Housing, Financial Crises and Macroprudential Regulation: The Case of Spain*

Preliminary and incomplete

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Abstract

Should macroprudential regulators who are concerned about preventing housing crises regulate mortgage borrowers or banks? We address this question in a dynamic stochastic general equilibrium (DSGE) model in which both banks and mortgage borrowers use non-contingent debt and can default. The equilibrium mortgage credit in the economy is determined either by a leverage constraint on banks or a loan-to-value (LTV) constraint on borrowers. Large drops in the house price (housing crises) occur endogenously and can lead to a run on the banking sector (banking crises). We calibrate the model to the Spanish economy, which experienced a severe financial crisis despite having stringent pre-crisis bank regulation. We find that both lower LTV ratios on borrowers and higher capital requirements on banks reduce the frequency of financial crises and mortgage default rates. The latter are less sensitive to house prices and therefore amplify business cycle volatility less. We also consider loan loss provisioning rules and find that a capital requirement that is high when expected default rates are high can reduce the level and cyclicality of default rates, but does not contribute to reducing the frequency of banking crises.

Keywords: House Prices, Credit Risk, Systemic Risk, Financial Crisis, Macroprudential Policy, Financial Stability.

JEL Classification: E37; E44; G21; G28.

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1 Introduction

Between 2008 and 2016, the Spanish economy witnessed a financial crisis unprecedented in its modern history. At the center of this financial crisis was a housing price bust intertwined with a severe disruption in the Spanish banking sector. What macroprudential policies to take to prevent such financial crises for the future has become a significant part of the policy agenda of financial regulators.

Our goal in this paper is to quantitatively assess the effectiveness of three different types of macroprudential policies in reducing the frequency and severity of financial crises resembling the one recently undergone in Spain. These policies are: (i) minimum bank capital requirement, (ii) provisioning against expected credit losses (aka dynamic provisioning), and (iii) maximum LTV ratio restrictions.

To achieve this goal, we first build a macroeconomic model to understand the interaction among house prices, mortgage loans, and bank runs in the context of the Spanish financial crisis. We calibrate the model to match dynamics of key financial and real variables during the crisis, including house prices, total output, leverage and credit spreads of both banks and households. With the calibrated model, we conduct counter-factual policy experiments with the aforementioned macroprudential policies and evaluate their effects in moderating housing and banking crises.

We study a non-linear DSGE model with heterogeneous agents, a housing market and a banking sector. Our model exhibits three key features: endogenous default on mortgage loans, dual constraints in mortgage credit, and endogenous banking crises in the form of bank runs. Mortgage borrowers can default on their loans, in which case the banks seize the houses of the defaulted households and foreclose these houses at the current price in the housing market. Importantly, the mortgage market is disciplined by two constraints. On the one hand, mortgage lending is restricted by a leverage constraint faced by banks as a result of a bank capital requirement. That is, given the level of bank equity, banks' lending ability is constrained. On the other hand, mortgage borrowers face a borrowing constraint, with a restriction on the maximum LTV ratio they can adopt. Whether the lending constraint or the borrowing constraint binds depends on the state of the economy and therefore varies over time. Banks experience a run from the depositors whenever the liquidation value of the banks in a run is lower than the value of their outstanding debt.

The model has two exogenous shocks: A productivity shock and a shock to the recovery values of mortgages and deposits. The productivity shock works through the real side of the economy and affects house prices primarily through housing demand, by lowering the income of households. The shock to the recovery values works through credit spreads and affects house prices through mortgage supply, by lowering the net worth of banks. Independently of whether the LTV constraint or the lending constraint binds, there is a financial accelerator effect: House price decreases tighten both constraints, reducing the equilibrium quantity of mortgages, which reduces house prices further. However, the quantitative strength of the financial accelerator depends on which constraint binds.

Bank runs occur more frequently if bank leverage is high or bank profitability low, e.g. due to a negative shock to recovery values on mortgages. A bank run leads to a collapse of the mortgage market, which lowers the housing demand of the borrowers substantially. The house price decreases dramatically, lowering the liquidation value of banks further. In that sense, the model is capable of generating intertwined housing and banking crises of the kind that happened in Spain.

We find that a higher capital requirement can actively eliminate bank runs in this economy. This substantially reduces the standard deviation of consumption of both borrowers and lenders. Moreover, a higher capital requirement leads to lower default rates on mortgages, since it reduces the aggregate amount of mortgage credit and hence, aggregate household leverage. Finally, a higher capital requirement is redistributive, since it transfers wealth from lenders to borrowers by reducing mortgage credit. This increases consumption of the borrowers and decreases that of the lenders in aggregate.

Provisioning against expected credit losses does not substantially affect leverage dynamics of either borrowers or banks. It does however lead to a reduction in average default rates and less cyclical default rates. Moreover, it leads to a slight decrease in recovery rates, because the average defaulted borrower will be worse, and a slightly more procyclical recovery rate. Overall, the macroeconomic impact of dynamic provisioning is limited.

Finally, imposing tighter LTV constraints can also reduce both default rates and the frequency of bank runs. However, since LTV constraints depend more strongly on house prices than bank capital requirements, they amplify consumption volatility of borrowers and house price volatility more. As was the case for higher capital requirements, tighter LTV constraints distribute wealth from lenders to borrowers by forcing lenders to save relatively more.

Our paper is closely related to the literature that studies financial distress and their real effects. There are two branches in this literature. The first one explores the financial accelerator effect, where weak balance sheet conditions of financial or non-financial firms undermine their access to credit, which impairs their balance sheet condition further, creating a negative feedback loop and amplifies business cycle fluctuations. This line of research is pioneered by Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). Since the global financial crisis, it has been an important mechanism in many research papers that try to link financial disruptions and the real effects of financial crises, such as Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), Boissay, Collard, and Smets (2016). The second branch studies bank run events, pioneered by Diamond and Dybvig (1983). There are two slightly different ways to model bank runs, one is as in Gertler and Kiyotaki (2015) and Gertler, Kiyotaki, and Prestipino (2016), where bank creditors suddenly stop rolling over their short-term investment in banks. The other way is to model a liquidity run due to mismatch of the liquidity from illiquid assets and liquid liabilities, such as in Martin, Skeie, and von Thadden (2014). Our model includes both a financial accelerator effect and the bank run mechanism as in Gertler and Kiyotaki (2015). One thing that this literature is silent about is the role of the house market boom and bust in the financial crisis. Our paper adds to this literature by adding the interaction of the housing market and the financial market.

There is an extensive empirical literature on financial crises. Examples include, but are not limited to Reinhart and Rogoff (2009), Jordà, Schularick, and Taylor (2011), Gorton and Metrick (2012), Laeven and Valencia (2012), Schularick and Taylor (2012), Mendoza and Terrones (2012), Romer and Romer (2017), Jordà, Richter, Schularick, and Taylor (2017), Krishnamurthy and Muir (2017) and Muir (2017). Relative to this literature, we document asset price and leverage dynamics during the Spanish financial crisis and study to what extent these dynamics are driven by financial as opposed to real shocks.

This paper is also related to the literature on macroprudential regulation. This literature explores how regulators can ensure that the market equilibrium internalizes the pecuniary externalities that arise if there are price-sensitive borrowing constraints and endogenous capital prices. Examples include Lorenzoni (2008), Bianchi (2011), Garcia-Macia and Villacorta (2016), Farhi and Werning (2016), Korinek and Simsek (2016), Dávila and Korinek (2017) and Gersbach and Rochet (2017). We discuss different regulatory policies in a concrete example, namely in the context of the Spanish housing crisis.

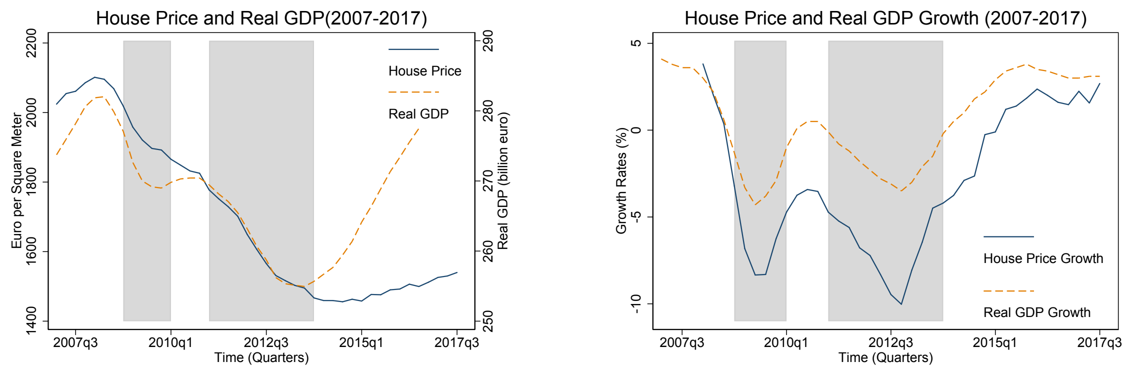
Finall, this paper relates to the literature which studies the interaction between housing crises and financial crises, e.g. Justiniano, Primiceri, and Tambalotti (2015) and Guerrieri and Uhlig (2016). Many of these papers study the US housing crisis, whereas we focus on the Spanish housing crisis.

The rest of the paper is organized as follows. In Section 2, we document some key dynamics of the financial crisis in Spain which we aim to match with our quantitative model. A description of the model economy can be found in Section 3. In Section 4, we provide a discussion of the binding constraints in the mortgage market. The calibration and model fit are discussed in Section 5. In Section 6, we conduct counter-factual policy experiments on three macroprudential policies. Section 7 concludes.

2 Financial Crisis & Macroprudential Regulation in Spain

In this section, we document important dynamics of the Spanish Financial Crisis and the pre-crisis macroprudential regulation policy in Spain, which we aim to match later with our banking crisis model and policy analysis. We look at aggregate data on GDP, house price, bank leverage, household leverage, and credit spread in Spain between 2007 and 2017. A detailed data description can be found in Appendix A.

Figure 1: House Prices and Growth Rate of GDP and House Prices in Spain



Source: House price: Spain Ministry of Construction. GDP growth: Eurostat.

2.1 The Spanish Financial Crisis

2.1.1 Housing Crisis and Economic Downturn

From mid 1990s, the real estate price in Spain embarked on an expansionary path, with the nominal house prices in Spain soaring 300% between 1995 and 2007 (see, for instance, Martín, Moral-Benito, and Schmitz (2018)). In the aftermath of the global financial crisis, the Spanish real estate market collapsed. As shown in the left panel of Figure 1, house price dropped from per square meter €2100 in 2008Q2 to €1450 in 2015Q1, putting an end to the Spanish housing boom.

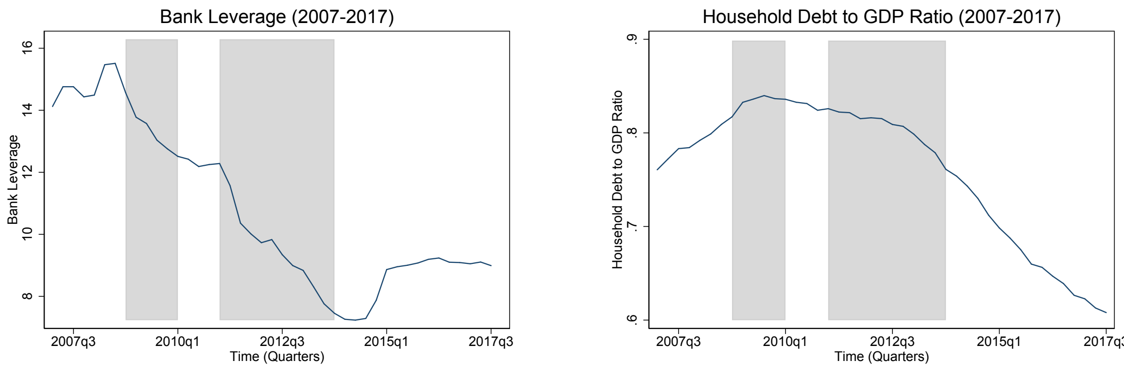
Following the conventional definition of economic recession as two consecutive quarters of decline in real GDP, there were two economic recessions in Spain: 2008Q4-2010Q1 and 2011Q1-2013Q4 (marked as shaded time spans in the figures), where the first recession corresponds to the 07-09 Global Financial Crisis, and the second one is corresponded with the 2009-2013 European Sovereign Debt Crisis.

From the right panel of Figure 1 we observe an interesting pattern: the GDP growth[1] (blue line) and house price growth (dashed orange line) have very similar shapes, suggesting a strong positive correlation between house price and real GDP in Spain during this time. However, the decrease in house price is much stronger than real GDP, with the lowest growth rate being -10% in 2012Q4.

In our model, occasional strong negative financial shocks (in the form of liquidity shocks) cause house price busts and real effects. We do not, however, model house price booms explicitly in our theoretical framework, as our main focus is to capture a financial crisis subsequent to a severe house price bust.

[1] The growth rates of GDP and house price are calculated as the percentage change compare to the same quarter in the previous year.

Figure 2: Bank Leverage and Household Leverage in Spain



Source: Aggregage bank balance sheet data and household debt: ECB Statistical Data Warehouse. Nominal GDP: Eurostat.

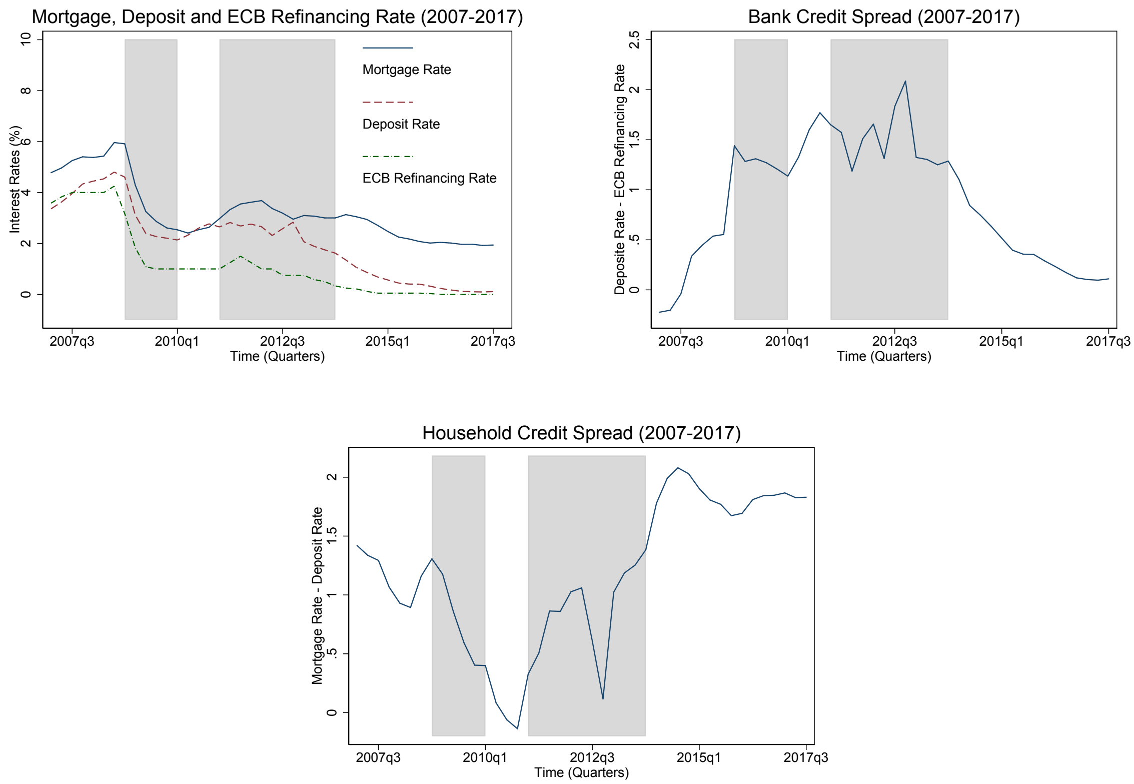
2.1.2 Banks and Households in the Financial Crisis

Leverage Over the course of the Spanish Financial Crisis, significant deleveraging took place in the banking sector. This trend can be seen in the left panel of Figure 2, where we define bank leverage as total asset over total equity. The average bank leverage in Spain decreased from the pre-crisis level of more than 15 in 2008 to around 7 in 2014. A simultaneous decrease in total bank asset and increase in bank equity contributed to this trend.

Deleveraging happened not only in the banking sector, but also in the household sector during the financial crisis. Figure 3 shows the household debt to GDP ratio as a measure of household leverage. Compare to the deleveraging of banks on the left, the progress of household deleveraging is much slower.

Credit Spread Another important feature of the financial crisis lies in credit spread, which captures the probability of default of the debt issuers. We calculate the bank credit spread is as the difference between the annual interest rate on bank deposit and the ECB refinancing rate (interest rate on the bulk of liquidity provided to the banking system by the ECB), and the household credit spread is calculated as the difference between the annual interest rate on bank deposit and the annual interest rate on *newly issued* mortgage loans. From Figure 3 we can see clearly that the credit spread of banks increased during the financial crisis, especially during the periods of economic recessions, suggesting that banks had to pay higher risk premium to compensate for higher risk of default. However, household credit spread decreased, suggesting a lower risk premium paid by households on mortgage loans. This can be explained by the change in credit quality of mortgage borrowers. During bad economic conditions, banks increased their lending standard and restrict loans to high-risk borrowers. Therefore the average probability of default on mortgage loans is lower, and the credit spread is lower during the crisis.

Figure 3: Deposit and Mortgage Rates and The Spread in Spain



Source: Nominal interest rate on household deposits and mortgage loans: ECB Statistical Data Warehouse. ECB refinancing rate: ECB statistics, official interest rates.

2.2 Macroprudential Policy in Spain

In July 2000, Banco de Espana, the Spanish Central Bank and banking supervisor, introduced dynamic provisioning (DP) in Spain, which requires banks to provision against *expected* loan losses (Saurina (2009) provides a detailed description of the policy rule). In the prevailing standards, loss identification was based on the "triggering events", e.g. decrease in collateral values, past-due status. The obvious drawback of this accounting rule is that loss recognition is too late, creating a pro-cyclical effect: banks provision less (lower capital buffer) during the boom when credit loss is low and provision more (higher capital buffer) during recessions when credit loss surges. On the contrary, under the dynamic provisioning regime, banks identify potential credit losses earlier and build up buffers in good times that can be used in bad times, creating an anti-cyclical effect, similar to the capital conservation buffer and counter-cyclical buffer requirement in Basel III.

In 2014, the International Accounting Standards Board (IASB) published IFRS 9 (the accounting standard for financial instruments), which includes a new accounting standard for provisioning against expected credit losses (see Cohen and Edwards (2017)). On that account, Spain is advanced in financial macroprudential regulation before the financial crisis.

3 Model

In this section, we introduce a DSGE model of the Spanish economy. The key features of the model are that both banks and households are leveraged and can default on their liabilities.

3.1 The Model Environment

We study a closed economy with discrete time and infinite horizon. The economy is populated by patient (fraction μ) and impatient households (fraction $1-\mu$). Households consume nondurables and housing services. They invest in new housing and either borrow from or lend to banks. Banks take deposits from households and make loans to households in the form of mortgages. To keep things simple, total house supply is fixed to $\mathcal{H}$, so that house prices are entirely demand driven.

We denote variables related to the patient households with "P", impatient households with "I", and banks with "B".

Figure 4 gives an overview of how resources flow in the economy.

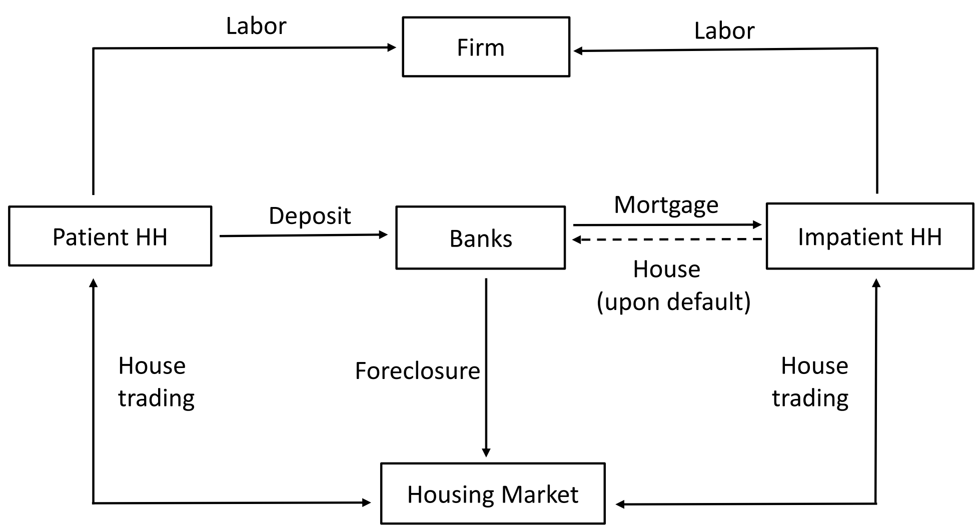


Figure 4: Overview of the Economy in equilibrium.

3.2 Households

3.2.1 Preferences and Housing

Preferences Households of type J, $J \in \{P, I\}$, maximize utility

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\beta^J\right)^t U^J \left(C_t^J, H_t^J\right) \right],$$

with the instantaneous utility function given by

$$U^J \left(C_t^I, H_t^I\right) = \chi^J \frac{(C_t^J)^{1-\sigma} - 1}{1-\sigma} + (1-\chi^J) \frac{(H_t^J)^{1-\sigma} - 1}{1-\sigma}.$$

C_t^J is consumption of household J in period t, H_t^J is housing at the beginning of period t, β^J is the discount factor, σ is the risk aversion and χ^J the weight of consumption in the utility function. Patient households and impatient households differ along two dimensions: i) patient households discount future utility less than impatient households, i.e. $\beta^P > \beta^I$. ii) patient households have a weaker preference for housing than impatient households: $\chi^P > \chi^I$.

Housing Each type of representative household holds a portfolio of houses H_{it}^J, with

$$H_t^J = \int_i H_{it}^J di$$

denoting the total housing stock of household J at the beginning of period t. P_t is the aggregate house price expressed in units of the consumption good. The idiosyncratic price of house i, P_{it}, is the product of the aggregate house price and an idiosyncratic shock ε_{it}:

$$P_{it} = P_t \varepsilon_{it}.$$

ϵ_{it} is distributed lognormal with standard deviation σ^ε and mean $-\frac{1}{2}(\sigma^\varepsilon)^2$, such that ε_{it} has a mean of 1, and the expected house price is equal to the average price P_t. Houses depreciate at rate δ. At the end of period t, the households sell off their housing stock $(1-\delta)H_t^J$ at price P_t and make a new purchase H_{t+1}^J for the next period at the same price.

3.2.2 Retail Banking Services

There is no direct financial market between patient and impatient households. Households can lend to banks in the form of bank deposits D_t^J, and borrow from banks in the form of mortgage loans M_t^J. In equilibrium, patient households are depositors and impatient households are mortgage borrowers.

Deposits Given that there is no bank run, households receive a non-contingent gross return R_t^D in period t on the deposit they made in period $t-1$, D_{t-1}. In the event of a run, only a share X_t^D of their deposit (including interest) can be recovered from the liquidation of the banks. The recovery rate of deposits X_t^D can be expressed as following:

$$X_t^D \begin{cases} = 1 & \text{without bank run,} \\ < 1 & \text{with bank run.} \end{cases}$$

Mortgages A representative household borrows a portfolio of mortgages $M_t^J = \int_i M_{it}^J di$ from banks. Each mortgage loan M_{it}^J is secured by a corresponding house H_{it}^J. Households may default on mortgages. If a household chooses not to default, a gross interest rate R_t^M on the mortgage loan is paid to the bank. If the household chooses to default, the bank will seize the house that serves as collateral and sells it at price P_{it} to make up the loss on the loan. If the proceeds from selling the house are not sufficient to cover the loss, the bank cannot seek the deficiency balance from the borrower, i.e. the recovery is limited to the value of the house. The recovery rate of the defaulted mortgage loan M_{it}^J is given by:

$$X_{it}^M = A_t \frac{P_{it} H_{it}^J (1-\delta)}{R_t^M M_{it}^J}.$$

A_t captures liquidation costs, with

$$\begin{aligned} A_t &= \min(\hat{A}_t, 1) \\ \ln \hat{A}_t &= \rho^A \ln \hat{A}_{t-1} + \varepsilon_t^A, \text{and } \varepsilon_t^A \sim N(0, \nu^A). \end{aligned}$$

In equilibrium, it will be optimal for the households to default on a mortgage if the value of the house is less than the outstanding liability to the bank, i.e. $X_{it}^M \leq 1$.

Borrowing Constraint The amount of mortgage loan impatient households are able to borrow is constrained by the value of their house, i.e. the collateral:

$$M_{t+1}^I \leq \kappa P_t H_{t+1}^I$$

where κ captures the maximum loan-to-value ratio that a household is allowed to take.

3.2.3 Capital Income and Government Transfer

Patient households own the consumption goods production firms and banks. They invest $\bar{E}_t^P$ as equity to banks every period. Each period they receive profit $\Pi_t^{F,P}$ from the firms and $\Pi_t^{B,P}$ dividends from banks. Capital income is taxed at rate τ.

Impatient households do not invest in bank equity and have no capital income, i.e. $\bar{E}_t^I = \Pi_t^{F,I} = \Pi_t^{B,I} = 0$, but receive a transfer T_t^I from the government each period. Patient households receive no transfer, i.e. $T_t^P = 0$.

3.2.4 Budget Constraint and Aggregation

Households provide labor inelastically at wage W_t, which is denoted with a bar above the labor supply variable, $\bar{L}^J$. The budget constraint of household $J \in \{P, I\}$ is given by

$$\begin{aligned} C_t^J + P_t \left[H_{t+1}^J - H_t^J(1-\delta)\right] + \left[1 - (1 - X_t^M)\Phi_t^M\right] R_t^M M_t^J + D_{t+1}^J + \bar{E}_t^J \\ = W_t \bar{L}^J + M_{t+1}^J + X_t^D R_t^D D_t^J + (1-\tau)(\Pi_t^{F,J} + \Pi_t^{B,J}) + T_t^J, \end{aligned}$$

where

$$X_t^M = \int_{X_{it}^M < 1} X_{it}^M dF(X_{it}^M),$$

is the expected recovery value in default with $F(X)$ denoting the cumulative distribution function of X. Φ_t^M is the mortgage default rate.

3.3 Banks

Banks function as financial intermediaries in the economy, who take deposits from some households and make mortgage loans to other households.

3.3.1 Bank Problem

Objective Function Banks are owned by patient households. As such, their objective is to maximize the discounted expected future dividend payouts at the discount rate of the patient households:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left[\underbrace{\frac{U_c(C_t^P, H_t^P)}{U_c(C_0^P, H_0^P)}}_{\text{stochastic discount factor}} \left[\beta^P (1-\eta)\right]^t \Pi_t^B \right] \right\},$$

where η is a constant exit probability and Π_t^B is the dividend payout. n_t^B is the net worth of an incumbent bank in period t, which is given by:

$$n_t^B = \tilde{R}_t^M M_t^B - R_t^D D_t^B.$$

with $\tilde{R}_t^M = \left[1 - (1 - X_t^M)\Phi_t^M\right] R_t^M$.

Entry and Exit As in Gertler and Kiyotaki (2015), every period there is a probability η that a bank exits the economy. This assumption makes sure that banks do not accumulate equity infinitely. To keep the aggregate number of banks constant, new banks enter at the same rate as bank exit.

Balance Sheet Banks face a balance sheet constraint, which requires that the mortgage loan (M^B_{t+1}) on the asset side of the bank must equate to the sum of deposits (D^B_{t+1}) and bank net worth (N^B_t) on the liability side of the bank:

$$M^B_{t+1} = n^B_t + D^B_{t+1}.$$

Capital Requirement / Lending Constraint Banks are subject to runs from depositors, and are thus regulated. We consider bank regulation in the form of bank capital requirement. If the capital requirement is binding, it means banks are constrained in their ability to finance lending with household deposits. Banks are required to keep a minimum asset to equity ratio, Γ_t, with :

$$\Gamma_t = \underline{\Gamma} + \gamma \left(\mathbb{E}_t \left[\Phi_{t+1}(1 - X^M_{t+1}) \right] \right).$$

This minimum capital ratio has two components. $\underline{\Gamma}$ is the minimum static capital ratio that a bank must satisfy. The second is a dynamic component that captures provisioning against expected future credit losses. That is, banks' capital ratio also depends on the expected losses on mortgage loans.

3.3.2 Bank Runs

Existence of the Bank Run Equilibrium We model bank runs as coordination failure of the bank creditors, i.e. the patient households, as in Gertler and Kiyotaki (2015).

In a bank run, the assets of the banks get liquidated. Patient households receive the liquidation value of the banks subject to some liquidation costs. The recovery rate on deposits for patient households when a bank run happens is given by:

$$X^D_t = A_t \frac{\tilde{R}^{M*}_t M^B_t}{R^D_t D_t}, \tag{3.1}$$

where $\tilde{R}^{M*}_t$ is the return on mortgages in a bank run. $\tilde{R}^{M*}_t$ depends on the house price in a bank run P^*_t. The liquidation cost captures asset liquidity risk, i.e. periods when banks face a high cost if they have to sell assets within a short period of time.

Transition Matrix A bank run equilibrium exists when:

$$X^D_t < 1. \qquad \text{(Bank-run Condition)}$$

That is, whenever patient households cannot fully recover their deposit from the liquidated assets of the bank when a run happens, there can be a bank run. Following Gertler and Kiyotaki (2015), bank runs occur with probability

$$\pi_t^{NoRun \to Run} = (1 - \min(\mathbb{E}_t \left[X_{t+1}^D\right], 1))^\zeta$$

We consider the case of a bank run on the whole banking sector. Once a bank run happens, banks' assets get liquidated and banks can no longer borrow or lend. After the bank run, banks reenter the economy with exogenous probability $\pi^{Run \to NoRun}$. The full transition matrix between the run state and the no run state is hence given by

$$\pi_t = \begin{bmatrix} 1 - \pi_t^{NoRun \to Run} & \pi_t^{NoRun \to Run} \\ \pi^{Run \to NoRun} & 1 - \pi^{Run \to NoRun} \end{bmatrix}.$$

3.4 Rest of the Model

Production Consumption goods producers hire labor from households to produce consumption goods. The production technology of the consumption goods producer is given by:

$$Y_t = Z_t L_t^\alpha,$$

where log productivity follows an AR(1) process: $\ln Z_t = \rho^Z \ln Z_{t-1} + \epsilon_t^Z$, and $\epsilon_t^Z \sim N(0, \nu^Z)$. Firms maximize profits:

$$\max_{L_t} \Pi_t^F = Z_t L_t^\alpha - W_t L_t,$$

which yields a first order condition:

$$W_t = \alpha Z_t L_t^{\alpha-1}. \tag{3.2}$$

During a bank run, a fraction of output gets lost. We model the output loss as a reduction in labor supply from L_t to $(1-\xi)L_t$.

Government The government runs a balanced budget every period. I.e. the capital income tax from the patient households equals the transfers made to the impatient households:

$$\tau \left(\Pi_t^{B,P} + \Pi_t^{F,P}\right) = T_t^I. \tag{3.3}$$

3.5 Equilibrium

3.5.1 Market Clearing

There are five markets in our model economy: consumption goods market, labor market, housing market, deposit market and mortgage market. The financial markets (deposit and mortgage) are only active in the no-run state.

Market Clearing in No-Run State In the case without bank runs, all five markets are active:

$$Y_t = \mu C_t^P + (1-\mu)C_t^I + \delta P_t \mathcal{H}, \tag{3.4}$$
$$L_t = \mu \bar{L}_t^P + (1-\mu)\bar{L}_t^I, \tag{3.5}$$
$$\mathcal{H} = \mu H_{t+1}^P + (1-\mu)H_{t+1}^I, \tag{3.6}$$
$$D_{t+1}^B = \mu D_{t+1}^P, \tag{3.7}$$
$$M_{t+1}^B = (1-\mu)M_{t+1}^I. \tag{3.8}$$

Market Clearing in Run State In the case of bank-run, the whole banking sector gets liquidated and stays excluded from the economy. Therefore, there are no credit markets in the economy. The market clearing conditions in a bank-run state are:

$$Y_t = \mu C_t^P + (1-\mu)C_t^I + \delta P_t \mathcal{H}, \tag{3.9}$$
$$\mu \bar{L}_t^P + (1-\mu)\bar{L}_t^I = L_t, \tag{3.10}$$
$$\mathcal{H} = \mu H_{t+1}^P + (1-\mu)H_{t+1}^I. \tag{3.11}$$

3.5.2 Equilibrium

An equilibrium is a sequence of prices

$$\{P_t, R_t^D, R_t^M, W_t\}_{t=0}^{\infty}$$

and allocations

$$\{C_t^P, C_t^I, H_{t+1}^P, H_{t+1}^I, D_{t+1}^B, D_{t+1}^P, M_{t+1}^B, M_{t+1}^I, L_t\}_{t=0}^{\infty}$$

such that, if the the economy is in a no-bank run state according to equation (Bank-run Condition), markets clear as described in equations 3.4 to 3.8 and agents solve their respective optimization problems described by equations C.5 to C.8 for the patient household, C.1 to C.4 for the impatient household, the Balance Sheet Constraint and the Bank Capital Requirement for the bank and equation 3.2 for the non-durable goods producer. If the economy is in a run state according to equation (Bank-run Condition), markets clear according to equations 3.9 to 3.11 and agents solve the optimization problems described by equations C.1 to C.8.

4 Theory

In the model, it will be either the borrowing constraint of households or the lending constraint of banks which is binding in the mortgage market. The financial accelerator effect is active in both cases, but it operates differently.

4.1 Binding Borrowing Constraint

If the borrowing constraint is binding, the equilibrium mortgage allocation is given by

$$M_{t+1} = \kappa P_t H^I_{t+1}.$$

A marginal increase in the house price increases mortgage credit by $\kappa H^I_{t+1} + \kappa P_t \frac{\partial H^I_{t+1}}{\partial P_t}$. There is a negative income effect, a negative substitution effect and a positive wealth effect which determine the sign of $\frac{\partial H^I_{t+1}}{\partial P_t}$. If the income and substitution effects dominate, then $\frac{\partial H^I_{t+1}}{\partial P_t} < 0$, which will reduce the strength of the financial accelerator effect.

4.2 Binding Lending Constraint

If the lending constraint is binding, the equilibrium quantity of mortgages is given by

$$M_{t+1} = \psi_t N^B_t.$$

Constant capital requirement Consider first the case of a constant capital requirement, $\psi_t = \psi$. A marginal increase in the house price hence increases mortgage credit ceteris paribus by $\psi \frac{\partial N^B_t}{\partial P_t}$, with

$$\frac{\partial N^B_t}{\partial P_t} = (X^M_t - 1)\frac{\partial \phi^M_t}{\partial P_t} + \phi^M_t \frac{\partial X^M_t}{\partial P_t}.$$

Hence, an increase in the house price has two effects on bank net worth: First, it decreases the default rate $\frac{\partial \phi^M_t}{\partial P_t} < 0$, which increases the net worth of the banks since $X^M_t < 1$. Second, it increases the recovery value of banks conditional on a default, $\frac{\partial X^M_t}{\partial P_t} > 0$, which also increases the net worth of banks. Overall, $\frac{\partial N^B_t}{\partial P_t} > 0$, such that a higher house price will increase the mortgage credit. Importantly, fluctuations in bank net worth translate into fluctuations in mortgage credit by a factor $\psi \gg 1$.

Dynamic Capital Requirement In the case of a dynamic capital requirement, the regulator can offset or amplify the effect of fluctuations in house prices on mortgage credit:

$$\frac{\partial M_{t+1}}{\partial P_t} = \frac{\partial \psi_t}{\partial P_t} N^B_t + \psi_t \frac{\partial N^B_t}{\partial P_t}.$$

Parameter	Description	Value	Target / Source
		Households	
β^P	Discount factor of patient households	0.9949	Annual real deposit rate = 0.3%
β^I	Discount factor of impatient households	0.9919	Annual real mortgage rate = 1.1%
χ^P	Patient HH consumption weight	0.9625	Value added of real estate activities/GDP = 5%
χ^I	Impatient HH consumption weight	0.6750	Share of houses with mortgage loans = 40%
μ	Share of patient households	0.6	Share of homeowners w/o mortgage = 60%
σ	Risk aversion	2	Kaplan, Mitman, and Violante (2017)
δ	Depreciation of housing stock	0.00625	Favilukis, Ludvigson, and Van Nieuwerburgh (2017)
κ	Borrowing constraint	0.8	Maximum LTV ratio = 80%
ν^ϵ	Volatility, idiosyncratic house value shock	0.25	Annual default rate = 2%
		Banks	
$\underline{\Gamma}$	Minimum bank capital requirement	0.08	Maximum bank leverage = 12.5
γ	Dynamic provisioning parameter	0	Correlation of bank leverage to GDP = 0.6621
η	Bank exit rate	0.1	Bank asset to quarterly GDP ratio = 9.761
$\pi^{Run \to NoRun}$	Bank run persistence	1/13	Average run length = 3.25 yrs
ζ	Bank run sensitivity to recovery	0.5	Gertler and Kiyotaki (2015)
		Production and Government	
α	Labor share of output	0.572	Labor share of output = 57.2%
ρ^Z	Autocorrelation, productivity	0.9704	Autocorrelation of detrended real GDP = 0.9704
ν^Z	Volatility, productivity	0.0145	Unconditional volatility of detrended real GDP = 6.13%
ρ^A	Autocorrelation, aggregate liquidation shock	0.95	Autocorrelation of bank equity = 0.9315
ν^A	Volatility, aggregate liquidation shock	0.01	Volatility of bank equity = 17.56%
τ	Capital income tax rate	0.2	Capital income tax rate = 20%
ξ	Labor supply loss in bank run	0.1	Unemployment Increase in Spain

Table 1: Parameters of the baseline model

If $\frac{\partial \psi_t}{\partial P_t} = -\frac{\psi_t}{N_t^B}\frac{\partial N_t^B}{\partial P_t}$, the regulator can offset the effect of fluctuations in bank net worth on mortgage credit completely.

4.3 Bank Runs

In general, a binding borrowing constraint implies $M_{t+1} < \psi_t N_t^B$, unless both constraints are binding at the same time. This means that banks have excess leverage capacity, which reduces the likelihood of a bank run.

5 Calibration

In this section, we describe the calibration of the model and explore the dynamics of key financial and real variables during the Spanish financial crisis. Our goal is to characterize quantitatively the behavior of the Spanish economy in the recent financial crisis. We solve for both the no-run and run equilibrium using global nonlinear methods. A detailed description of the solution algorithm can be found in Appendix D.

5.1 Parameters

The choice of parameters values of the model are listed in Table 1. Each model period corresponds to one quarter. For the baseline calibration, we use Spanish data between 1997Q3 and 2017Q3. A detailed description of the dataset can be found in Appendix A.

There are overall 19 parameters in the model to be calibrated. We begin with the parameters related to the household sector. The discount factors of the households, β^P and β^I, are set to match the average real interest rate on deposit and mortgage loan (new business) in Spain. The consumption weight in utility for impatient households, χ^I, is set to match the share of houses financed by mortgage loans. The consumption weight for patient households, χ^P, is calibrated to match the value added of real estate activities as share of GDP. The share of homeowners without mortgage loans in Spain is 60%, therefore the share of patient households μ is set to 0.6. The households risk aversion is set at $\sigma = 2$, a standard value in the literature. The depreciation rate of housing is calibrated to 2.5% per year following the literature. The borrowing constraint parameter, κ, is set to match the maximum LTV ratio in 1997-2017, which is 80%. Finally the volatility of idiosyncratic house value shock is set to match an annual default rate of 2%.

The parameters related to banks are calibrated with the following strategies. The constant component of bank capital requirement $\underline{\Gamma}$ is set to be 6.5%, corresponding to the maximum bank leverage of 15.5 in 1997-2017. The bank exit rate η is set to match the average bank asset to quarterly GDP ratio of 9.761. The bank run persistence rate π is set to match an average length of bank runs of 3.25 years[2].

The production function parameter α is chosen to match the labor share of output of 57.2%. The autocorrelation and volatility of productivity are calibrated to match the data counterpart of detrended real GDP. And the autocorrelation and volatility of the aggregate liquidation shock are calibrated to match the autocorrelation and volatility of bank equity. We choose the value of capital income tax rate to match the capital income tax in Spain of 20%.

6 Macroprudential Policy

6.1 Higher Capital Requirement

As a first policy experiment, we double the minimum capital requirement from 8 to 16 percent. We report the results of this exercise in column 3 of Table 2.

A higher capital requirement increases the average house price. It also increases consumption of impatient households and decreases the consumption of patient households. The reason for this is that with a higher capital requirement, impatient households borrow less and hence accumulate

[2]We use financial crises data of OECD countries after WWII to calculate the average length of bank runs in OECD countries.

more wealth. Since they own a higher share of the housing stock of the economy, and since they value houses more than patient households, house prices increase.

Under a higher capital requirement, the volatility of house prices and consumption is lower as well. This is mostly due to the lower frequency of bank runs.

A higher capital requirement reduces the leverage of both households and banks. Note however that household leverage becomes slightly more countercyclical, while bank leverage becomes strongly procyclical.

A higher capital requirement lowers default rates, since household leverage is lower. Hence, the spread between mortgages rates and deposit rates will also decrease. The recovery rate on defaulted mortgages is 100 percent, since there is almost no more default.

Finally, doubling the capital requirement from 8 to 16 percent can reduce the frequency of bank runs to 0.

	Baseline	$\underline{\Gamma} = 0.16$	$\gamma = 1$	$\kappa = 0.4$
Real Economy				
Avg House Price (% Dev from Baseline)	0.000	0.866	0.037	0.504
Avg Consumption, Patient HH (% Dev from Baseline)	0.000	-3.101	-0.628	0.384
Avg Consumption, Impatient HH (% Dev from Baseline)	0.000	7.396	3.945	3.554
StDev(House Price) (%)	8.652	8.085	8.446	8.297
StDev(Consumption, Patient HH) (%)	5.103	4.130	4.609	4.276
StDev(Consumption, Impatient HH) (%)	11.086	3.972	6.212	5.471
Leverage				
Leverage, Impatient HH	1.811	1.231	1.803	1.608
Leverage, Banks	11.660	6.219	12.081	10.102
Corr(Household Leverage,GDP)	-0.305	-0.936	-0.585	-0.403
Corr(Bank Leverage,GDP)	0.330	0.353	0.193	0.645
Default Rates and Asset Prices				
Spread $R^M - R^D$ (% per year)	1.933	1.396	1.375	1.570
HH Default Rate (% per year)	2.905	0.040	1.497	0.223
Mortgage Recovery Rate (%)	86.263	100.000	85.317	90.674
Corr(Spread, GDP)	-0.089	-0.401	-0.215	-0.562
Corr(HH Default Rate, GDP)	-0.252	0.000	-0.677	-0.595
Corr(Recovery Rate, GDP)	0.206	-	0.405	0.538
Bank Runs				
Bank Runs per 100 years	1.564	0.000	0.411	0.149

Table 2: Model Statistics. In the baseline model, $\underline{\Gamma} = 0.08$, $\gamma = 0$ and $\kappa = 0.8$.

6.2 Loan Loss Provisioning

As a second policy experiment, we maintain a capital requirement of 8 percent, but allow it to increase one for one whenever expected default rates are high. Such a capital requirement is weakly procyclical, i.e. lower if GDP is low. It corresponds roughly to the dynamic provisioning that existed in Spain, which, despite having a counter-cyclical component was overall pro-cyclical.

Such a procyclical capital requirement slightly reduces house prices and redistributes wealth from patient to impatient households, decreasing the consumption of the former and increasing the consumption of the latter. In addition, the standard deviation of consumption for both patient and impatient households as well as the standard deviation of the house price increase.

Average leverage is essentially unaffected, but household leverage becomes less countercyclical as in the baseline model.

Credit spreads decrease slightly, as does the default rate. The recovery rate on mortgages increases. Moreover, the cyclicality of default and recovery rates and hence the credit spreads decreases.

Finally, the frequency of bank runs decreases slightly due to such a procyclical capital buffer.

6.3 Lower LTV Ratio

For the third and last policy experiment, we halve the maximum permissible LTV ratio of households from 80 to 40 percent.

As a consequence, house prices increase, and wealth is again redistributed from patient to impatient households, as the latter borrow less. Hence, consumption of impatient households is higher and consumption of patient households is lower.

Moreover, the volatility of consumption of the impatient households increases substantially. This is because mortgage credit under a tight LTV limit is more often determined by the LTV limit instead of the capital requirement of the bank, and the LTV constraint depends more strongly on the very volatile house price. The volatility of consumption of patient households as well as the volatility of the house price do not change substantially.

Leverage of households and banks decreases slightly. Moreover, household leverage becomes less countercyclical and bank leverage more procyclical.

Default rates are lower, recovery rates higher and hence the credit spread is lower. It is also less cyclical than the credit spread in the baseline model.

Finally, a higher LTV constraint reduces the frequency of bank runs slightly, since it leads to a non-binding bank capital requirement and lower bank leverage ratios.

7 Conclusion

We build a tractable macroeconomic model to study a joint housing and financial crisis, as was experienced by Spain in 2008-2016. We empirically document key features of such a crisis, namely a fall in house prices, a reduction in bank and household leverage and a rise in credit spreads for banks as well as borrowers. The model aims to capture these key features in a parsimonious way. Key features of the model are that both borrowers and banks use non-contingent debt and can either default or are subject to bank runs. Dynamics in the model are driven by both productivity shocks and financial shocks.

We find that both capital requirements and LTV constraints are effective policies to both decrease household default rates and the frequency of bank runs. Dynamic loan loss provisioning is effective at reducing default rates, but not at eliminating bank runs.

In future work, we plan to use the model as a framework to disentangle to what extent financial risk and productivity risk contributed to the housing crisis in Spain. Furthermore, we want to characterize the optimal policy mix between rule-based capital requirements as well as LTV constraints from a welfare point of view.

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A Data

Table 3: Variables Used in Calibration

Variables	Description	Unit	Source
Banking Sector			
Asset	Total bank assets	Trillion Euro	ECB Statistical Data Warehouse
Liability	Total bank liabilities	Trillion Euro	ECB Statistical Data Warehouse
Bank Leverage	Asset/Equity	1	ECB Statistical Data Warehouse
Credit Spread	Spread between bank deposit and German gov bond	per cent	Gilchrist&Mojon 2017
Share NPL	Bank nonperforming loans to total gross loans	per cent	World Bank
Housing Sector			
House Price	House price index for newly built and exiting houses	2005=100	Eurostat
Home Ownership	Distribution of population by tenure status	per cent	Eurostat
Labor Market			
Hourly Earnings	Average total earnings paid per employed person per hour, Private sector, SA	2010=100	OECD
Hours Worked	Average number of usual weekly hours of all professions all employed persons both part and full time	Hours	Eurostat
Economy			
Real GDP	Chain linked volumes 2010, seasonally and calendar adjusted	Million Euro	Eurostat
GDP Growth	Chain linked volumes, percentage change compared to same period in previous year	Per Cent	Eurostat
CPI	Harmonised index of consumer prices (HICP)	2015=100	Eurostat

B Steady State of the Model

B.1 Steady State Conditions

Prices:

$$R^D = \frac{1}{\beta^P} \tag{B.1}$$

$$W = \alpha Z L^{\alpha-1} \tag{B.2}$$

Patient households (P):

$$P = \beta^P \left(P + \frac{1-\chi}{\chi} \frac{C^P}{H^P} \right) \tag{B.3}$$

$$C^P = W\bar{L^P} + (R^D - 1)D^P + (1-\tau)\frac{O}{\mu} \tag{B.4}$$

Impatient households (I):

$$R^M = \max\left(\frac{1 - \beta^I \frac{PH^I}{M}\Phi}{\beta^I(1-\Phi)}, \frac{1}{\beta^I} \right) \tag{B.5}$$

$$X^M = \min\left(\frac{PH^I}{R^M M^I}, 1 \right) \tag{B.6}$$

$$P = \beta^I \left(P + \frac{1-\chi}{\chi} \frac{C^I}{H^I} \right) \tag{B.7}$$

$$C^I + \left[\left(1 - (1 - X^M)\Phi\right) R^M - 1 \right] M^I = W\bar{L^I} + \tau \frac{O}{(1-\mu)} \tag{B.8}$$

$$R^M M^I \leq \kappa P H^I \tag{B.9}$$

Banks (B):

$$N^B + D^B = M^B \tag{B.10}$$

$$n^B = \left[1 - (1 - X^M)\Phi\right] R^M M^B - R^D D^B \tag{B.11}$$

$$N^B = n^B(1-\eta) + E\eta \tag{B.12}$$

$$N^B \geq \Gamma M^B \tag{B.13}$$

$$\left[\left[1 - (1 - X^M)\Phi\right] R^M - R^D \right] D \geq 0 \tag{B.14}$$

Firms:

$$Y = Z L^\alpha \tag{B.15}$$

Market clearing:

$$Y = \mu C^P + (1-\mu)C^I \tag{B.16}$$

$$L = \mu \bar{L^P} + (1-\mu)\bar{L^I} \tag{B.17}$$

$$1 = \mu H^P + (1-\mu)H^I \tag{B.18}$$

$$\mu D^P = D^B \tag{B.19}$$

$$(1-\mu)M^I = M^B \tag{B.20}$$

Steady state variables to be determined:
Patient households: C^P, H^P, D^P;
Impatient households: C^I, H^I, M^I;
Banks: N^B, D^B, M^B;
Aggregate: H.

B.2 Comparative Statics in Steady State

In Figures 5 to 8, we show comparative statics for the leverage constraint Γ, the bank exit rate η, the discount factor of impatient households β^I and steady state productivity μ^Z. Γ is the main policy parameter we are interested in. We mark the baseline parametrization of the model with a red vertical line.

B.2.1 Varying Γ

Consider first the effects of raising the leverage constraint Γ displayed in Figure 5. A higher leverage allows banks to use more deposits to finance a given amount of lending. As long as the interest rate differential $\left[1-(1-X^M)\Phi\right] R^M - R^D$ is positive, taking on more leverage is profitable and increases the net worth of the bank. To see this, we substitute B.10 and B.13 into B.11:

$$n^B = \left[\left[\left[1-(1-X^M)\Phi\right] R^M - R^D\right]\Gamma + R^D\right] N^B.$$

A higher bank net worth in turn implies more deposits and more mortgages, which raises the consumption of both consumption goods and housing of patient households, since they will overall save more. Similarly, it lowers consumption of consumption goods and housing of impatient households, since they save less. Moreover, a higher mortgage coupled with less housing of impatient households means that mortgages become more risky, since their recovery rate decreases and the mortgage default rate increases. This leads to a higher required return on mortgages. Finally, since a larger share of housing is now held by patient households who value housing less than impatient households, the house price index decreases.

Above a certain threshold, it is no longer the leverage constraint of banks, but the borrowing constraint of impatient households which will constrain the amount of mortgages in the economy. Beyond that threshold, raising the leverage constraint no longer has any effect on the economy.

Figure 5: Comparative statics with respect to the leverage constraint ψ.

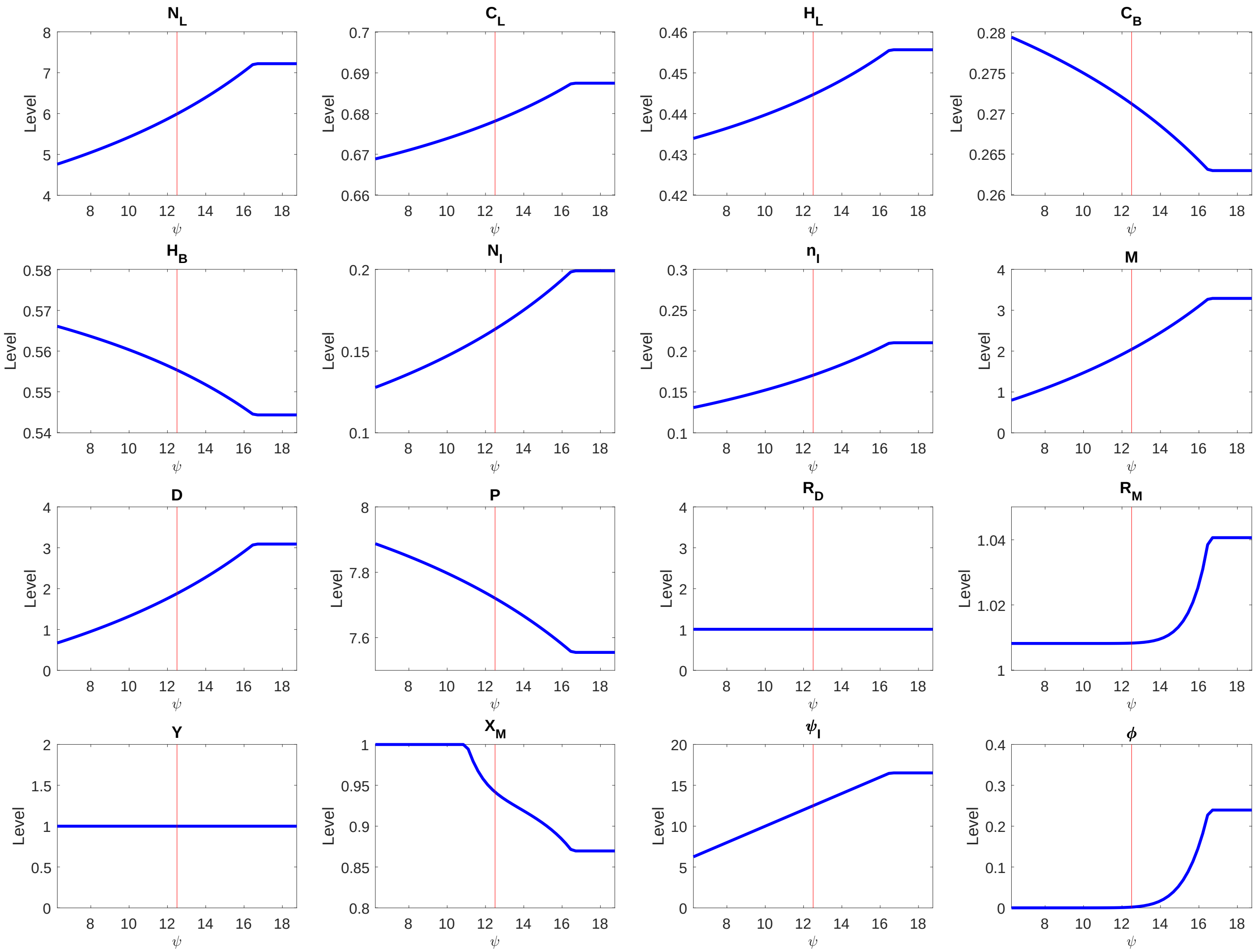


Figure 6: Comparative statics with respect to the exit rate of banks η.

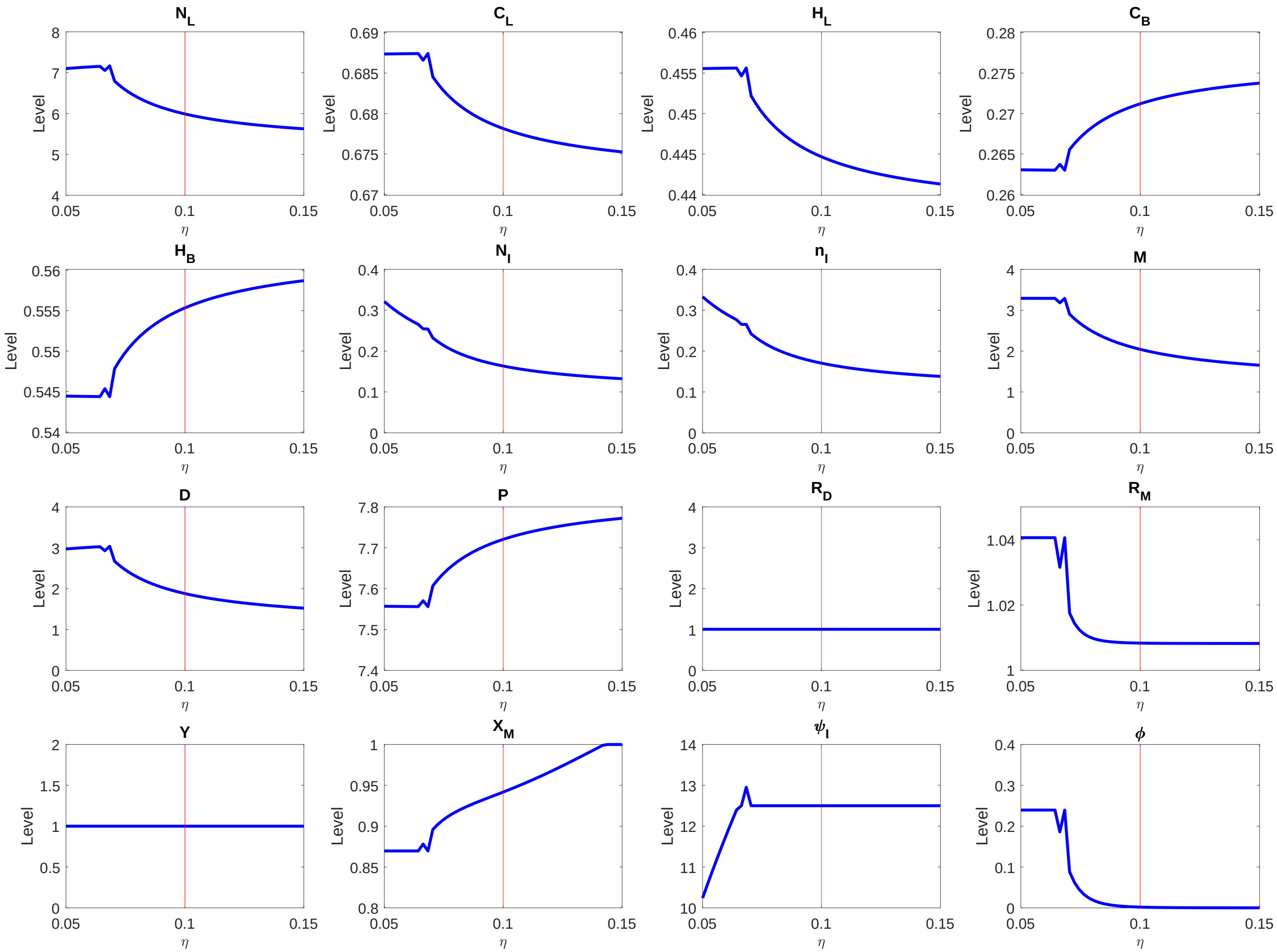


Figure 7: Comparative statics with respect to the discount factor of impatient households β^I.

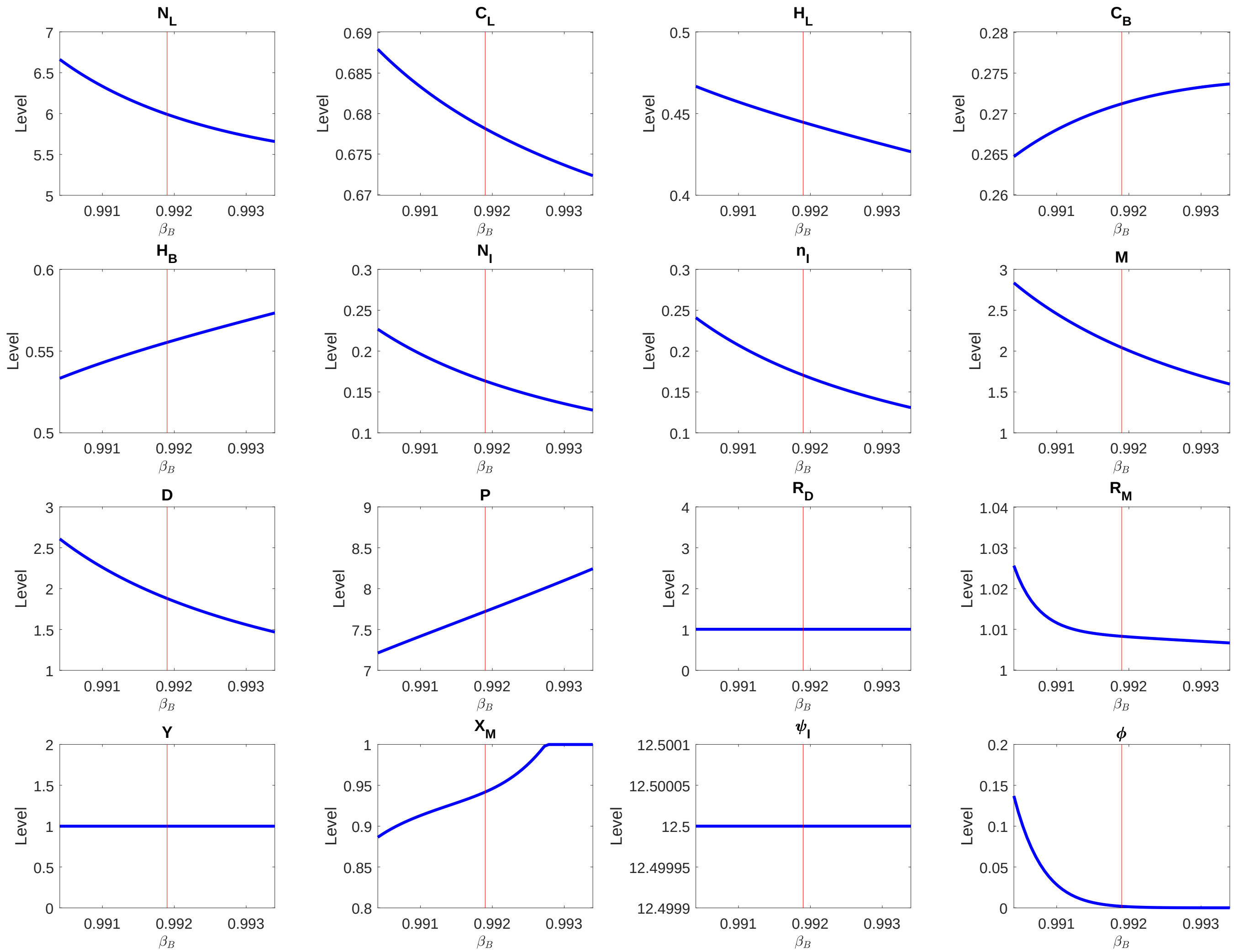
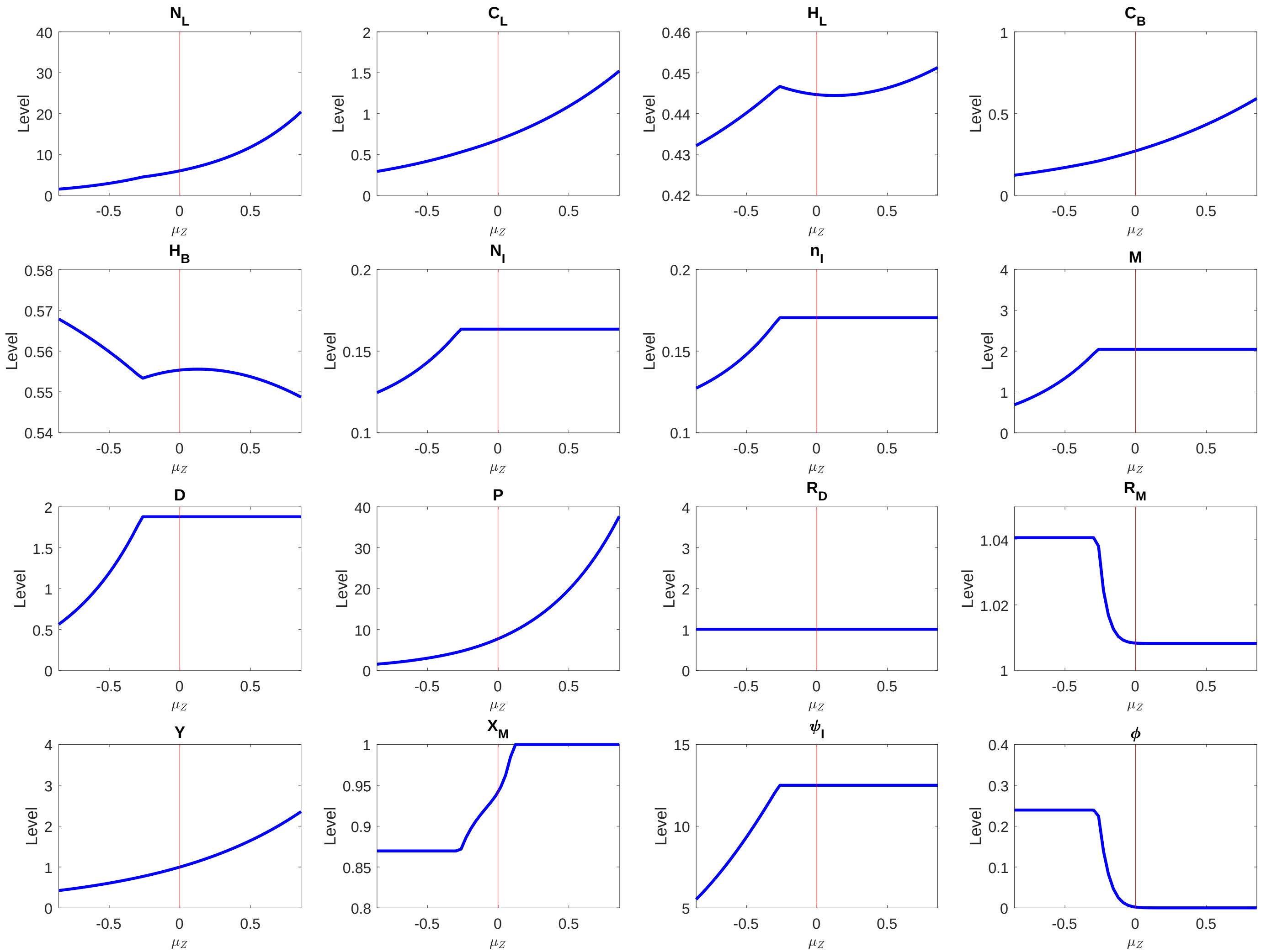


Figure 8: Comparative statics with respect to the steady state productivity μ^Z.



C Complete Statement of the Model

C.1 Impatient Households

The impatient households' problem can be summarized as:

$$\max_{\{C_t^I, H_{t+1}^I, M_{t+1}^I\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\beta^I\right)^t U^I \left(C_t^I, H_t^I\right) \right],$$

s.t.

$$C_t^I + P_t(H_{t+1}^I - (1-\delta)H_t^I) + \left[1 - (1 - X_t^M)\Phi_t\right] R_t^M M_t^I = W_t \bar{L}_t^I + M_{t+1}^I + T_t,$$

$$M_{t+1}^I \leq \kappa P_t H_{t+1}^I,$$

$$C_t^I, H_{t+1}^I, M_{t+1}^I \geq 0.$$

The FONC of the impatient household's problem are:

$$U_1(C_t^I, H_t^I) = \lambda_t^I, \tag{C.1}$$

$$\lambda_t^I P_t = \beta^I \mathbb{E}_t \left[\lambda_{t+1}^I P_{t+1} + \mathbf{U}_2(\mathbf{C}_{t+1}^I, H_{t+1}^I)\right], \tag{C.2}$$

$$\lambda_t^I = \beta^I \mathbb{E}_t \left[\lambda_{t+1}^I [1 - (1 - X_{t+1}^M)\Phi_{t+1}] R_{t+1}^M\right], \tag{C.3}$$

$$C_t^I + P_t(H_{t+1}^I - (1-\delta)H_t^I) + \left[1 - (1 - X_t^M)\Phi_t\right] R_t^M M_t^I = W_t \bar{L}_t^I + M_{t+1}^I + T_t. \tag{C.4}$$

C.2 Patient Households

The patient households face a standard consumption-and-saving problem with housing:

$$\max_{\{C_t^P, H_{t+1}^P, D_{t+1}^P\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\beta^P\right)^t U \left(C_t^P, H_t^P\right) \right],$$

s.t.

$$C_t^P + P_t(H_{t+1}^P - (1-\delta)H_t^P) + D_{t+1}^P = W_t \bar{L}_t^P + X_t^D \left(R_t^D D_t^P\right) + (1-\tau)O_t,$$

$$C_t^P, H_{t+1}^P, D_{t+1}^P \geq 0,$$

The FONC of the patient household's problem are:

$$U_1(C_t^P, H_t^P) = \lambda_t^P, \tag{C.5}$$

$$\lambda_t^P P_t = \beta^P \mathbb{E}_t \left[\lambda_{t+1}^P P_{t+1} + \mathbf{U}_2(\mathbf{C}_{t+1}^P, H_{t+1}^P)\right], \tag{C.6}$$

$$\lambda_t^P = \beta^P \mathbb{E}_t \left[\lambda_{t+1}^P R_{t+1}^D\right], \tag{C.7}$$

$$C_t^P + P_t(H_{t+1}^P - (1-\delta)H_t^P) + D_{t+1}^P = W_t \bar{L}_t^P + R_t^D D_t^P + (1-\tau)O_t. \tag{C.8}$$

C.3 Banks

Full statement of the bank's problem The surviving bank's maximization problem is:

$$\max_{\left\{M_{t+1}^B, D_{t+1}^B\right\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left[\frac{U_c(C_t^P, H_t^P)}{U_c(C_0^P, H_0^P)} \left[\beta^P (1-\eta)\right]^t \eta n_t^B \right] \right\},$$

s.t.

$$M_{t+1}^B = D_{t+1}^B + n_t^B, \qquad \text{Balance Sheet Constraint}$$

$$N_t^B \geq \Gamma_t M_{t+1}^B, \qquad \text{Bank Capital Requirement}$$

where net worth of the surviving banks n_t^B is given by:

$$n_t^B = \left[1 - (1 - X_t^M)\Phi_t\right] R_t^M M_t^B - R_t^D D_t^B. \tag{C.9}$$

Conditional on the leverage constraint being binding, the banker's problem is fully described by the leverage constraint, the balance sheet constraint and the non-negative profit margin constraint.

C.4 Bank's Problem: Non-binding Optimality Conditions

Consider the case of a bank that faces a non-binding capital requirement today. Re-writing the bank's problem in recursive form yields

$$V_t = \max_{M_{t+1}^B, D_{t+1}^B} \beta^P \mathbb{E}_t \left[\frac{U_c^P(C_{t+1}^P, H_{t+1}^P)}{U_c^P(C_t^P, H_t^P)} \bar{V}_{t+1} \right]$$

s.t.

$$M_{t+1}^B = D_{t+1}^B + n_t^B$$

where

$$\bar{V}_t = \eta n_t^B + (1-\eta) V_t,$$

and

$$n_t^B = \tilde{R}_t^M M_t^B - R_t^D D_t^B.$$

Define $\Omega_t \equiv \frac{\bar{V}_t}{n_t}$. In general, Ω_t depends only on aggregate variables and is hence exogenous from the point of view of an individual bank.[3] Rewriting the bank's problem yields:

$$\begin{aligned} V_t &= \max_{M_{t+1}^B, D_{t+1}^B} \beta^P \mathbb{E}_t \left[\frac{U_c^P(C_{t+1}^P, H_{t+1}^P)}{U_c^P(C_t^P, H_t^P)} \Omega_{t+1} n_{t+1} \right] \\ \bar{V}_t &= \eta n_t^B + (1-\eta) V_t \\ &\text{s.t.} \\ M_{t+1}^B &= D_{t+1}^B + n_t^B \\ n_t^B &= \tilde{R}_t^M M_t^B - R_t^D D_t^B \end{aligned}$$

Optimality of Deposit Funding Consider a bank that uses deposit financing. In this case, the problem of the bank can be simplified to

$$V_t = \max_{D_{t+1}^B} \beta^P \mathbb{E}_t \left[\frac{U_c^P(C_{t+1}^P, H_{t+1}^P)}{U_c^P(C_t^P, H_t^P)} \Omega_{t+1} (\tilde{R}_{t+1}^M (D_{t+1}^B + n_t^B) - R_{t+1}^D D_{t+1}^B) \right]$$

A bank will use some deposit funding in addition to lending out its equity whenever

$$\frac{\partial V_t}{\partial D_{t+1}^B} = \beta^P \mathbb{E}_t \left[\frac{U_c^P(C_{t+1}^P, H_{t+1}^P)}{U_c^P(C_t^P, H_t^P)} \Omega_{t+1} (\tilde{R}_{t+1}^M - R_{t+1}^D) \right] \geq 0,$$

i.e. whenever the net benefit of raising an additional unit of deposits and lending it out in the form of mortgages is positive.

Optimality of Mortgage Lending A bank will lend out its net worth in the form of mortgages whenever

$$\frac{\partial V_t}{\partial n_t^B} = \beta^P \mathbb{E}_t \left[\frac{U_c^P(C_{t+1}^P, H_{t+1}^P)}{U_c^P(C_t^P, H_t^P)} \Omega_{t+1} \tilde{R}_{t+1}^M \right] \geq 1,$$

i.e. when the benefit of reinvesting an additional dollar of net worth is higher than the benefit of simply paying it out to the households.

[3] This is straightforward to prove in the case of an always binding lending constraint. Noting that

$$\begin{aligned} M_{t+1}^B &= 1/\Gamma_t n_t^B \\ D_{t+1}^B &= (1/\Gamma_t - 1) n_t^B \\ n_t^B &= \tilde{R}_t^M 1/\Gamma_t n_{t-1}^B - R_t^D (1/\Gamma_t - 1) n_{t-1}^B, \end{aligned}$$

we see that n_t^B / n_{t-1}^B depends only on aggregate returns. Plugging this into the bank's value function and iterating forward gives the result.

D Numerical Solution Algorithm

Collect the exogenous states in $\mathcal{Y} = (A, Z)$. Collect the endogenous states in $\mathcal{S} = (H^P, N^P, N^I)$. The state space of the model without wage rigidity is completely characterized by $(\mathcal{S}, \mathcal{Y})$. Adding wage rigidity adds the lagged wage as a state variable. The state space in a bank run is characterized by $\left(H^P, \mathcal{Y}\right)$.

We need to find four unknown non-linear policy functions, namely $c^P_{NoRun}(\mathcal{S}, \mathcal{Y})$, $c^I_{NoRun}(\mathcal{S}, \mathcal{Y})$, $c^P_{Run}(H^P, \mathcal{Y})$ and $c^I_{Run}(H^P, \mathcal{Y})$ and laws of motion for net worth $N^{P'}_{NoRunToNoRun}$, $N^{P'}_{NoRunToRun}$, $N^{P'}_{RunToNoRun}$, $N^{P'}_{RunToRun}$ and $N^{I'}_{NoRunToNoRun}$ as functions of the endogenous and exogenous states.

The general idea is to approximate the unknown functions on a sparse state grid and then solve the model by backward iteration. The outline of the algorithm is as follows:

1. Find the steady state of the model. See above for details.
2. Set up grid: Adaptive sparse grid as in Brumm and Scheidegger (2017).
   (a) Bounds:
      - Exogenous processes: +- 4 unconditional standard deviations.
      - Endogenous processes: around steady state.

   (b) Grid level: 6, meaning that we use up to six nested sets of basis functions.
3. Initial guess for the unknown functions and the laws of motion for net worth
4. Compute expectations: Mixture of Gauss-Hermite and Gauss-Legendre quadrature. 11 quadrature nodes on each shock. Gauss-Hermite quadrature is standard to approximate the expectation over normally distributed variables. Gauss-Legendre quadrature is useful, since it allows simple integration over a bounded interval, which is what we want to do to work with the exact bank run cutoff.
5. Find new policy functions: Solving a system of non-linear equations. Need to find $H^{B'}, H^{L'}, P$ for both the run- and no-run equilibrium.
6. Update unknown functions and net worth laws of motion
7. Check convergence: Specify tolerances

We provide details on the critical steps below.

D.1 Grid

The policy functions in the no-run case are approximated on a five-dimensional grid, the laws of motion for net worth in the no-run case on a seven-dimensional grid. The functions in the run case do not have N^B as a state variable.

D.2 Expectations

We want to numerically approximate expectations of the kind

$$
\begin{aligned}
&\mathbb{E}\left[f(H^{P'}, N^{P'}, N^{I'}, A', Z')|A, Z\right] = \\
&= \int_0^\infty \left(\int_0^{A^*} f^{Run}(H^{L'}, A', Z')dG(A'|A) + \right. \\
&\left. \int_{A^*}^\infty f^{NoRun}(H^{P'}, N^{P'}, N^{I'}, A', Z')dG(A'|A) \right) dF(Z'|Z),
\end{aligned}
$$

where A denotes productivity and Z is the liquidation cost. For productivity, we simply use Gauss-Hermite quadrature with integration nodes x^a and integration weights w^a. Since we want to compute the expectations using the exact thresholds for the liquidation cost shock, we use Gauss-Legendre quadrature, using integration nodes $x^z \in [-1, 1]$ and integration weights w^z, with $\sum_z w^z = 2$. We assume that $\ln Z$ is bounded between $\ln \underline{Z}$ and $\ln \overline{Z}$. This procedure essentially follows Hatchondo, Martinez, and Sosa-Padilla (2016). The integration consists of four steps:

1. Find the exact bank run cutoffs $\varepsilon^{Z*}(H^P, R^D D, R^M M, A, Z_{-1})$. The cutoffs can be found by solving the non-linear equation

$$
\begin{aligned}
&Z_{-1}^{\rho^A} \exp \varepsilon^{Z*} \frac{(1 - \pi(P^*))R^M M + \pi(P^*)P^* H^I}{R^D D} = 1, \\
&P^* = p^{Run}(H^P, A, Z_{-1}, \varepsilon^Z).
\end{aligned}
$$

2. Determine the integration nodes for Z. We distinguish two cases:

   (a) $Z^* \leq \underline{Z}$ or $Z^* \geq \overline{Z}$. In this case, there is no interior bank run cutoff. We compute the integration nodes to be equally spaced in probability: Define

$$
\overline{\varepsilon} = \frac{\ln \overline{Z} - \rho^A \ln Z}{\eta^A}
$$

   and

$$
\underline{\varepsilon} = \frac{\ln \underline{Z} - \rho^A \ln Z}{\eta^A}
$$

   Then, we compute the adjusted integration nodes ε^z as

$$
cdf(\varepsilon^z) = cdf(\underline{\varepsilon}) + \frac{1 + x^z}{2}(cdf(\overline{\varepsilon}) - cdf(\underline{\varepsilon})).
$$

   $cdf(.)$ is the cumulative distribution function of a standardized normal distribution.

   (b) $\underline{Z} < Z^* < \overline{Z}$. In this case, there is an interior bank run cutoff. Hence, we compute two sets of adjusted integration nodes. Define

$$\varepsilon^* = \frac{\ln Z^* - \rho^A \ln Z}{\eta^A}.$$

Then, the first set of integration nodes is given by

$$cdf(\varepsilon^z_{Run}) = cdf(\underline{\varepsilon}) + \frac{1+x^z}{2}(cdf(\varepsilon^*) - cdf(\underline{\varepsilon})),$$

and the second set by

$$cdf(\varepsilon^z_{NoRun}) = cdf(\varepsilon^*) + \frac{1+x^z}{2}(cdf(\overline{\varepsilon}) - cdf(\varepsilon^*)).$$

3. Integrate piecewise in the No-Run and Run-region of the state space. Define

$$Z^z_{Run} = \exp(\rho^A \ln Z + \eta^A \varepsilon^z_{Run}).$$

and

$$A^a = \exp(\rho^Z \ln A + \eta^Z x^a).$$

Then, the run expectation can be approximated as

$$\begin{aligned}\mathbb{E}\left[f^{Run}|A, Z\right] &= \int_0^\infty \int_0^{Z^*} f^{Run}(H^{L'}, A', Z, \varepsilon^Z)dG(\varepsilon^Z)dF(A'|A) \\ &\approx \sum_a \sum_z w^a \frac{w^z}{2} f^{Run}(H^{L'}, A^a, Z, \varepsilon^z_{Run})\end{aligned}$$

The same applies to the no-run expectation $\mathbb{E}\left[f^{NoRun}|A, Z\right]$ and the expectation in the case of no interior cutoff, in which we can directly compute $\mathbb{E}\left[f|A, Z\right]$, subject to the adjustment below.

4. Sum up over the piecewise integrals. Note that since the probability mass between $\underline{Z}$ and $\overline{Z}$ is not one, we need to adjust the expectation:

$$\mathbb{E}\left[f|A, Z\right] = \frac{\mathbb{E}\left[f^{NoRun}|A, Z\right] + \mathbb{E}\left[f^{Run}|A, Z\right]}{cdf(\overline{\varepsilon}) - cdf(\underline{\varepsilon})}.$$

D.3 Nonlinear Equation Systems

We need to solve for the no run and the run equilibrium.

D.3.1 No Run System

Take the laws of motion of consumption at iteration $i-1$ as given. In iteration i, we compute the following expectations:

$$C^{P'} = c^P_{(i-1)} \left(h^{P'}(\mathcal{S},\mathcal{Y}), d^{'}(\mathcal{S},\mathcal{Y}), m^{'}(\mathcal{S},\mathcal{Y}), \mathcal{Y}' \right)$$

takes the law of motion of aggregate net worth as given.

With these expressions for the expectations, it's straightforward to compute the solution to the first-order conditions:

Patient Household:

$$\begin{aligned}
U_1(C^P, H^P)P &= \beta^P \mathbb{E}\left[U_1(C^{L'}, H^{L'})P' + U_2(C^{L'}, H^{L'}) | A, Z \right] \\
U_1(C^P, H^P) &= \beta^P R^{D'} \mathbb{E}\left[U_1(C^{L'}, H^{L'}) | A, Z \right] \\
C^P + PH^{L'} + D^{'} &= WL^P + PH^L + R^D D + \Pi + \eta \frac{N^B - \eta E}{1-\eta} - \eta E
\end{aligned}$$

Bank:

$$\begin{aligned}
N^B + D^{'} &= M^{I'} \\
M^{'} &= \psi N^B \\
N^B &= (R^M M(1-\phi(P)) + P(\mathcal{H} - H^P)\phi(P) - R^D D)(1-\eta) + E\eta
\end{aligned}$$

Impatient Household:

$$\begin{aligned}
U_1(C^I, \mathcal{H} - H^P)P &= \beta^I \mathbb{E}\left[U_1(C^{B'}, H^{B'})P' + U_2(C^{B'}, H^{B'}) | A, Z \right] \\
U_1(C^I, \mathcal{H} - H^P) &= \beta^I R^{M'} \mathbb{E}\left[U_1(C^{B'}, H^{B'}) | A, Z \right] \\
C^I &= AL^\alpha - C^P
\end{aligned}$$

where C^I comes from the aggregate resource constraint.

Firms:

$$\begin{aligned}
A\alpha L^{\alpha-1} &= \frac{W}{1+\xi(R^K - 1)} \\
\Pi &= AL^\alpha - WL
\end{aligned}$$

Market Clearing:

$$\mathcal{H} = H^{L'} + H^{B'}$$

Effectively, this system of equations can be boiled down to solving a simple nonlinear system of equations in two variables, $H^{L'}, P$, which we do using MATLAB's fzero routine. All other variables can either be calculated explicitly or determined residually.

D.3.2 Run System

Patient Household:

$$\begin{aligned} U_1(C^P, H^P) &= \beta^P V_H^P(\mathcal{S}, A, Z) \\ C^P + PH^{L'} &= WL^P + XR^D D + \Pi + \eta \frac{N^B - \eta E}{1-\eta} - \eta E + (ZR^K - 1)K \\ X &= Z \frac{R^M M(1-\phi(P)) + P(\mathcal{H} - H^P)\phi(P)}{R^D D} \end{aligned}$$

Impatient Household:

$$\begin{aligned} U_1(C^I, \mathcal{H} - H^P) &= \beta^I V_H^I(\mathcal{S}, A, Z) \\ C^I &= AL^\alpha - C^P \end{aligned}$$

where C^I comes from the aggregate resource constraint.

Firms:

$$\begin{aligned} A\alpha L^{\alpha-1} &= \frac{W}{1+\xi(R^K - 1)} \\ \Pi &= AL^\alpha - WL(1+\xi(R^K - 1)) \\ K &= \xi WL \end{aligned}$$

Market Clearing:

$$\mathcal{H} = H^{L'} + H^{B'}$$