European University Institute
Background Course on Mathematics
Academic Year 2010 -11

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Office Hours: 2:30-4:30 after classes, plus additional office hours TBA
Teaching Assistants: Michal Markun
Teaching Assistant Hours: TBA.

The (ambitious) goal of the course is to provide a mathematical background for the first year courses in microeconomics and macroeconomics

Prerequisites.

Students are supposed to be familiar with the content of the chapters listed below.
Simon C.P. and L. Blume, (1994), *Mathematics for Economists*, Norton, New York. Chapters 1-12 included, Chapters 26, 27 and 28. Appendices: all of them.

Content of the course

I. Linear algebra
Review of basic concepts. The Euclidean Space $\mathbb{R}$; vector spaces; matrices.
Linear functions. Definition; kernel and image of a linear function; nonsingular functions and isomorphisms.
Linear functions and matrices. The space $\mathcal{L}(V, U)$; from a linear function to the associated matrix; from a matrix to the associated linear function; $\mathbb{M}(m, n)$ and $\mathcal{L}(V, U)$ are isomorphic; some facts on $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$.
Systems of linear equations. Rouchè-Capelli's and Cramer's theorems.
Eigenvalues and eigenvectors. Review of basic properties of complex numbers; linear functions represented by diagonal matrices; eigenvalues and eigenvectors of a linear function; characteristic polynomial; similar matrices.
II Some topology in metric spaces.
Metric spaces. Definitions and examples; sequences; open and closed sets; compactness.
Functions. Limits; continuous functions; continuous functions on compact sets.
Banach spaces. Definition and examples; the contraction mapping theorem.
Correspondence and fixed point theorems. Continuous correspondences; the maximum theorem; fixed point theorems.
III Differential calculus in Euclidean spaces
Partial derivatives and directional derivatives.
Differentiability. Total derivative and differentiability; total derivatives in terms of partial derivatives.
Some theorems. The chain rule; mean value theorem; a sufficient condition for differentiability; a sufficient condition for equality of mixed partial derivatives; Taylor's theorem for real valued functions.
Implicit function theorem. Some intuition; functions with full rank square Jacobian; the inverse function theorem; the implicit function theorem.
IV Nonlinear programming.
Concavity. Convex sets; different kinds of concave functions; relationships among different kinds of concavity; Hessians and concavity.
Maximization Problems. The case of inequality constraints; Kuhn-Tucker theorems; on uniqueness of the solution; the case of equality constraints: Lagrange theorem; the case of both equality and inequality constraints; main steps to solve a (nice) maximization problem; some difficulties and some solutions; the implicit function theorem and comparative statics analysis; the envelope theorem and the meaning of multipliers.

V Discrete dynamics.

Difference equations. Ordinary difference equations; existence and uniqueness; linear difference equations; second-order equations with constant coefficients; $n-th$ order equations with constant coefficients; the method of variation of parameters; systems of difference equations.

Discrete dynamic programming under certainty. A deterministic model of optimal growth; the principle of optimality; the case of bounded returns.

Reading material

Mandatory. Villanacci, A., (2010), Notes for the Course on Mathematics.
Notes will be available on line.

Main sources of the Notes.
I. Linear algebra

Lang S. (1971), *Linear Algebra*, second edition, Addison Wesley, Reading.

Lipschutz, S., (1991), *Linear Algebra*, 2nd edition McGraw-Hill Company, New York.

Smith, L.,(1992), *Linear Algebra*, 2nd edition, Springer-Verlag, New York, NY.

II Some topology in metric spaces.

Lipschutz S. (1965), *General Topology,* McGraw-Hill Company, New York.

Ok. E. A., (2007), *Real Analysis with Economic Applications*, Princeton University Press, Princeton NJ.

Simmons, G. F., (1963), *Introduction to Topology and Modern Analysis*, McGraw-Hill, New York.

III Differential calculus in Euclidean spaces

Apostol, T. M., (1974), Mathematical Analysis, 2nd edition, Addison-Wesley Publishing Company, Reading, MA.

IV Nonlinear programming.

Cass D., (1991), Nonlinear Programming for Economists, University of Pennsylvania, Class Notes.

V Discrete dynamics.

Azariadis, C., (2000), *Intertemporal Macroeconomics*, Blackwell Publisher Ltd, Oxford, UK.

Galor, O., (2007), *Discrete Dynamical Systems*, Springer-Verlag, Berlin.

Stokey, N. C., Luca, E. C. (with Prescott, E. C.), (1989), *Recursive methods in Economic Dynamics*, Harvard University Press, Cambridge, MA.

Sydsaeter, K., (1981), *Topics in Mathematical Analysis for Economists*, Academic Press, London, UK.

Teaching and Review Sessions

The course is organized on 14 lectures and 6 review sessions. Review sessions will be devoted to the discussion and solutions of exercises.

Exam Requirements

There will be a final exam, weighted 80 per cent of the final grade, and 5 homework assignments, weighted 20 per cent.