

# Error Cascades in Observational Learning: An Experiment on the Chinos Game

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## Abstract

The paper reports an experimental study based on a variant of the popular Chinos game, which is used as a simple but paradigmatic instance of observational learning. There are three players, arranged in sequence, each of which wins a fixed price if she manages to guess the total number of coins lying in everybody's hands. Our evidence shows that, despite the remarkable frequency of equilibrium outcomes, deviations from optimal play are also significant. And when such deviations occur, we find that, for any given player position, the probability of a mistake is increasing in the probability of a mistake of her predecessors. This is what we call an *error cascade*, which we measure by evaluating the (heterogenous) Quantal Response Equilibrium which better suits our data. We also check the robustness of our findings when we allow for belief heterogeneity by applying Kübler and Weizsäcker's (2004) cognitive frame of *limited depth of reasoning*.

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## 1 Introduction

There are many situations of economic interest that involve *public sequential decisions* – that is, choices perfectly observed by others and made in a sequential order in which the position and identity of each player is well anticipated.<sup>1</sup> This is often the case of financial markets daily routine (where the moves of at least some “big players” are known to the market), or the choice of firms on technological adoptions under uncertain market conditions.<sup>2</sup> As both examples suggest, in these situations agents may have some private but incomplete information on which is the profitable decision. Therefore, the

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<sup>1</sup>Take, for example, Kennedy (2002) on how firms shape their business strategy; Welch (1992) on consumer behavior; Glaeser *et al.* (1996) and Kahan (1997) on the spread of crime and Lohmann (1994) on political action.

<sup>2</sup>Models of positional learning inspired by the phenomenon of speculative bubbles in financial markets are those of Lee (1998), Chari and Kehoe (2004) and Avery and Zemsky (1998). This latter paper has been tested experimentally by Cipriani and Guarino (2005).

action they take (as well as their identity and reputation) may implicitly convey some of this private information to late movers, who can use it as input in their own decisions. Then, it may occur that the higher the number of agents who have already taken their decision, the lower the level of uncertainty faced by those who still have to do it. If this happens, the population enjoys what is often labelled as *observational (or positional) learning*, a phenomenon that has been the object of recent attention, both on the theoretical and the experimental side.<sup>3</sup>

This paper reports an experimental study on observational learning based on a traditional parlour game played in many countries, which in Spain is known as *Chinos*.<sup>4</sup> In this game, players start by hiding in their hands a number of coins (or pebbles), from zero to a certain maximum number (often three). Then, in some pre-specified order, each player produces a guess on the *sum* of coins in the hands of every player. When doing so, a player is informed of her own number of coins as well as the guesses produced by all others who preceded her.

Formally, this yields a multi-stage game with incomplete information. In its simplified version played in the lab, the number of coins in the hands of each player is the outcome of an exogenous random mechanism, i.e., a stochastic move by Nature. We further simplified matters by considering just three players and restricting the number of coins in the hands of each player to be either zero or one. Finally, concerning payoffs, we design the game so that players' incentives do not conflict. Specifically, we allow players to submit the same guess, and the same fixed price is awarded to *all* subjects who guess the sum of coins right.<sup>5</sup>

As a consequence of this payoff structure, our modified Chinos game has a unique Perfect Bayesian Equilibrium (PBE). In it, after observing any given

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<sup>3</sup>In the context of positional learning, *herd behavior* and *information cascades* have been first analyzed theoretically in the seminal papers of Banerjee (1992) and Bikhchandani *et al.* (1992). Their models have been tested experimentally by Anderson and Holt (1997) and Allsopp and Hey (2000), respectively.

<sup>4</sup>The word “chinos” is a slight modification of the Spanish word “chinas”, which refers to the pebbles that players may hide in their hands when playing the game. This game was first analyzed by Pastor-Abia *et al.* (2000).

<sup>5</sup>In this respect, the paper by Çelen and Kariv (2004) is the closest to ours. They analyse a situation where each agent receives a signal from the continuous space  $[-10, 10]$  with uniform probability and players have to guess sequentially whether the sum over the signals of all players is “positive” or “negative”. Their objective, however, is to differentiate *information cascades* from *herd behavior* in the lab.

player’s guess, each subsequent player infers exactly the number of coins lying in the formers’ hands. Therefore, the probability of winning increases with player position, with the last player in the sequence guessing the correct answer with certainty.

In this light, the main objectives of our experiment are now advanced. *First*, we want to check whether, as theory would unambiguously prescribe, earlier movers choose clear-cut signalling guesses *and* followers are perfectly able to “decipher” the predecessors’ actions and react accordingly. Qualitatively, we find that this is indeed the case, and that there are also systematic deviations from the equilibrium pattern – in particular, it appears that the probability that a late mover makes a mistake increases with the frequency of mistakes by her predecessors.

Therefore, as a *second* step in our analysis, we propose a decision-theoretic model that explicitly allows for mistakes. This model builds upon the by-now standard notion of Quantal Response Equilibrium (QRE) proposed by McKelvey and Palfrey (1995, 1998) and generalizes it to allow for player heterogeneity: this gives rise to the concept that Rogers *et al.* (2009) label Heterogenous QRE (HQRE). We estimate such generalized model and indeed confirm, as conjectured, that our experimental evidence displays *error cascades*. That is, we find that the probability that a particular player in a some late position (2 or 3) makes an error is positively correlated with the estimated probability that predecessors have made an error as well.

Finally, as a *third* step in our discussion, we test the robustness of our findings by pursuing an approach that follows Kübler and Weizsäcker’s (2004) account of Stahl and Wilson’s (1995) classic model of *Limited Depth of Reasoning* (LDR).<sup>6</sup> This approach relaxes the accurate (or “rational”) expectation assumption that underlies the (H)QRE concept and allows for the possibility that agents’ beliefs may be incorrect – in particular, concerning the opponents’ mistake probabilities. We again estimate our model within this extended framework and confirm the persistence of error cascades. Interestingly enough, we also find that two of Kübler and Weizsäcker’s (2004) main findings apply to our context. First, players consistently underestimate the degree of rationality of their opponents, and this may provide some explanation for the systematic errors found in our experiments. Second, subjects’ reasoning on the behavior of others becomes more uncertain (i.e. their predictions more noisy) as higher-order considerations/beliefs are involved.

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<sup>6</sup>See also Nagel (1995).

The considerations at work are akin to those that, in the context studied by De Marzo et al. (2003), give rise to what they call “persuasion bias”. For, in essence, their model shares with ours the feature that, when an agent observes the behavior of some other, she ignores the fact that such behavior is itself a reaction to what that agent has previously observed. This, in general, can induce severe distortions on how agents learn from observation. And, within our context, it can indeed underlie the “cascading errors” observed in the lab: later movers do not internalize enough of their predecessors’ behavior in their considerations and this indeed exacerbates their tendency to committing errors.

The remainder of the paper is organized as follows. Section 2 provides a description of the experimental design. Section 3 relates descriptive statistics to PBE. Next, Section 4 analyzes the experimental data using the HQRE and LDR models. Finally, Section 5 concludes.

## 2 Experimental design

In what follows, we describe the features of the experiment in detail.

1. *Sessions.* The 4 experimental sessions were run in a computer lab. A total of 48 students (12 per session) were recruited among the student population of the Universidad de Alicante – mainly, undergraduate students from the Economics Department with no (or very little) prior exposure to game theory. Instructions were provided by a self-paced, interactive computer program that introduced and described the experiment. Subjects were also provided with a written copy of the experimental instructions, identical to what they were reading on the screen.<sup>7</sup>
2. *Matching.* In each session, subjects played 20 rounds of the Chinos game described below. And, in all 20 rounds, subjects played anonymously in groups of 3 players. Each group consisted of the same subjects throughout (that is, group composition was kept constant) and each of them occupied the same position. Both of these important

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<sup>7</sup>The experiment was programmed and conducted using the software *z-Tree* (Fischbacher, 2007). A copy of the instructions, translated into English, can be found in the supplementary material for the paper.

features of the experimental design were publicly announced at the beginning of each session. Given these experimental conditions, we were able to collect 16 independent observations of our experimental environment.<sup>8</sup>

3. *The game.* The three players of each game, indexed by  $i \in N = \{1, 2, 3\}$ , privately receive an i.i.d. signal  $s_i \in \{0, 1\}$ , with  $s_i = 1$  chosen with probability equal to  $p = 0.75$ , uniform across players. Players act in sequence, as indicated by their indices, and have to guess the sum of signals,  $\sigma \equiv s_1 + s_2 + s_3$ . By the time player  $i$  makes her guess  $g_i \in G \equiv \{0, 1, 2, 3\}$ , she knows her own signal ( $s_i$ ) and the guesses of those players  $j < i$  who acted before her. All players who guess correctly (i.e., those for which  $g_i = \sigma$ ) receive a fixed prize, while any incorrect guess yields a payoff of 0.
4. *Payoffs.* Monetary payoffs in the experiment were expressed in Spanish pesetas (1 euro is approx. 166 pesetas).<sup>9</sup> All subjects received 1000 pesetas just to show up. The fixed prize for each round was equal to 50 pesetas.
5. *Ex-post information.* After each round, each subject was informed of all payoff-relevant information, that is, the correct guess (and, therefore, their own payoff), as well as the individual guesses and signals of all subjects in their group. In addition, they were provided with an *history table*, to better track the sequence of signals and guesses of their group members in all previous rounds.

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<sup>8</sup>Since subjects interact with each other within groups but not across groups, each group can be considered as a statistically independent observation.

<sup>9</sup>It is standard practice, for all experiments run in Alicante, to use (obsolete) Spanish pesetas as experimental currency. The reason for this design choice is twofold. First, it mitigates integer problems, compared with other currencies (USD or euros, for example). On the other hand, although Spanish pesetas are no longer in use (substituted by the euro in the year 2002), Spanish people still use pesetas to express monetary values in their everyday life. Thus, by using a “real” (as opposed to artificial) currency, we avoid the problem of framing the incentive structure of the experiment using a scale (*e.g.* “experimental currency”) with no cognitive content.

## 3 PBE

### 3.1 Theory

In the Chinos game described above all players have an unambiguous incentive to maximize their chances of guessing correctly, hence revealing their private signal to later movers.<sup>10</sup> We shall first characterize *players' optimal behavior*. Since the sum of  $k$  signals is binomially distributed as  $\text{Bin}(k, 0.75)$ ,  $k \leq 2$ , all relevant distributions are unimodal, with the modes of  $\text{Bin}(1, 0.75)$  and  $\text{Bin}(2, 0.75)$  being 1 and 2, respectively. This greatly simplifies the analysis: given the realized vector of signals  $s \equiv (s_1, s_2, s_3)$ , we can “solve forward” for the unique equilibrium outcome – i.e. sequence of guesses  $\bar{g}_i(\cdot)$  along the equilibrium paths – common to all the PBE of the game, which only differ in terms of out-of-equilibrium beliefs:

$$\bar{g}_1(s_1) = s_1 + 2, \tag{1}$$

$$\bar{g}_2(g_1, s_2) = s_2 + 1 + \mathbb{I}(g_1 = 3) \tag{2}$$

$$\bar{g}_3(g_2, s_3) = s_3 + \mathbb{I}(g_2 = 2) + 2 \mathbb{I}(g_2 = 3), \tag{3}$$

where  $\mathbb{I}(\cdot)$  is an indicator function that is 1 if condition  $(\cdot)$  holds, and 0 otherwise. Thus, in equilibrium, each player is perfectly informed of the signal received by her predecessors, computes her guess by taking expectations over the signals of her successors and, in doing so, perfectly reveals her own signal. This implies that, the higher the player position, the higher the chances to win the prize. In particular, player 3 guesses right with probability one, while player 2 does so with probability  $\Pr(s_3 = 1) = 0.75$ , and player 1 with probability  $\Pr(s_2 + s_3 = 2) = 0.5625$ . Finally, note that, when player 3 computes her optimal guess in (3) she does not need to look at player 1's guess, since all the relevant information (including that regarding  $s_1$ ) is subsumed in player 2's guess,  $g_2$ .

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<sup>10</sup>Ponti and Carbone (2009) analyze a variation of the game-form proposed here, in which one of the players in the sequence, in case of a correct guess, receives the prize only with some positive probability (strictly smaller than one). This additional feature does not modify the basic structure of the game (i.e. the strict incentive to full revelation). This is in contrast with the traditional version of the Chinos game, where agents' incentives are opposed because guesses have to be distinct, i.e., no player allowed to mimic the guess of a predecessor. This fundamental variation of the game is analyzed in a companion work, Feri *et al.* (2010).

## 3.2 Evidence

Table 1 reports players’ winning frequencies (i.e., the fraction of times when their guess coincides with the sum of signals). Within brackets are the corresponding theoretical predictions – that is, for each player position, the probability of guessing right (or, equivalently, winning the prize) if all players conformed to the equilibrium strategy (1)-(3).

Player	Frequency of guessing right
1	40.51 <b>(56)</b>
2	50.32 <b>(75)</b>
3	61.08 <b>(100)</b>

**Table 1.** Winning distribution

First, we can observe that these probabilities, although lower than the corresponding equilibrium levels, are qualitatively consistent with PBE play, since the probability of winning is increasing with player position. We also observe that the difference between theoretical and actual frequency is increasing with player position: 15.49 for player 1, 24.68 for player 2 and 38.92 for player 3. In this respect, *the deviation from the equilibrium outcome also increases with player position.*<sup>11</sup>

We now turn our attention to subjects’ aggregate behavior. Since we run 4 sessions of 20 rounds each, with 4 groups of 3 players in each session, our panel database contains  $4 \times 4 \times 20 = 320$  guessing sequences for  $4 \times 4 = 16$  independent observations. The focus here will be on behavioral strategies along the (PBE) equilibrium path, while we postpone to Sections 4 the analysis of out-of-equilibrium behavior.

Tables 2.1*a*), 2.2*a*), and 2.3 report behavioral strategies of players 1, 2, and 3, respectively. In all tables, each row corresponds to a different information set (i.e., a specific combination of own signal and predecessors’ guesses), while each column corresponds to one of the four possible guesses,  $g_i$ . In each cell, we report absolute (top) and relative (bottom) frequency of use of a particular guess – where the latter can be seen as the “aggregate behavioral strategy” empirically observed. In all tables, we also highlight in light

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<sup>11</sup>This conclusion would not change if we considered *ratios*, instead of *differences*, as deviations from optimal behavior correspond, respectively, to 27.66%, 32.91% and 38.92% of the corresponding theoretical frequencies.

(dark) grey the equilibrium path corresponding to  $g_1 = 2$  ( $g_1 = 3$ ). Tables 2.1*b*) and 2.2*b*), instead, report relative frequencies of signals conditional on a particular guess, i.e., they look at aggregate choice frequencies of players 1 and 2 from the perspective of information decoding. In a strict sense, the latter are the only relevant regularities players need to extract from their predecessors' strategies. For, indeed, such conditional frequencies reflect the extent to which *players' guesses reveal their private signals*.<sup>12</sup>

Let us now consider in turn each of the three player positions in some detail. First, observe in Table 2.1*a*) that player 1 guesses consistently with equilibrium 58.43% of the time  $((78 + 109)/320)$ . Also notice that the equilibrium guess corresponds to the modal choice in both information sets, although this frequency is higher when player 1 gets signal 0 (72% *vs.* 52%). However, despite the fact that player 1 tends to play optimally more often when  $s_1 = 0$  (thus choosing  $g_1 = 2$ ), her “message” is much clearer when her guess is  $g_1 = 3$ . For, as the right-bottom cell of Table 2.1*b*) shows, it is *always* the case that  $s_1 = 1$  when  $g_1 = 3$ , and this should be taken into account by players 2 and 3 when attempting to decode the informational content of player 1's guess.

$s_1 \backslash g_1$	0	1	2	3
0	1	29	78	0
%	<b>0.93</b>	<b>26.85</b>	<b>72.22</b>	<b>0.00</b>
1	0	20	79	109
%	<b>0.00</b>	<b>9.62</b>	<b>37.98</b>	<b>52.40</b>

a)

$s_1 \backslash g_1$	0	1	2	3
0	1	29	78	0
%	<b>100</b>	<b>59.18</b>	<b>49.68</b>	<b>0.00</b>
1	0	20	79	109
%	<b>0.00</b>	<b>40.82</b>	<b>50.32</b>	<b>100.00</b>

b)

**Table 2.1.** Player 1's behavioral strategy

We now move to player 2, whose aggregate behavior is reported in Table 2.2. First, Table 2.2*a*) shows that player 2 conforms to the equilibrium strategy 65.78% of the time,  $((20 + 61 + 22 + 72)/266)$ , where 266 is the number of times player 1 submitted an equilibrium guess  $g_1 \geq 2$  (i.e. a guess that is made at equilibrium for some signal). The distortion detected for player 1

<sup>12</sup>Since player 3 is the last in line, what would be Table 2.3*b*) is omitted here. Also notice that player 2 never guessed 0. The symbol “N/A” in Table 2.2*b*) simply indicates that, in this case, conditional probabilities cannot be calculated.

(namely, that the frequency of equilibrium behavior depends on her own signal,  $s_1$ ) also occurs for player 2, but in the opposite direction: conformity to equilibrium is now higher when  $s_2 = 1$ . On the other hand, we also find that conformity to equilibrium behavior is much stronger after  $g_1 = 3$  than otherwise. This may be due to the fact that, in that case, player 1’s message is “crystal clean”, as explained above. We may then conjecture that experience should lead player 2 and 3 to reach this same conclusion. As a consequence, adherence to equilibrium behavior on behalf of player 2 is higher when  $g_1 = 3$  (86.23%) than when  $g_1 = 2$  (51.59%).

Obviously, for player 1, guessing according to equilibrium behavior (1) always coincides with optimal behavior, independently of her successors’ guesses. In fact, the same happens for player 2 when she reacts as in (2) after observing  $g_1 = 3$ , since this guess always happens to be an accurately revealing message, just as in equilibrium. Concerning the optimality of player 2’s response to  $g_1 = 2$ , however, matters are more intricate. For, as we observe in Table 2.1*b*), this guess was almost equally likely to be delivered for  $s_1 = 0$  and  $s_1 = 1$ . This entails a message-decoding problem different from that at equilibrium, which therefore needs to be learned by player 2 through experience. But such a learning renders player 2’s decision problem more difficult, possibly leading to her behaving suboptimally in some cases. And this, intuitively, should tend to increase the complexity of player 3’s decision problem and its consequent probability of behaving suboptimally. The former chain of considerations is what we shall call an error cascade in Section 4.

		$g_2$			
		$s_2$	0	1	2
2	0	0	20	31	0
	%	<b>0.00</b>	<b>39.22</b>	<b>60.78</b>	<b>0.00</b>
	1	0	8	61	37
	%	<b>0.00</b>	<b>7.55</b>	<b>57.55</b>	<b>34.91</b>
3	0	0	6	22	1
	%	<b>0.00</b>	<b>20.69</b>	<b>75.86</b>	<b>3.45</b>
	1	0	0	8	72
	%	<b>0.00</b>	<b>0.00</b>	<b>10.00</b>	<b>90.00</b>

a)

		$g_2$			
		$s_2$	0	1	2
2	0	0	20	31	0
	%	<b>N/A</b>	<b>71.43</b>	<b>33.70</b>	<b>0.00</b>
	1	0	8	61	37
	%	<b>N/A</b>	<b>28.57</b>	<b>66.30</b>	<b>100.00</b>
3	0	0	6	22	1
	%	<b>N/A</b>	<b>100.00</b>	<b>73.33</b>	<b>1.37</b>
	1	0	0	8	72
	%	<b>N/A</b>	<b>0.00</b>	<b>26.67</b>	<b>98.63</b>

b)

**Table 2.2.** Player 2’s behavioral strategy (along the equilibrium path)

Finally, Table 2.3 summarizes player 3’s aggregate behavior. Here we find that player 3 follows the equilibrium strategy (3) with a frequency of

63.23%. And, similarly to player 2, player 3 is more likely to play the equilibrium strategy when her predecessors' guesses are higher. Overall, player 3's likelihood to mimic equilibrium behavior grows as *i*) her own signal,  $s_3$ , is higher and *ii*) players 1 and 2's guesses,  $g_1$  and  $g_2$ , are also higher. The effect in *i*) is analogous to the systematic distortion tailored to the player's own signal that was already encountered for player 2, while the effect in *ii*) appears to reflect the same considerations which underlay player 2's behavior: higher guesses give rise to clearer messages. As a stark case in point, consider the situation in which both players 1 and 2 produce a common guess  $g_1 = g_2 = 3$ . Then, it is intuitive that player 3 must expect (as will also be confirmed by ongoing evidence) that both player 1 and 2 received a signal equal to 1. This shows in the very large conformity to equilibrium behavior displayed by player 3 in this case (87.5% and 100% for the low and high signals, respectively). But, in general, to carry out a proper discussion of the problem, either for player 2 or 3, the average statistics contained in Table 2.3 are clearly not enough. We need a model where the probability of error itself be explicitly accounted for. This is the objective of the next two sections.

$g_1$	$g_2$	$s_3 \backslash g_3$	0	1	2	3	
2	1	0	1	8	2	0	
		%	<b>9.09</b>	<b>72.73</b>	<b>18.18</b>	<b>0.00</b>	
	1	1	0	5	10	2	
		%	<b>0.00</b>	<b>29.41</b>	<b>58.82</b>	<b>11.76</b>	
	2	2	0	0	5	24	0
			%	<b>0.00</b>	<b>17.24</b>	<b>82.76</b>	<b>0.00</b>
1		1	0	0	45	18	
		%	<b>0.00</b>	<b>0.00</b>	<b>71.43</b>	<b>28.57</b>	
3	2	0	0	6	9	1	
		%	<b>0.00</b>	<b>37.50</b>	<b>56.25</b>	<b>6.25</b>	
	1	1	0	0	9	5	
		%	<b>0.00</b>	<b>0.00</b>	<b>64.29</b>	<b>35.71</b>	
	3	0	0	1	21	2	
		%	<b>0.00</b>	<b>4.17</b>	<b>87.50</b>	<b>8.33</b>	
1	1	0	0	0	49		
	%	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>100.00</b>		

**Table 2.3.** Player 3's behavioral strategy (along the equilibrium path)

## 4 “Noisy” equilibria

### 4.1 Theory

Since our aim is to build up a statistical model of error cascades, we need to compute players’ optimal responses, in and out of the equilibrium path. We focus on behavioral strategies, defined in the conventional fashion as a mapping from information sets to (possibly probabilistic) guesses. Next, we define players’ beliefs as systems of probabilities of signals conditional on guesses. We estimate subjects’ beliefs within a general framework that extends McKelvey and Palfrey’s (1995, 1998) QRE to the case of heterogenous agents and, also, allows for the possibility that agents’ expectations concerning the behavior of their group partners might not be accurate (or, as sometimes called, “rational” or “consistent”).

A quantal response is, basically, a smoothed-out best response, in the sense that agents are assumed to select each strategy with a probability that is proportional to some exponentially increasing function of its corresponding expected payoff. We follow Rogers *et al.* (2009) by assuming that each player  $i$  in a given group is independently assigned by nature a response sensitivity,  $\lambda_i$ , drawn from a distribution  $F_i(\lambda_i)$ . Here, for simplicity, we shall consider the extreme formulation where each  $F_i(\lambda_i)$  is fully concentrated on a particular value. For each player  $i > 1$ , we also specify beliefs on the response sensitivities of her predecessors  $(\lambda_j)_{j < i}$ . Assuming again point distributions, we denote by  $(\lambda_{ij}^I)_{j < i}$  the values on which those beliefs are concentrated – thus, for all  $j < i$ , player  $i$  believes with probability 1 that  $\lambda_j$  is equal to some  $\lambda_{ij}^I$ . Regarding player 3, we also consider some (point-)beliefs on her part as to what player 2 believes on the response sensitivity of player 1. These beliefs by player 3 (i.e. the value on which they are concentrated) are denoted by  $\lambda_{31}^{II}$  – thus, player 3 believes with probability 1 that  $\lambda_{21}^I$  is equal to  $\lambda_{31}^{II}$ .

In this section, we first provide the expressions for the profile of behavioral strategies,  $\gamma = (\gamma_1, \gamma_2, \gamma_3)$  and, then, define the variables that will be used in Subsections 4.2 and 4.3 to detect and measure error cascades in our experiment. In Subsection 4.2, we follow Rogers *et al.* (2009) in assuming that the beliefs on the response sensitivities of predecessors are consistent. This assumption will be removed in Subsection 4.3, where we follow Kübler and Weizsäcker (2004) in allowing for belief inconsistencies.

### Behavioral strategies

Let  $\mathcal{H}_i$  denote the collection of player  $i$ 's information sets. For player 1, we can simply write  $\mathcal{H}_1 \equiv \{h_1 = s_1\}$ , since she has only two information sets that can be associated with each of the possible realizations of  $s_1$ . For players 2 and 3, information sets can be defined as  $\mathcal{H}_2 \equiv \{h_2 = (g_1, s_2)\}$  and  $\mathcal{H}_3 \equiv \{h_3 = (g_1, g_2, s_3)\}$ , respectively. Player  $i$ 's behavioral strategy is denoted by  $\gamma_i : \mathcal{H}_i \rightarrow \Delta(G)$ , where  $\gamma_i(g_i, h_i)$  stands for the probability of guessing  $g_i$  at information set  $h_i$ . Given the parameter array  $\Lambda \equiv \{\lambda_1, (\lambda_2, \lambda_{21}^I), (\lambda_3, \lambda_{31}^I, \lambda_{32}^I, \lambda_{31}^{II})\}$  prevailing in any given group, the induced behavioral strategies  $(\gamma_i)_{i=1,2,3}$  can be obtained as follows. First, for player 1, information set  $h_1$ , and guess  $g_1$ , a behavioral strategy is derived as

$$\gamma_1(g_1, h_1) = \frac{\exp[\lambda_1 \pi_1(g_1, h_1)]}{\sum_{k=0}^3 \exp[\lambda_1 \pi_1(k, h_1)]}, \quad (4)$$

where  $\pi_1(g_1, h_1)$  is player 1's expected payoff associated with guess  $g_1$  in information set  $h_1$ .<sup>13</sup> Note that, if we replace  $\lambda_1$  by  $\lambda_{21}^I$  (resp.  $\lambda_{31}^I$ ) in (4), we obtain the subjective belief on  $\gamma_1$  held by player 2 (resp. 3), that we denote by  $\gamma_{21}^I$  (resp.  $\gamma_{31}^I$ ). Likewise, if we replace  $\lambda_1$  by  $\lambda_{31}^{II}$  in (4), we obtain the belief on  $\gamma_{21}^I$  held by player 3, that we denote by  $\gamma_{31}^{II}$ .

The point here is that player 1 does not need to form beliefs on the choices of others in order to evaluate the optimality of her own strategy. But this is not the case for players 2 and 3. Each of these players may hold (generally different) beliefs  $\{\mu_{i1}^I(g_1)\}_{g_1 \in G}$  specifying their subjective probability that player 1 has received signal  $s_1 = 1$  conditional on the observed guess  $g_1 \in G$ . These beliefs are induced by the point-beliefs  $\lambda_{i1}^I$  each of them has on  $\lambda_1$ , i.e.,  $\mu_{i1}^I(g_1)$  can be computed by applying Bayes rule to  $\gamma_{i1}^I$ .<sup>14</sup> For player 2,  $\{\mu_{21}^I(g_1)\}_{g_1 \in G}$  are the sole relevant beliefs. That is, they are enough to define her behavioral strategy  $\gamma_2$  as

$$\gamma_2(g_2, h_2) = \frac{\exp[\lambda_2 \pi_2(g_2, h_2 | \mu_{21}^I(g_1))]}{\sum_{k=0}^3 \exp[\lambda_2 \pi_2(k, h_2 | \mu_{21}^I(g_1))]}, \quad (5)$$

<sup>13</sup>As  $\lambda_1 \rightarrow \infty$ , the probability of choosing the guess with the highest expected payoff goes to 1. Instead, low values of  $\lambda_1$  are associated with "noisy" equilibria. As  $\lambda_1 \rightarrow 0$ , the density function in (4) becomes flat over the entire support and turns to being essentially random.

<sup>14</sup>Formally,  $\mu_{i1}^I(g_1) = \frac{\frac{3}{4}\gamma_{i1}^I(g_1, 1)}{\frac{3}{4}\gamma_{i1}^I(g_1, 1) + \frac{1}{4}\gamma_{i1}^I(g_1, 0)}$ .

where  $\pi_2(g_2, h_2 \mid \mu_{21}^I(g_1))$  is player 2's expected payoff associated with guess  $g_2$  in information set  $h_2$ , conditional on player 2's belief  $\mu_{21}^I(g_1)$ .

In order to compute  $\gamma_3$ , player 3 also needs to entertain *additional* beliefs  $\{\mu_{31}^{II}(g_1)\}_{g_1 \in G}$  specifying her subjective probabilities attributed to  $\{\mu_{21}^I(g_1)\}_{g_1 \in G}$ . They are induced by the point-beliefs  $\lambda_{31}^{II}$  she has on  $\lambda_{21}^I$ , i.e.,  $\mu_{31}^{II}(g_1)$  can be computed by applying Bayes rule to  $\gamma_{31}^{II}$ . Note that, if we replace  $\lambda_2$  and  $\mu_{21}^I(g_1)$  by  $\lambda_{32}^I$  and  $\mu_{31}^{II}(g_1)$  in (5), we obtain the belief on  $\gamma_2$  held by player 3, that we denote by  $\gamma_{32}^I$ . Using this belief, we can now define  $\{\mu_{32}^I(g_1, g_2)\}_{g_1, g_2 \in G}$ , specifying player 3's subjective probability that player 2 has received signal  $s_2 = 1$  conditional on the observed guesses  $g_1, g_2 \in G$ . These beliefs are induced by her point-beliefs  $\lambda_{32}^I$  on  $\lambda_2$ , i.e.,  $\mu_{32}^I(g_1, g_2)$  can be computed by applying Bayes rule to  $\gamma_{32}^I$ . Hence, player 3's behavioral strategy  $\gamma_3$  is defined as

$$\gamma_3(g_3, h_3) = \frac{\exp[\lambda_3 \pi_3(g_3, h_3 \mid \mu_{31}^I(g_1), \mu_{32}^I(g_1, g_2))]}{\sum_{k=0}^3 \exp[\lambda_3 \pi_3(k, h_3 \mid \mu_{31}^I(g_1), \mu_{32}^I(g_1, g_2))]}, \quad (6)$$

where  $\pi_3(g_3, h_3 \mid \mu_{31}^I(g_1), \mu_{32}^I(g_1, g_2))$  is player 3's expected payoff associated with guess  $g_3$  in information set  $h_3$ , conditional on player 3's beliefs  $\mu_{31}^I(g_1)$ ,  $\mu_{32}^I(g_1, g_2)$ .

### *Error cascades*

In order to detect and measure error cascades in our experiment, which shall be the object of Subsections 4.2 and 4.3, we need first to define what is a “mistake.” Intuitively, we want to conceive it as a situation in which a player fails to choose a suitable best response. But, given that the beliefs held by a player on the behavior of others can be (very) wrong, there are different ways of formulating this idea. For example, how should one label a player  $i$  who responds almost optimally (i.e., with a large  $\lambda_i$ ) to very distorted expectations of what others do? To address this issue, the notion of error will be grounded on the strategy profile actually played. This provides an objective basis for the concept and frees it from subjective considerations such as players' expectations.<sup>15</sup> Specifically, we shall declare any specific guess to be a mistake if it would appear as suboptimal if the actual (quantal-response) strategies followed by all players were commonly known, i.e. if players' expectations were fully correct.

<sup>15</sup>This is not an issue in Section 4.2, in which rational expectations are imposed.

More formally, let  $(\gamma_1, \gamma_2, \gamma_3)$  be the quantal-response response strategies used (or estimated) on the part of players. Then, one can readily apply Bayes Rule to obtain the conditional beliefs that, after each possible sequence of guesses, the corresponding players have signal 0 or 1. In particular, for each  $g_1 \in G$ , we may obtain the posterior probability  $\mu_1(g_1)$  that player 1 has received signal  $s_1 = 1$  conditional on the observed guess  $g_1$ . And, similarly, we can compute the posterior probability  $\mu_2(g_1, g_2)$  that player 2 has received signal  $s_2 = 1$  after observing guesses  $g_1, g_2 \in G$ .

Then, on the basis of these probabilities (interpreted as the beliefs agents would hold under correct expectations), it is easy to identify what is the optimal response by each player  $i$  after every possible relevant history  $h$  (i.e. information set, which includes both her own signal and all prior guesses). Given any history (or information set)  $h_i \in \mathcal{H}_i$ , the optimal response, generically unique, is defined by

$$\varphi_i(h_i) = \arg \max_{g_i} \pi_i \left( g_i, h_i \mid (\mu_j(\cdot))_{j < i} \right). \quad (7)$$

Now we compute more specifically, for each player and every possible contingency, the optimal response given by (7). We start with player 1, for whom matters are straightforward since beliefs play no role. Optimal behavior is therefore perfectly aligned with the PBE equilibrium (1) and thus we have:

$$\varphi_1(h_1) = s_1 + 2. \quad (8)$$

Player 2's optimal response, in turn, depends upon her own signal,  $s_2$ , plus the most likely realization of the sum of the other group members' signals,  $s_1 + s_3$ , conditional on every particular  $h_2 \in \mathcal{H}_2$  being reached. This sum is a number from 0 to 2, whose corresponding probabilities are  $\frac{1 - \mu_1(g_1)}{4}$ ,  $\frac{3 - 2\mu_1(g_1)}{4}$  and  $\frac{3\mu_1(g_1)}{4}$ , respectively.<sup>16</sup> This, in turn, implies:

$$\varphi_2(h_2) = s_2 + 1 + \mathbb{I}(\mu_1(g_1) > 0.6). \quad (9)$$

By analogy with (9), player 3's optimal guess, depends upon her own signal,  $s_3$ , plus the most likely realization of the sum of the predecessors' signals,  $s_1 + s_2$ , conditional on the particular  $h_3 \in \mathcal{H}_3$  being reached. This sum is a number from 0 to 2, whose corresponding probabilities are  $(1 - \mu_1(g_1))(1 - \mu_2(g_1, g_2))$ ,

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<sup>16</sup>Note that  $3 - 2\mu_1(\cdot) > 1 - \mu_1(\cdot)$  for all  $\mu_1(\cdot)$ , and that  $3\mu_1(\cdot) > 3 - 2\mu_1(\cdot)$  if and only if  $\mu_1(\cdot) > 0.6$ .

$\mu_1(g_1)(1 - \mu_2(g_1, g_2)) + \mu_2(g_1, g_2)(1 - \mu_1(g_1))$  and  $\mu_1(g_1)\mu_2(g_1, g_2)$ , respectively.<sup>17</sup> This then leads to:

$$\begin{aligned} \varphi_3(h_3) = & s_3 + \mathbb{I} \left[ \begin{array}{c} 3\mu_1(g_1)\mu_2(g_1, g_2) < \\ \min\{\mu_1(g_1) + \mu_2(g_1, g_2), 2(\mu_1(g_1) + \mu_2(g_1, g_2)) - 1\} \end{array} \right] \\ & + 2\mathbb{I}(3\mu_1(g_1)\mu_2(g_1, g_2) > \mu_1(g_1) + \mu_2(g_1, g_2) > 1). \end{aligned} \quad (10)$$

Expressions (8), (9), and (10) embody what it means for each player  $i$  to behave optimally, after every possible sequence of prior guesses  $(g_j)_{j < i}$  and her own signal  $s_i$ . But let us now take the perspective of the subsequent players  $j' > i$  who know the underlying strategy profile  $(\gamma_1, \gamma_2, \gamma_3)$  and the sequence of prior guesses but, at their own point of choice, *ignore* the signal received by player  $i$ . Under these conditions we ask: given any particular guess  $g_i$  that player  $i$  may have put forward, what is the probability  $\beta_i((g_j)_{j \leq i})$  that those other players should assign to such a guess being optimal? Of course, their answer to this question must be probabilistic, since optimality in this case hinges on what (unobserved) signal player  $i$  may have received. In this context, the notion of *error cascade* is captured in a nutshell by the following condition: the lower is the probability  $\beta_i$ , the lower is also the probability that subsequent players  $j' > i$  themselves behave optimally.

For our purpose, it suffices to focus on the probabilities  $\beta_1$  and  $\beta_2$  associated to the behavior of players 1 and 2, since these are the only players who have any successors whose behavior they may affect. First, for player 1, let us define by  $\beta_1(g_1)$  the probability that any particular guess  $g_1$  be optimal, i.e. consistent with (8). When her guess is  $g_1 = 2$  (resp.  $g_1 = 3$ ), the implication is clear: the signal received must have been  $s_1 = 0$  (resp.  $s_1 = 1$ ). Thus, we have  $\beta_1(3) = \mu_1(3)$  and  $\beta_1(2) = 1 - \mu_1(2)$ . Instead, for any  $g_1 < 2$ , matters are less clear, for there is no way to rationalize any such guess as optimal (i.e., payoff maximizing), for any  $s_1$ . For these cases, we naturally extend our approach and label any guess  $g_1 < 2$  as *conditionally optimal* if it is submitted under the signal  $s_1$  that delivers the highest conditional payoff (i.e.,  $s_1 = 0$ ). Thus, in sum, a compact specification of  $\beta_1(g_1)$  is:

$$\beta_1(g_1) = \begin{cases} \mu_1(g_1) & \text{if } g_1 = 3, \\ 1 - \mu_1(g_1) & \text{otherwise.} \end{cases} \quad (11)$$

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<sup>17</sup>Hence, if we denote by  $P(k)$  the probability that  $s_1 + s_2$  is  $k$ , we get: (i)  $P(1) > P(0)$  if and only if  $3\mu_1(\cdot)\mu_2(\cdot) < 2(\mu_1(\cdot) + \mu_2(\cdot)) - 1$ ; (ii)  $P(1) > P(2)$  if and only if  $3\mu_1(\cdot)\mu_2(\cdot) < \mu_1(\cdot) + \mu_2(\cdot)$ ; (iii)  $P(2) > P(0)$  if and only if  $\mu_1(\cdot) + \mu_2(\cdot) > 1$ ; and (iv)  $P(2) > P(1)$  if and only if  $\mu_1(\cdot) + \mu_2(\cdot) < 3\mu_1(\cdot)\mu_2(\cdot)$ .

Let us now turn to player 2 and, correspondingly, define by  $\beta_2(g_1, g_2)$  the estimated probability that, after a prior guess  $g_1$  by the preceding player 1, her own guess  $g_2$  is optimal. In analogy with (11), that probability can be specified as follows:

$$\beta_2(g_1, g_2) = \begin{cases} \mu_2(g_1, g_2) & \text{if either } [g_2 = 2 \text{ and } \mu_1(g_1) < 0.6] \text{ or } [g_2 = 3], \\ 1 - \mu_2(g_1, g_2) & \text{otherwise.} \end{cases} \quad (12)$$

## 4.2 HQRE

In this section, we follow Rogers *et al.* (2009) in assuming that the beliefs on the response sensitivities of predecessors are correct. In other words, we assume that, for each  $i$  and  $j > i$ , the (first and second order) beliefs  $\lambda_{ji}^I$  and  $\lambda_{ji}^{II}$  of player  $j$  on the response sensitivity of player  $i$  coincide with  $\lambda_i$ . Thus, beliefs and strategies are statistically consistent, which provides a fixed-point (equilibrium) condition. Operationally, this implies that  $\lambda_i$  enters directly in the evaluation of  $i$ 's behavioral strategy “perceived” by  $j$  when formulating beliefs  $\mu_{ji}^I$ , i.e.,  $\gamma_{ji}^I = \gamma_{ji}^{II} = \gamma_i$  and, therefore,  $\mu_{ji}^I = \mu_{ji}^{II} = \mu_i$ . Players’ behavior, therefore, is uniquely induced by the vector of responsiveness parameters  $(\lambda_1, \lambda_2, \lambda_3)$ .

Our first task will be to estimate, independently, for each of our 16 matching groups, by maximum likelihood, the parameter profile  $(\lambda_1, \lambda_2, \lambda_3)$  which defines the HQRE which better suits our experimental evidence.<sup>18</sup> Our estimation strategy proceeds sequentially, as follows. First, we estimate the group-specific value  $\lambda_1$  which refers to each player in position 1 in each matching group. Such estimated  $\lambda_1$ , assumed thereafter to be common knowledge within the group, is subsequently used to define the beliefs  $\mu_1$  of the same matching group. This allows the estimation of the corresponding values of  $\lambda_2$ , which, in turn, induce beliefs  $\mu_2$ . Finally, the procedure is closed by estimating the values for  $\lambda_3$  in each group.

Once estimates for each group are (independently) obtained, we estimate the probabilities that any given player  $i$  in information set  $h_i$  selects at round  $t$  her optimal guess – i.e. the magnitudes  $\Pr(g_{it} = \varphi_i(h_i))$ , where optimal

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<sup>18</sup>Individual group estimates are not reported here, but are available upon request. We have also carried out pooled estimates (aggregated across groups) for each round. These estimations show no significant evidence of learning effects (i.e. correlation between rounds and  $\lambda_i$ ), for all  $i$ .

responses  $\varphi_i(h_i)$  are obtained from (8)-(10). These probabilities are assumed to follow a logit distribution, conditional on  $s_i$  and the corresponding estimated  $\beta_j(\cdot)$ ,  $j < i$ , as defined by (11) and (12). In case of player 3, we also include in the regression an interaction term,  $\beta_1(g_1)\beta_2(g_1, g_2)$ , to capture possible non-linearities. Table 3 reports the estimated coefficients of these regressions.<sup>19</sup>

	<b>Var.</b>	<b>Coef.</b>	<b>Std. Err.</b>	<b>z</b>	<b>P&gt;z</b>
<b>1)</b>	$s_1$	-1.337	0.371	-3.61	0.000
	<i>Cons.</i>	1.051	0.742	1.42	0.157
<b>2)</b>	$\beta_1$	4.453	0.717	6.21	0.000
	$s_2$	0.182	0.345	0.53	0.598
	<i>Cons.</i>	-3.055	0.795	-3.84	0.000
<b>3)</b>	$\beta_1$	-12.092	3.171	-3.81	0.000
	$\beta_2$	-3.599	1.796	-2.00	0.045
	$\beta_1\beta_2$	14.817	3.574	4.15	0.000
	$s_3$	1.640	0.389	4.22	0.000
	<i>Cons.</i>	1.455	1.620	0.90	0.369
	$dProb/d\beta_1$	-0.101	0.191	-0.53	0.598
$dProb/d\beta_2$	1.183	0.266	4.45	0.000	

**Table 3.** Error cascades

The main conclusion to be gathered from Table 3 is that, in line with our conjecture and former heuristic discussion, the estimated probability that a player's predecessors have played optimally yields a positive and significant effect on that player's own probability of playing optimally. This effect is manifest both on regression 2 (for player 2) and on regression 3 (for player 3). In the first case, we find a strong (and positive) dependence of  $\beta_1(g_1)$  on  $\Pr(g_{2t} = \varphi_2(h_2))$  across all  $h_2 \in \mathcal{H}_2$ . Concerning regression 3, the interaction term,  $\beta_1(g_1)\beta_2(g_1, g_2)$ , requires us to focus on the marginal effects (evaluated at average values of  $\beta_1(g_1)$  and  $\beta_2(g_1, g_2)$ ). In this respect, while the marginal effect of  $\beta_1(g_1)$  on  $\Pr(g_{3t} = \varphi_3(h_3))$  across histories  $h_3 \in \mathcal{H}_3$  is not significantly different from 0, the marginal effect of  $\beta_2(g_1, g_2)$  is indeed positive and highly significant. This result suggests that the impact of predecessors'

<sup>19</sup> All regressions in Tables 3 and 5 also include round dummies, whose estimated coefficients are not reported here.

mistakes vanishes as one moves along the sequence of play.<sup>20</sup>

Table 3 also shows that the effect of an agent's own signal on the probability of optimal play is as suggested in Subsection 3.2. That is, the estimated coefficient of  $s_1$  is negative and significant for player 1, while those of  $s_2$  and  $s_3$  are positive for players 2 and 3 (although only the coefficient associated to  $s_3$  is statistically significant).

### 4.3 LDR

We now relax the equilibrium assumption, allowing for the possibility that agents' expectations concerning the behavior of their group partners may not be accurate. The motivation here is that each player displays a limited ability to anticipate the full implications of her partners' rationality. To formalize matters, we follow an approach similar to that proposed by Kübler and Weizsäcker (2004).<sup>21</sup> In contrast with the HQRE model studied in Subsection 4.2, the present one posits that later movers form (first and second-order) beliefs, not necessarily correct, over the response sensitivities of their predecessors.

Formally, within the general framework presented in Subsection 4.1, every player  $i$  is assumed to hold a common first (resp. second) order belief  $\lambda_i^I$  (resp.  $\lambda_i^{II}$ ) on the response parameters of all her predecessors, i.e., we assume that  $\lambda_{21}^I = \lambda_2^I$ ,  $\lambda_{31}^I = \lambda_{32}^I = \lambda_3^I$ , and  $\lambda_{31}^{II} = \lambda_3^{II}$ .<sup>22</sup> Hence, the parameter array  $\Lambda$  can be simplified to  $\{\lambda_1, (\lambda_2, \lambda_2^I), (\lambda_3, \lambda_3^I, \lambda_3^{II})\}$ .

In essence, here we extend the former approach to allow for non consistent beliefs. The aim of this exercise is two-fold. First, we want to investigate whether the sizable errors detected in the previous sections can be rationalized as an outcome of incorrect expectations. And, in that case, we want

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<sup>20</sup>If we include the predecessors' guesses in the estimations of Table 3, all the results remain qualitatively unaffected, i.e., the coefficients of these new variables are not significant and the sign and significance of all the other variables remain the same.

<sup>21</sup>See also Weizsäcker (2003) and Goeree and Holt (2004), who use related behavioral models in the context of normal form games.

<sup>22</sup>Kübler and Weizsäcker (2004) consider an experimental design in which, at every round, each subject is randomly assigned a player position in the sequence (of their game). Since all players experience every player position, they impose that each order of beliefs is the same for each player position. In contrast, in our design each subject only experiences one player position, and we allow each order of beliefs to differ by player position. However, we share their formulation that players hold each order of beliefs common for all their predecessors.

to identify (empirically) the nature of such distortions. For example, one may conjecture that the agents systematically underestimate the rationality of their partners (i.e. the magnitude of their decision parameters), which then would become a possible explanation for the observed deviations from optimality. To check this conjecture, we need a separate estimation of the sensitivity parameters and the relevant point-beliefs. Second, we want to assess the robustness of the error-cascade phenomenon to the relaxation of the rational-expectations principle.

Relying on the above modifications, our empirical strategy follows closely that of Subsection 4.2. For each of our experimental subjects in player position  $i \in \{1, 2, 3\}$ , we estimate – by maximum likelihood (48 estimations) – the full array of parameters,  $\Lambda$ , embodying players’ decisions and beliefs.<sup>23</sup> Note that, since now we do not assume that players have correct beliefs, the estimations within each group are independent across players/positions  $i \in \{1, 2, 3\}$ . For each such player, however, all her parameters  $(\lambda_i, \lambda_i^I, \lambda_i^{II})$  must be estimated simultaneously. We have also performed a pooled estimation, whose results (with the specification of all the parameters corresponding to each player position) are reported in Table 4.

	<b>Var.</b>	<b>Coef.</b>	<b>Std. Err.</b>	<b>z</b>	<b>P&gt;z</b>
<b>1)</b>	$\lambda_1$	4.893	0.346	14.160	0.000
	$\lambda_2$	6.415	0.481	13.340	0.000
<b>2)</b>	$\lambda_2^I$	4.081	0.352	11.600	0.000
	$\lambda_3$	7.377	0.605	12.200	0.000
<b>3)</b>	$\lambda_3^I$	3.567	0.193	18.530	0.000
	$\lambda_3^{II}$	1.238	0.042	29.220	0.000

**Table 4**

First note that, for all  $i$ ,  $\lambda_i > \lambda_i^I > \lambda_i^{II}$ .<sup>24</sup> In this respect, our evidence confirms a regularity already highlighted by Kübler and Weizsäcker (2004): the *perceived* payoff responsiveness of players falls as the order in reasoning (or beliefs) increases. Or, somewhat differently expressed, one can say that

<sup>23</sup>The individual estimates of the parameters for each experimental group (which are the basis of the regression presented in Table 5) are not reported here but are available upon request.

<sup>24</sup>All these differences are statistically significant at 1%.

subjects' reasoning on the rationality of others becomes more noisy as higher-order considerations are involved. As indicated in the Introduction, this phenomenon is reminiscent of what De Marzo *et al.* (2003) label “persuasion bias,” i.e. the fact that agents do not correctly anticipate that the information conveyed by an agent is partly a reflection of what this agent herself has observed.<sup>25</sup> In our context, a manifestation of this effect occurs when, say, player 3 does not take sufficiently into account that the guess issued by player 2 reflects not only her own signal but also 2's observation of 1's guess. And this will happen if, as our evidence indeed suggests, player 3's attributes a low weight (as given by  $\lambda_3^{II}$ ) to the weight that 2 associates to 1 being responsive to her own (i.e. 1's) signal.

Relatedly, a second observation following from Table 4 is that players tend to *underestimate* their predecessors' payoff responsiveness. That is,  $\lambda_2^I < \lambda_1$ ,  $\lambda_3^I < \lambda_1$  and  $\lambda_3^I < \lambda_2$ .<sup>26</sup> Moreover, player 3 also underestimates player 2's ability to take into account player 1's responsiveness ( $\lambda_3^{II} < \lambda_2^I$ ).

Let us now revisit the phenomenon of error cascades in light of these findings, proceeding in a way parallel to that of Table 3. The results are reported in Table 5, which summarizes for the present framework the error-cascade analysis reported for the HQRE framework. Since the regressions for players 1 and 2 are identical to those reported on Table 3, we focus in Table 5 on the results for player 3 alone.

	Var.	Coef.	Std. Err.	z	P>z
<b>3)</b>	$\beta_1$	-8.291	2.707	-3.06	0.002
	$\beta_2$	-2.812	1.408	-2.00	0.046
	$\beta_1\beta_2$	10.629	2.943	3.61	0.000
	$s_3$	1.558	0.368	4.24	0.000
	<i>Cons.</i>	1.124	1.344	0.84	0.403
	$dProb/d\beta_1$	-0.042	0.188	-0.22	0.824
	$dProb/d\beta_2$	0.844	0.201	4.20	0.000

**Table 5**

We find that the preceding analysis of Section 4.2 is here basically confirmed, as all the coefficients of Table 3 maintain the same sign and signifi-

<sup>25</sup> We thank the Advisory Editor in charge for pointing out the connection to this reference.

<sup>26</sup>  $\lambda_2^I$  is significantly lower than  $\lambda_1$  at 10% confidence level. All the remaining differences are statistically significant at 1% confidence.

cance. We may conclude, therefore, that the presence of error cascades is a phenomenon robust to the relaxation of equilibrium analysis that allows for inaccurate beliefs of any order on others' behavior.<sup>27</sup>

## 5 Conclusion

This paper provides experimental evidence that could prove useful for the design of dynamic models of information transmission. It shows that such models should not only accommodate the fact that players make errors, but also the possibility that such errors may accumulate, or grow, along the decision sequence. Moreover, our results can be viewed as a warning for real-world contexts where these considerations might be important. The fact that, in our simple setup – with binary signals and an exogenous guessing sequence – players' mistakes amplify quite sharply lead us to conjecture that similar phenomena could also be quite prevalent in more complex situations. In financial markets, for example, this issue should be probably taken into account by both participants and regulators.

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<sup>27</sup>As a further proof of robustness of the error cascade phenomenon, a previous version of this paper, Feri *et al.* (2010), considers a very different model where the optimality of agents' behavior is assessed in terms of their empirically observed strategies, in a way which is analogous to how players form their beliefs in the classic fictitious-play dynamics. It is remarkable that, despite the marked methodological contrast with the present model, such an alternative learning framework yields similar conclusions.

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## Supplementary Material for “Error Cascades in Observational Learning: An Experiment on the Chinos Game”

### Experimental Instructions

Welcome to the experiment! This is an experiment to study how people solve decision problems. Our unique goal is to see how people act on average; not what you in particular are doing. That is, we do not expect any particular behavior of you. However, you should take into account that your behavior will affect the amount of money you will earn throughout the experiment. These instructions explain the way the experiment works and the way you should use your computer. Please do not disturb the other participants during the course experiment. If you need any help, please, raise your hand and wait quietly. You will be attended as soon as possible.

How to get money! This experimental session consists of 20 rounds in which you and two additional persons in this room will be assigned to a group, that is to say, including you there will be a total of three people in the group. You, and each of the other two people, will be asked to make a choice. Your choice (and the choices of the other two people in your group) will determine the amount of money that you will earn after each round. Your group will remain the same during the whole experiment. Therefore, you will be always playing with the same people. During the experiment, your earnings will be accounted in pesetas. At the end of the experiment you will be paid the corresponding amount of Euros that you have accumulated during the experiment.

The game. Notice that you have been assigned a player number. Your player number is displayed at the right of your screen. This number represents your player position in a sequence of 3 (player 1 moves first, player 2 moves after player 1 and player 3 moves after players 1 and 2). Your position in the sequence will remain the same during the entire experiment. At the beginning of each round, the computer will assign to each person in your group (including yourself) either 0 tokens or 1 token. Within each group, each player is assigned 0 tokens with a probability of 25% and is assigned 1 token with a probability of 75%. The fact that a player is assigned 0 tokens or 1 token is independent of what other players are assigned; that is to say, the above probabilities are applied separately for each player.

At each round, the computer executes again the process of assignment of tokens to each player as specified above. The number of tokens that each player is assigned at any particular round does not depend at all on the assignments that he/she had in other rounds. You will only know the number of tokens that you have been assigned (0 or 1), and you will not know the number of tokens assigned to the other persons in your group. The same rule applies for the other group members (each of them will only know his/her number of tokens).

At each round you will be asked to make a guess over the total number of tokens distributed among the three persons in your group (including yourself) at the current round. The other members of your group will also be asked to make the same guess. The order of the guesses corresponds to the sequence of the players in the group. That is to say: player 1 makes his/her guess first, then player 2 makes his/her guess and, finally, player 3 makes his/her guess. Moreover, you will make your guess knowing the guesses of the players in your group that moved before yourself. Therefore, player 2 will know player 1's guess and player 3 will know both player 1 and player 2's guesses.

At each round, if you make the correct guess you will win a prize of 50 pesetas and if your guess is not the correct one you will earn nothing.