B. Taub

Dynamic Contract Theory and Related Topics in Continuous TIme
Advanced Macro Block II
Course Handbook

Summary. The course will examine how dynamc economic phenomena are affected by informational and commitment constraints.

- In a collection of heterogeneous agents with private information, how can a contract be structured to share risk but also to be incentive compatible? How does incentive compatibility requirement affect the time pattern of consumption, and the distribution of wealth?
- Theory suggests that countries should insure each other against country-specific output fluctuations. Empirically they don't. Is this explained by their inability to dynamically commit to risk sharing arrangements?
- In stock markets, if large traders receive separate signals about firm values, how should they trade on their information over time?
- Firms want to collude but cannot fully observe each others' actions in the course of that collusion. Is collusion nevertheless possible? What is the dynamic pattern of collusive output?

In order to explore these questions, we will develop and use continuous time stochastic control methods. These methods are used in both finance and economics. The course will assume prior familiarity with similar topics in discrete time and will connect to this literature often.

The course content has significant overlap with Ramon Marimon's Block I course and with Arpad Abraham's course; the emphasis here will be on revisiting these and other topics in continuous time to develop methodological building blocks. Because of the rapid pace the emphasis will be on intuitive and illustrative examples.

Assessment. The assessment will be via an individual presentation using the tools of the course to analyse a paper, and a LaTeX/Beamer summary of the presentation. Exercises will be distributed and discussed in class but these will not count toward the grade.

Lecture outline

Note that basic readings are listed here but additional references will be provided at the start of the course. The basic materials will be presented in lecture notes that will be posted online.

1. Basic continuous time stochastic control methods and tools

   (a) Review of basic deterministic differential equations and optimal control (if necessary)

   (b) Basic Brownian motion ideas, stochastic differential equations, Ito's lemma

   (c) Basic stochastic control

2. Fundamental applications

   (a) Portfolio choice and equilibrium

   - Background readings: Merton (1971), Kamien and Schwartz (1981), Radner and Majumdar (1991)

   Learning outcomes: Students will learn basic continuous-time control methods and apply them in the classic Merton model of portfolio choice.

   (b) Asset pricing and equilibrium

   - Background readings: Stokey, Lucas and Prescott (1989), initial chapters

   Learning outcomes: The Merton model will be used to model general equilibrium, noting the implications of risk on the equilibrium interest rate. Dynamic programming methods, including its mathematical underpinnings, will be introduced in order to further develop the dynamic general equilibrium model. CAPM and equity premium in continuous time.

3. Additional fundamental applications:

   (a) Martingale measure and market equilibrium

   - Background readings: Pliska (1997)

   Learning outcomes: Risky assets generate a higher return, potentially driving low-risk, low-return assets out of existence; the martingale measure converts preferences to risk-neutral and asset return processes to a common mean, leading to equilibrium with existence for assets that are high-risk, high-return. This can all be done in continuous time.

4. Dynamic contracts with risk sharing and private information in continuous time

   (a) Methodological: Dual processes

   - Primary readings: Green (1985), Sannikov (2008)

- Background readings: He and Pages (1993), Atkeson and Lucas (1992), Mele (2014), Rogerson (1985)

Learning outcomes: The connection between private-information contracts, asset markets, and related elements such as risk sharing and the distribution of wealth will be explored using both continuous-time methods and discrete-time dynamic programming methods.

5. Dynamic risk sharing contracts with limited commitment in continuous time

   (a) Methodological: Singular control

   - Primary readings: Miao-Zhang (2015), Sannikov (2007)
   - Background readings: Marcet and Marimon (2019), Kehoe and Perri (2002), Sannikov (2007), Chade and Taub (1992), Alvarez and Jermann (2000)

   Learning outcomes: Continuous time methods will be used to explore how risk-sharing contracts are affected by the requirement that the contract be structure such that no participant defects from the contract for any stochastic realisation, with applications to risk-sharing between countries.

6. Market microstructure (if time permits)

   - Primary readings: Kyle (1985), Back (1992), Boulatov and Taub (2014)
   - Background readings: Taub (2018), Glosten and Milgrom (1985)

   Learning outcomes: The basic models of the microstructure literature will be introduced using continuous time methods. The role of information in asset pricing in strategic environments will be explored.

7. Duopoly in continuous time with private information and limited commitment (if time permits)

   - Primary readings: Sannikov (2007)
   - Background readings: Abreu, Pearce and Stacchetti (1986)

   Learning outcomes: Continuous time methods will be used to explore how firms can partially enforce cartel arrangements when they cannot perfectly observe the actions of other firms.

8. Other potential topics (if time permits)

   (a) Option pricing, real options (optimal stopping), optimal stopping with learning (bandit problems), Kalman filter in continuous time

Books

Here are some books from which limited selections will be used to drive lecture material.

Kamien, Morton, and Nancy Schwartz (1981), *Dynamic Optimization*. New York: North Holland.

Karatzas, Ioannis and Steven Shreve, *Brownian Motion and Stochastic Calculus*. New York: Springer-Verlag (1991).

Malliaris, A. G. and William Brock, *Stochastic Methods in Economics and Finance*. Amsterdam: North-Holland (1990).

Oksendal, B. (2007) *Stochastic Differential Equations: An Introduction with Applications*. Sixth Edition. Springer.

Pliska (1997) *Introduction to Mathematical Finance: Discrete Time Models*. Blackwell Publishing.

Stokey, Nancy, Robert E. Lucas, Jr., with Edward Prescott (1989), *Recursive Methods in Economic Dynamics*. Cambridge: Harvard University Press.

Articles

Here is an incomplete list of some of the articles we will examine. Specific references will be provided in context.

Abreu, Dilip, David Pearce, and Ennio Stacchetti (1986), "Optimal cartel equilibria with imperfect monitoring," *Journal of Economic Theory* 39, 251-269.

Alvarez, Fernando and Urban Jermann (2000), "Efficiency, equilibrium, and asset prices with risk of default," *Econometrica* 68, 775-797.

Atkeson, Andrew and Robert E. Lucas, Jr. (1992), "On efficient distribution with private information," *Review of Economic Studies*.

Back, K. (1992), "Insider trading in continuous time," *Review of Financial Studies* 5(3), 387-409.

Chade, H. and Taub, B. (2002), "Segmented risk sharing in a continuous-time setting," *Economic Theory* 20, 645-675.

Glosten, L. and P. Milgrom (1985), "Bid, ask and transaction prices in a specialist market with heterogeneously informed traders," *Journal of Financial Economics* 1, 71-100.

Green, Edward (1987), "Lending and the smoothing of uninsurable income," in *Contractual Arrangements for Intertemporal Trade*, ed. by E. Prescott and N. Wallace. Minneapolis: University of Minnesota Press.

He, Hua and Henri Pagès (1993), "Labor income, borrowing constraints and equilibrium asset prices," *Economic Theory* 3(4), 663-696.

Harrison, Michael, and David Kreps, "Martingales and arbitrage in multiperiod securities markets," *Journal of Economic Theory* 20, 381-408 (1979).

Kehoe, Patrick, and Fabrizio Perri (2002), "International business cycles with endogenous incomplete markets," *Econometrica* 70(3), 907-928.

Kyle, Albert (1985), "Continuous auctions and insider trading," *Econometrica*

Merton, Robert (1971), "Optimum consumption and portfolio rules in a continuous time model," *Journal of Economic Theory* 3, 373-413.

Mele, Antonio (2014) "Repeated moral hazard and recursive Lagrangeans," *Journal of EconomicDynamics & Control* 42, 69–85.

Miao, Jianjun, and Yuszhe Zhang (2015), "A Duality Approach to Continuous-Time Contracting Problems with Limited Commitment," *Journal of Economic Theory.*

Radner, Roy and Mukul Majumdar (1991), "Linear models of economic survival under production uncertainty," *Economic Theory* 1, 13-32.

Rogerson, W. (1985), "Repeated moral hazard," *Econometrica* 53(1), 69-76.

Sannikov, Yuliy (2007),"Games with imperfectly observable actions in continuous time," *Econometrica*

Sannikov, Yuliy (2008), "A Continuous-Time Version of the Principal–Agent Problem," *Review of Economic Studies* (2008) 75, 957–984

Schättler, Heinz, and Jaeyoung Sung (1993), "The first-order approach to the continuous-time principal-agent problem with exponential utility," *Journal of Economic Theory.*

Taub, Bart (2018), "Inconspicuousness and obfuscation: How large shareholders dynamically manipulate output and information for trading purposes," *Annals of Finance*, 14(4), pp. 429-464 (2018)