rate fluctuations in response to country-specific shocks at downstream level. This result is a generalization of Duarte and Obstfeld [2007], who also stress nontradability as a reason for nominal exchange rate flexibility. However, they include nontradables as a separate sector in the economy (as they abstract from vertical interactions), and focus on the case of one period preset prices (hence abstract from forward looking price setting).

The above discussion is clearly reflected in the results in Table 2. When we focus on downstream shocks only, the variability of CPI inflation is not zero, and remains remarkably stable for different degrees of home bias in consumption. What instead varies considerably with the degree of home bias is the variability of markups across sectors, since home bias shifts the weight of monetary stabilization away from imported goods. Precisely, observe that in the third column — the case of no home bias — markups are equally stabilized at the retail level, for both domestic and foreign goods. In the second column, instead, the markup of final producers is much less volatile if they sell domestic goods, than if they sell imported goods.

Relative to the case of upstream shocks, there are two notable differences as regards exchange rate volatility. First, because of nontradability, the real exchange rate is now much more volatile than the terms of trade, even in the flexible price allocation. Second, relative to the flexible price allocation, an economy with nominal rigidities and the optimal policy in place will be characterized by more volatility in both the real exchange rate and the terms of trade. The fact that these patterns are quite different from those discussed in the previous subsection makes it clear that optimal monetary policies do not translate into any general prescription about the relative volatility of these international prices.

7.3 Baseline economy

We now have all the basic elements to analyze our baseline economy with all shocks combined. Results are shown in Table 3. Observe that the combination of downstream and upstream shocks raises the volatility in our artificial economy reasonably close to the data for the US and other large industrial economies: for instance, the standard deviations of real GDP is (realistically) around 2 percent, regardless of nominal rigidities.

Consider first the flexible-price benchmark, shown in the first column of the table. With flexible prices, the demand elasticity facing producers, and thus the markups they charge, are constant; therefore the law of one price holds at the dock (the volatility of deviations from the law of one price at the dock is correspondingly zero). Nonetheless, the law of one price (cannot and) does not hold for final goods: country-specific productivity shocks hitting the downstream firms drive a wedge between final goods' prices across countries (expressed in a common currency). As a result, and in accord to stylized facts, the real exchange rate is more volatile than the terms of trade; the correlation between the real (and nominal) exchange rate is high and positive — despite the fact that upstream and downstream technology shocks are assumed not to be correlated. Recall that, in our flexible price economy, we posit that monetary policy keeps the CPI constant: consistent with this monetary regime, sectoral inflation rates are more volatile at producer level than at the final level, and for imported goods than for domestically produced goods, respectively. The latter result clearly reflects the low weight of foreign goods in the CPI.

The second column of Table 3 displays results for our sticky-price economy with the optimal policy in place. In order to reduce inefficiencies due to price stickiness, monetary policy mitigates fluctuations in the major components of consumer price inflation. However, it falls short of completely stabilizing either the CPI or domestic intermediate prices inflation. Key to understanding this result are the different trade-offs discussed in the text above. First, as in Corsetti and Pesenti [2005], LCP at upstream level leads benevolent monetary authorities to attach a positive weight to stabilizing the consumer price of imports, and thus to deviate from perfect stabilization of the final prices of domestic goods. Second, downstream technology shocks prevent perfect stabilization of all consumer prices, because of the heterogeneity of marginal costs implied by vertical interactions. This effect is of course worse in an open economy setting, because of the response of the intermediate price of imports to exchange rate fluctuations. Third, the elasticity of the producer's demand curve depends on the industry's dispersion of final goods prices, motivating policy emphasis on final price stabilization. The implications of these trade-offs for the volatilities of prices and markups, real exchange rates and terms of trade are discussed below, together with a comparative analysis of the optimal policy relative to other policy rules.

Prices and markups Because of limited price adjustment, it is not surprising that real variables generally display more volatility in the sticky-price economy (with the optimal monetary policy in place), than in the flexible-price economy.[17] Notable exceptions are the terms of trade and hours worked. The reduced volatility of the terms of trade is a consequence of LCP at the intermediate level, as discussed in Section 7.1 above. A reduced volatility of hours worked is already a feature of optimal monetary policy with downstream shocks only in Table 2, and is essentially a consequences of our assumption that downstream firms do not employ labor.

What is most interesting, instead, is the very large discrepancy in volatilities of producers' and distributors' average markups, which are constant in the flexible-price allocation. The markup of domestically produced goods is two and a half times as volatile at the upstream level as at the downstream level. This is remarkable in light of the fact that, in our experiments, we assume the same degree of nominal rigidities at either level. The volatility differential reflects the real components of markup movements in producers' prices, arising from vertical interactions. Conversely, the markup of imported goods is more volatile at the downstream level than at the upstream level — almost twice as much. Such differential reflects the fact that optimal policy attaches a large weight to stabilizing domestically produced goods at the retail level — the bulk of households' consumption.

We should stress here that fluctuations in markups translate into inefficient deviations in the law of one price, both at the border and at the consumer level. Observe that the volatility of deviations from the law of one price in final prices is quite similar to the one in the economy with

[17]The volatility differential between our economies with and without nominal rigidities is by no means uniform across sectors. Namely, for domestically produced goods, the ratio in volatility of upstream and downstream prices is 4 in the flexible price allocation, but it falls down to 2.5 with nominal rigidities. A similar drop can be found in the ratio of volatility of imports prices to domestic goods prices. Conversely, the volatility of the producer price of imported goods, though lower than in the flexible price economy, is now twice that of domestically produced goods.

flexible prices, notwithstanding that, per effect of the exchange rate movements, the markups of Home downstream firms selling imported goods have the highest volatility.

Real exchange rates and terms of trade A notable international dimension of the optimal policy in Table 3 is that the real exchange rate is more volatile in the economy with nominal frictions than under flexible prices, while the terms of trade is less volatile, reflecting the effects of LCP and nontradability discussed above. These findings clearly caution against suggestions to drastically curb the volatility of nominal and real exchange rates. For instance, they caution against the strong policy prescription derived by by Devereux and Engel [2007], who argue that under pervasive LCP the optimal stabilization policy should reduce the variability of the real exchange rate significantly below that of the terms of trade. In these authors' view, the fact that we observe the opposite pattern in the data suggests that policymakers around the world fail to stabilize currency movements efficiently. As we argued above, the problem with this and similar views is that, while there are good theoretical reasons to expect optimal policies under LCP to contain the volatility of the terms of trade, these reasons cannot be mechanically applied to the real exchange rate, whose volatility is bound to depend on a number of structural features of the economy.[18]

Simple rules The last three columns of Table 3 report results for alternative policy rules, namely, CPI inflation targeting, GDP inflation targeting and a standard Taylor rule. Compared to the optimal policy, these alternative simple rules bring about noticeably larger volatility in most real variables, particularly in the markups and the deviations from the law of one price for both consumer prices and prices at the dock.

Focus first on the strict CPI inflation targeting regime, presented in the third column of Table 3: such monetary policy regime leads to more volatility in the upstream prices of all goods (imported and domestically produced). Relative to the optimal policy, the economy displays higher volatility of markups, terms of trade and the real exchange rate. This is so because complete stabilization of headline consumer price inflation brings about suboptimally

[18] As discussed above, in our model the optimal ranking of volatility between the real exchange rate and the terms of trade depends, among other things, on the relative degree of price stickiness among upstream and downstream producers. If producer prices are assumed to be completely flexible, the real exchange rate becomes less volatile than the terms of trade — in line with the case discussed by Devereux and Engel [2007].

large movements in sectoral (i.e. domestic and imported goods) inflation rates at retail level, which ultimately affect the desired markups by upstream producers.

Likewise, stabilizing the prices of domestically produced goods — the case dubbed GDP inflation targeting in the fourth column of Table 3 — also leads to too much volatility in sectoral inflation rates, especially in inflation of imported goods at the border as well as at the consumer level. Interestingly, consumption and real GDP are less volatile than under the optimal policy; but this is achieved by generating more volatility in all other real variables, especially in hours worked and in the terms of trade, because of the suboptimally low weight attached to stabilizing export and import goods prices.

Finally, a Taylor rule (following a quite standard parameterization) improves on the strict CPI inflation target, by producing less volatility in consumption and hours. However, relative to the optimal policy, both the CPI and its individual components are too volatile, since too much importance is attached to output stabilization. As a result, the volatility of consumption is excessive, that of hours is too low.

8 Concluding remarks

The literature in international economics and open macro has so far pursued two distinct explanations of the observed stability of import prices in local currency. According to one modeling strategy, this is the result of optimal markup adjustment by monopolistic firms, which optimize profits through price discrimination across national markets. In this case, market segmentation is attributed to real factors. According to an alternative modeling strategy, local-currency price-stability reflects nominal rigidities, which imply suboptimal variations in firms profits in response to shocks. By considering vertically integrated firms, our paper emphasizes that a rigid distinction between these two approaches is unwarranted, since optimal markup adjustments and nominal frictions are likely to act as intertwined factors in causing stable import prices in local currency. Specifically, we build a model where, because of market-specific nominal rigidities at the downstream level, different dynamics in final prices provide an incentive for upstream producers to price discriminate across countries, exacerbating the distortions from monopoly power. At the same time, the use of local nontradable inputs by firms selling goods to final users mutes the response of final prices to exchange rate movements.

There are at least three potentially important implications of our findings for policymaking. First, by creating price discrimination at the border, consumer price movements feed back to deviations from the law of one price across markets. The transmission mechanism from consumer price inflation to price discrimination provides monetary authorities with an additional reason to stabilize final prices. In this respect, our analysis sheds light on one possible reason why the progressive stabilization of inflation in the last decade may have contributed to the observed fall in exchange-rate pass-through. By reducing movements in consumer prices, policymakers indirectly affect the demand elasticity faced by upstream producers, reducing opportunities for exercising monopoly power through price discrimination.

Yet, complete CPI stabilization will never be desirable in our economies, because of both international and domestic policy trade-offs. Specifically, in addition to the international dimensions of monetary policy already discussed in the literature, we show that, with vertical interactions among industries adjusting prices in a staggered fashion, domestic price stability is actually unfeasible. This is due to the fact that nominal rigidities inducing staggered pricing by upstream producers, inherently lead to cost heterogeneity among downstream firms.

Finally, as shown by the literature, nominal rigidities in local currency induce positive comovements in the optimal monetary stance across countries, which tend to curb the volatility of the terms of trade. However, our results make it clear that, at an optimum, the real exchange rate can be more or less volatile than the terms of trade, depending on a number of structural features of the economy, like home bias in expenditure and local components of marginal costs in consumer goods. In this sense, the empirical regularity that real exchange rates are typically more volatile than the terms of trade does not automatically suggest that policymakers fall short of stabilizing exchange rates efficiently.

9 Appendix

In this appendix we provide details on the derivation of a few results used extensively in the text.

9.1 Price dispersion

We can write the within-industry price dispersion of consumer prices as:

$$\begin{aligned} S_t(h) &\equiv \int \left(\frac{P_t(h, r_h)}{\mathbb{P}_t(h)} \right)^{-\eta} dr_{h.} \\ &= \mathbb{P}_t^{\eta}(h) \sum_{j=0}^{\infty} (1-\theta)\theta^j P_{t-j}(h)^{-\eta} \\ &= (1-\theta) \left(\frac{P_t(h)}{\mathbb{P}_t(h)} \right)^{-\eta} + \theta \pi_t^{\eta}(h) S_{t-1}(h) \end{aligned}$$

Similarly, we can express the across-industry dispersion in consumer prices as follows:

$$\begin{aligned} \overline{S}_t &= \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} S_t(h) dh \\ &= \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} (1-\theta) \left(\frac{P_t(h)}{\mathbb{P}_t(h)} \right)^{-\eta} dh + \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} \theta \pi_{t-1}^{\eta}(h) S_{t-1}(h) dh \\ &= (1-\theta) \int \left(\frac{P_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} dh + \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} \theta \pi_t^{\eta}(h) S_{t-1} dh \\ &= (1-\theta) \left[(1-\overline{\theta}) \left(\frac{P_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} + \overline{\theta} \left(\frac{\widetilde{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} \right] + \theta \pi_{\mathrm{H},t}^{\eta} \overline{S}_{t-1} \end{aligned}$$

9.2 The derivative of the optimal downstream price with respect to upstream prices

We now show that $\dfrac{\partial P_t^o(h)}{\partial \overline{P}_t(h)} = \dfrac{P_t^o(h)}{\overline{P}_t(h)}$. First, take the derivative

$$\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} = \frac{\eta}{\eta - 1} \left\{ \begin{array}{c} \dfrac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k \left[\dfrac{\partial MC_{t+k}(h)}{\partial \overline{P}_t(h)} C_{t+k}(h, r_h) + \dfrac{\partial C_{t+k}(h, r_h)}{\partial \overline{P}_t(h)} MC_{t+k}(h) \right]}{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k C_{t+k}(h, r_h)} + \\ - \dfrac{\left[E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k C_{t+k}(h, r_h) MC_{t+k}(h) \right] \left[E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k \dfrac{\partial C_{t+k}(h, r_h)}{\partial \overline{P}_t(h)} \right]}{\left[E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k C_{t+k}(h, r_h) \right]^2} \end{array} \right\};$$

Noting that $\dfrac{\partial MC_{t+k}(h)}{\partial \overline{P}_t(h)} = \dfrac{1}{Z_{t+k}} = \dfrac{MC_{t+k}(h)}{\overline{P}_t(h)}$, and

$$\begin{aligned}\frac{\partial C_{t+k}(h,r_h)}{\partial \overline{P}_t(h)} &= -\eta\left(\frac{P_t(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta} C_{\mathrm{H},t+k}\frac{1}{P_t(h)}\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} \\ &= -\eta\frac{C_{t+k}(h,r_h)}{P_t(h)}\frac{\partial P_t(h)}{\partial \overline{P}_t(h)},\end{aligned}$$

we obtain

$$\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} = \frac{\eta}{\eta-1}$$

$$\left\{\begin{array}{c}\dfrac{\frac{1}{\overline{P}_t(h)}E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)MC_{t+k}(h) - \eta\frac{\partial P_t(h)}{\partial \overline{P}_t(h)}\frac{1}{P_t(h)}E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)MC_{t+k}(h)}{E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)} + \\ -\dfrac{\left[E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)MC_{t+k}(h)\right]\left[-\eta\frac{\partial P_t(h)}{\partial \overline{P}_t(h)}\frac{1}{P_t(h)}E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)\right]}{\left[E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)\right]^2}\end{array}\right\};$$

which after further simplification becomes

$$\begin{aligned}\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} &= \left\{\frac{P_t(h)}{\overline{P}_t(h)} - \eta\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} + \eta\frac{\partial P_t(h)}{\partial \overline{P}_t(h)}\right\} \\ &= \frac{P_t(h)}{\overline{P}_t(h)}.\end{aligned}$$

The result in Section 4.4 that $\dfrac{\partial P_t^o(h)}{\partial \overline{P}_t(h)} = \dfrac{\eta}{\eta-1}$ in the Leontief case could be derived in a similar way.

Finally, in the derivation of (10) in the same section we used the following property of $\widetilde{S}_{t+k}(h)$:

$$\begin{aligned}\widetilde{S}_{t+k}(h) &\equiv \int\left(\frac{P^o_{t+k}(h,r_h)}{\mathbb{P}_{t+k}(h)}\right)^{-\eta-1}dr = (1-\theta)\mathbb{P}^{\eta+1}_{t+k}(h)\sum_{j=0}^{\infty}\theta^j P^o_{t+k-j}(h)^{-\eta-1} \\ (1-\theta)\mathbb{P}^{\eta+1}_{t+k}(h)\sum_{j=0}^{k}\theta^j P^o_{t+k-j}(h)^{-\eta-1} &= \widetilde{S}_{t+k}(h) - (1-\theta)\mathbb{P}^{\eta+1}_{t+k}(h)\sum_{j=k+1}^{\infty}\theta^j P_{t+k-j}(h)^{-\eta-1} \\ &= \widetilde{S}_{t+k}(h) - (1-\theta)\frac{\mathbb{P}^{\eta+1}_{t+k}(h)}{\mathbb{P}^{\eta+1}_{t-1}(h)}\mathbb{P}^{\eta+1}_{t-1}(h)\theta^{k+1}\sum_{j=0}^{\infty}\theta^j P_{t-j-1}(h)^{-\eta-1} \\ &= \widetilde{S}_{t+k}(h) - \theta^{k+1}\left(\frac{\mathbb{P}_{t+k}(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta+1}\widetilde{S}_{t-1}(h).\end{aligned}$$

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Table 1. Volatility Under Optimal Policy: Upstream Shocks Only (in percent)

Standard deviation	*Economies*		
	With Home Bias		Without Home Bias
	Flexible Prices	Optimal Policy	Optimal Policy
Inflation Rates			
CPI	0	0.01	0
Domestic final goods	0.19	0.11	0.25
Imported final goods	0.74	0.43	0.25
Domestic intermediate prices	0.19	0.19	0.50
Import intermediate prices	0.74	0.87	0.50
Export intermediate prices	0.74	0.87	0.50
International Prices			
Real exchange rate (CPI based)	1.38	1.62	0
Terms of trade	2.30	2.10	2.06
Deviations from the LOP			
Home goods at producer level	0	0.57	0
Home goods at consumer level	0	0.39	0
Home Markups			
Domestic intermediate goods	0	0.06	0.17
Exported intermediate goods	0	0.26	0.17
Domestic final goods	0	0.24	0.65
Imported final goods	0	1.12	0.65
Quantities			
Home consumption	0.87	0.90	0.81
Home hours	0.41	0.43	0.47
Real GDP	1.42	1.43	1.54

Table 2. Volatility Under Optimal Policy: Downstream Shocks Only (in percent)

Standard deviation	*Economies* With Home Bias Flexible Prices	*Economies* With Home Bias Optimal Policy	*Economies* Without Home Bias Optimal Policy
Inflation Rates			
CPI	0	0.12	0.13
Domestic final goods	0.02	0.13	0.12
Imported final goods	0.09	0.11	0.12
Domestic intermediate prices	0.69	0.38	0.39
Import intermediate prices	0.77	0.44	0.39
Export intermediate prices	0.77	0.44	0.39
International Prices			
Real exchange rate (CPI based)	2.62	2.91	2.75
Terms of trade	0.27	0.72	0.78
Deviations from the LOP			
Home goods at producer level	0	0.27	0.29
Home goods at consumer level	2.46	2.68	2.66
Home Markups			
Domestic intermediate goods	0	0.58	0.57
Exported intermediate goods	0	0.56	0.57
Domestic final goods	0	0.04	0.04
Imported final goods	0	0.11	0.04
Quantities			
Home consumption	0.99	1.08	1.05
Home hours	0.51	0.47	0.42
Real GDP	1.21	1.30	1.46

Table 3. Volatility Under Alternative Policies: Baseline calibration (in percent)

Standard deviation	*Policies* Flexible Prices	Optimal Policy	CPI Inflation Targeting	GDP Inflation Targeting	Taylor Rule
Inflation Rates					
CPI	0	0.12	0	0.41	0.38
Domestic final goods	0.19	0.17	0.11	0.41	0.43
Imported final goods	0.75	0.45	0.44	0.64	0.48
Domestic intermediate prices	0.71	0.49	0.95	0	0.51
Import intermediate prices	1.09	0.99	1.13	1.04	0.76
Export intermediate prices	1.09	0.99	1.13	1.04	0.76
International Prices					
Real exchange rate (CPI based)	2.97	3.35	3.78	3.14	3.62
Terms of trade	2.31	2.21	2.55	2.26	2.04
Deviations from the LOP					
Home goods at producer level	0	0.63	0.78	1.41	1.21
Home goods at consumer level	2.47	2.71	3.14	2.76	3.19
Home Markups					
Domestic intermediate goods	0	0.60	1.06	0.87	0.82
Exported intermediate goods	0	0.63	0.96	0.90	0.89
Domestic final goods	0	0.25	0.61	0.59	0.72
Imported final goods	0	1.13	1.24	1.38	0.97
Quantities					
Home consumption	1.39	1.47	1.58	1.44	1.57
Home hours	0.61	0.60	0.61	0.62	0.54
Real GDP	1.93	2.00	1.98	1.90	1.98

Figure 1. Productivity Shock to Home Upstream Production

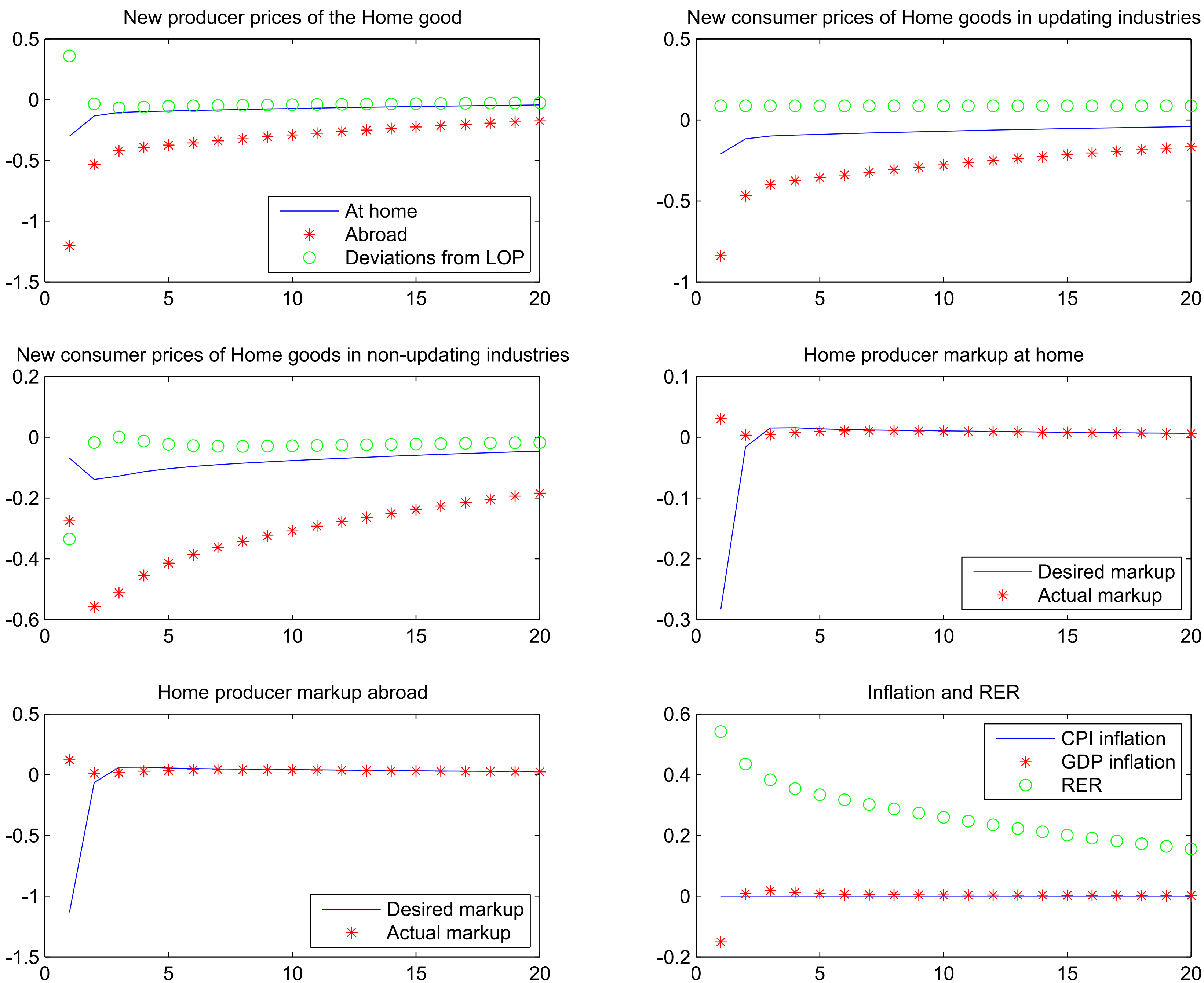

Figure 2. Productivity Shock to Home Downstream Production