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Endogenous trading constraints with incomplete asset markets ☆

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Abstract

This paper endogenizes the borrowing constraints on capital in a production economy with incomplete markets. We find that these limits get looser with income, a property that is consistent with US data on credit limits. The framework with endogenous limits is then used to study the effects of a revenue neutral tax reform that eliminates capital income taxes. Our results illustrate that it is very important to take into account the effects of tax policies on the limits. Throughout the transition, these effects can be big enough to change the overall conclusion about the desirability of a tax reform.

JEL classification: E23; E44; D52

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1. Introduction

This paper endogenizes the borrowing constraints on capital holdings in an infinite horizon incomplete markets model with production. This is done by introducing the possibility of default on financial liabilities. In particular, we assume that households can break their trading contracts every period. In this case, individual liabilities are forgiven and agents are excluded from future

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trade forever. The endogenous trading limits are then set at the level at which households are indifferent between honoring their debt and defaulting.

Recently, models with heterogeneous agents and uninsurable income shocks have become one of the main economic tools to analyze important issues, such as the shape of the wealth distribution, the degree of risk sharing and the welfare implications of different economic policies. One of the appealing features of these models is that they are able to generate a realistic wealth distribution. However, the fact that there is a significant proportion of individuals in debt in the data implies that a realistic model of incomplete markets should also be able to generate enough borrowing. Clearly, these two aspects are interrelated through the borrowing constraints, since they are one of the key determinants of the (equilibrium) level of debt and, in general, of the wealth distribution in these type of economies. In the present paper, we determine these constraints endogenously and we calibrate the model so that the distribution of assets and the amount of debt matches the one in the data.

The first important advantage of our approach is that we are able to provide a characterization of the endogenous borrowing limits that we can compare to the behavior of credit limits in the data. First, we show analytically that the level of debt which makes individuals indifferent between defaulting and paying back is monotonically increasing with individual labor income if the income shocks are i.i.d. Moreover, this relationship between income and debt limits also holds in our calibrated economy, which assumes persistent income shocks. At first sight, this result might seem surprising, since a higher level of income increases the value of the outside option and therefore the incentives to default. Notice, however, that income also increases the value of paying back due to the fact that markets are incomplete. It turns out that this latter gain is higher than the rise in the outside option, implying that the endogenous borrowing limits get looser with individual income. Here, we should point out that this analytical reasoning only relies on the fact that the consumption allocation does not display perfect risk sharing in equilibrium, implying that (at least in the i.i.d. case) a similar result would also hold in models with complete markets and limited commitment. Second, our analytical results for the i.i.d. case and the quantitative findings with persistent shocks show that the endogenous limits as a fraction of labor income get tighter with income. Using data from the 2004 Survey of Consumer Finances, we document that both this latter property and the fact that there is a positive relationship between income and credit limits are consistent with the behavior of credit limits in the US data. While this provides an external validation of the way we endogenize the limits, it also implies that our framework serves as a good tool for quantitative analysis.

The second appealing property of endogeneizing the borrowing limits becomes more apparent when we consider policy applications. In a framework in which the equilibrium allocations exhibit imperfect risk sharing, changes in economic policy typically affect the wealth distribution. In the presence of limited commitment, these changes also affect the relative value of default and consequently the endogenous borrowing constraints. This is particularly important in models with capital accumulation, generating quantitatively important general equilibrium effects that interact with the borrowing limits. In order to illustrate these effects, we use a calibrated version of the model to analyze the long run welfare implications of a revenue neutral tax reform that eliminates capital income taxes. We consider two variants of this reform. The first one replaces the lost revenue by simply increasing the linear labor income tax rate, while the second one achieves the same objective by making labor taxes progressive. Under such a reform, the relative value of default with respect to paying back changes directly (through taxes) and indirectly (through capital accumulation). This implies that the endogenous limits respond as well. In fact, our results show that the welfare effects and overall desirability of the particular tax reform we consider vary depending on wether one takes into account the effects of the reform on the borrowing constraints or not.

More precisely, the elimination of capital taxes leads to more capital accumulation and this makes borrowing cheaper and default less attractive, leading to looser borrowing limits. It turns out that ignoring this latter effect has a large quantitative welfare impact. In the two reforms we consider, the aggregate welfare gain would be around 1.5 percentage points lower without this effect, and this difference is big enough to change the overall conclusion about the desirability of the reform when labor taxes become progressive. The key intuition behind this result is that looser limits improve welfare for low income and low asset agents, who are typically borrowing constrained. These welfare gains may be offset by the higher labor income taxes. However, when the system becomes progressive, the increase in taxes affects the (income) poor only indirectly through exogenous possible future productivity (wage) increases. When all this is taken into account, it turns out that they benefit from the reform overall.

Our work builds a bridge between several important strands of literature. First, it contributes to an increasingly growing literature in which a number of authors have introduced limited enforceability of risk-sharing contracts in models with complete markets, implicitly resulting in agent and state specific trading constraints. Among others, Kehoe and Levine [22], Alvarez and Jermann [6,7] and Krueger and Perri [23] introduce these type of limits in exchange economies, whereas Kehoe and Perri [20,21] study a production economy where investors are interpreted as countries. Since the lack of commitment leads to equilibrium allocations that exhibit imperfect risk sharing, these models are labelled endogenous incomplete market economies.

Apart from the fact that this literature does not characterize the endogenous borrowing limits, the imperfect risk sharing result may not be robust to the introduction of capital accumulation in closed economy models. For example, Ábrahám and Cárceles-Poveda [2] show that the equilibrium of a two agent model with endogenous production exhibits full risk sharing in the long run for standard parameterizations. Further, Krueger and Perri [24] show that a model with a continuum of agents and endogenous incomplete markets is not able to account for the increase in US consumption inequality due to the fact that there is too much risk sharing.[1] Since the implications of models with full or close to full risk sharing are clearly at odds with the data, this provides a strong motivation to study limited commitment in economies with incomplete markets, where risk sharing is always limited. While the number of assets traded is still exogenous in this case, the presence of limited commitment endogenizes the amount that households can borrow. In this sense, the degree of market incompleteness becomes partially endogenous in the present paper.

Second, our work is also related to the traditional incomplete market models where the borrowing limits are ad hoc. Some examples are Heaton and Lucas [17], Telmer [33], Aiyagari [3,4], Huggett [18] and Krusell and Smith [25,26]. Whereas the previous authors have often argued that the ad hoc trading constraints are tighter than the natural borrowing limits to avoid default in equilibrium, the present work is one of the few formalizing this argument in a setting with incomplete markets. It therefore provides a deeper foundation of the trading limits. An exception

[1] Cordoba [12] also obtains full risk sharing in a production economy with a continuum of agents, complete markets and collateral constraints. It is important to note that Kehoe and Perri [20] obtain imperfect risk sharing in an open economy with complete markets and production. However, one of the key differences is that their idiosyncratic shocks are interpreted and calibrated as country specific *aggregate* productivity shocks, whereas they are shocks to *individual* labour productivity in our economy. In addition, Bai and Zhang [9] calibrate a similar economy to the one of Kehoe and Perri differently, and they also find extensive risk sharing under complete markets.

is the work by Zhang [34,35], who derives endogenous borrowing limits resulting from the possibility of default in a Lucas type exchange economy with two agents and trade in one asset. In contrast, we allow for the possibility of capital accumulation in an economy with a continuum of agents, two features that have important effects on the incentives to default. Finally, Zhang uses his model to study asset pricing implications, while we study the welfare effect of eliminating capital income taxes.

Third, our work is related to the recent literature studying the welfare effects of capital income taxation in a context with heterogeneous agents. For example, Aiyagari [4] studies the optimal capital income tax in a model with incomplete markets and no borrowing. In contrast to the seminal papers of Chamley [10] and Judd [19], who show that the optimal long run capital income tax is zero for a wide class of infinite horizon models with complete markets, the author shows that the optimal long run capital income tax is always strictly positive. Further, in a model with no borrowing but with a more realistic calibration, Domeij and Heathcote [16] find that eliminating capital income taxes may be welfare improving in the long run, while it decreases welfare in the short run. Similarly to Domeij and Heathcote [16], we show that the elimination of capital income taxes increases welfare in the long run and decreases welfare when taking into account the transition for the case in which the endogenous borrowing limits are not allowed to change and the tax system becomes progressive. However, if the limits change endogenously, this tax reform turns out to be desirable both in the long run and along the transition, and it also gains substantial public support. Note that Domeij and Heathcote [16] do not consider a reform with progressive labor taxes but instead increase the linear labor tax rate for all agents. When we do this, welfare is reduced both in the short run and the long run, with the main difference in the long run arising most likely from the fact that the previous authors do not allow for borrowing. Overall, these findings illustrate that the long run welfare effects of the tax reform we consider also depend on whether one allows for borrowing and on whether the effects of tax changes on the borrowing limits are taken into account or not.

The fact that a different tax policy can affect the incentives to default has been already noted by Krueger and Perri [23], who study the optimality of progressive income taxation in a model with endogenous incomplete markets. In their case, moving from a progressive labour income tax (which in principle should lead to a higher degree of risk sharing) to a proportional tax can actually increase welfare by decreasing the value of defaulting and by allowing for a looser limit and a higher level of risk sharing. However, the authors assume that markets are complete. In addition, they do not have capital accumulation and they focus on progressive labour income taxation. In contrast, we study a model with capital accumulation and incomplete markets, and we focus on capital income taxation. As explained earlier, the presence of capital accumulation allows us to also take into account how different levels of aggregate capital affect the value of default indirectly through factor prices.

We should note that the presence of endogenous trading limits considerably complicates our computations, since we have to extend the usual policy (or value) iteration algorithm to incorporate an endogenous and non-rectangular grid for some of the states, introducing an additional fixed point problem. In spite of the computational difficulties, however, the methods developed in the present work could be fruitfully applied to study a wide set of interesting incomplete market models with endogenous limits. Our results suggests that fiscal policy and social insurance programs can have significant effects on the level of the endogenous trading constraints. Given this, a welfare analysis of any policy reform should take these effects into account.

The rest of the paper is organized as follows. Section 2 presents the general model with incomplete markets and it characterizes the endogenous trading limits that prevent equilibrium default.

Section 3 documents a couple facts regarding the relationship between credit limits and income in the data. Section 4 presents the calibration of the benchmark model together with the quantitative characterization of the endogenous limits. Section 5 analyzes the welfare implications of a tax reform. Finally, Section 6 summarizes and concludes.

2. The model

We consider an infinite horizon economy with endogenous production, idiosyncratic income shocks and sequential asset trade subject to borrowing restrictions. The economy is populated by a government, a representative firm and a continuum (measure 1) of infinitely lived households that are indexed by $i \in I$.

Households

Households have identical additively separable preferences over sequences of consumption $c_i \equiv \{c_{it}\}_{t=0}^{\infty}$ of the form:

$$U(c_i) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}), \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor and E_0 denotes the expectation conditional on information at date $t = 0$. The period utility function $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$ is assumed to be strictly increasing, strictly concave and continuously differentiable, with $\lim_{c_i \to 0} u'(c_i) = \infty$ and $\lim_{c_i \to \infty} u'(c_i) = 0$.

Each period, households can only trade (borrow or save) in physical capital to insure against uncertainty. The after-tax gross return on capital is equal to $1 + r_t(1 - \tau_k(k_{it}))$, where k_{it} represents the beginning of period individual capital holdings and τ_k is the tax rate on interest income. We assume that only savers pay tax on their interest income. Given this, capital income taxes depend on the level of assets in the following way:

$$\tau_k(k_{it}) = \begin{cases} \boldsymbol{\tau}_k & \text{if } k_{it} \geqslant 0, \\ 0 & \text{if } k_{it} < 0. \end{cases}$$

Apart from asset income, household $i \in I$ receives a stochastic labour endowment ϵ_i. This shock is i.i.d. across households and it follows a Markov process with transition matrix $\Pi(\epsilon' \mid \epsilon)$ and S_ϵ possible values that are assumed to be strictly positive. The after-tax individual labor income is equal to $w_t(1 - \tau_l)\epsilon_{it}$, where w is the aggregate wage rate and τ_l is the tax rate on labor income. The households' budget constraint can thus be expressed as:

$$c_{it} + k_{it+1} = w_t(1 - \tau_l)\epsilon_{it} + \big(1 + r_t\big(1 - \tau_k(k_{it})\big)\big)k_{it}. \tag{2}$$

At each date, household $i \in I$ also faces a possibly endogenous and state-dependent trade restriction on the end of period capital holdings k_{it+1}. Throughout the paper, we assume that households cannot commit on the trading contracts and we determine the borrowing constraint endogenously at the level that prevents default in equilibrium. In case of default, we assume that individual liabilities are forgiven and households are excluded from future asset trade. This implies that their only source of income from the default period on is their labor income. Following Livshits, MacGee and Tertilt [27], we also assume that there is an additional penalty λ that reduces labour income by $(1 - \lambda)$ after default. This penalty can be interpreted as a reduced form for different monetary and non-monetary costs of defaulting, such as the fraction of income that is garnished by creditors, the disutility (stigma) of default, the fixed monetary costs of filing, and the increased cost of consumption.[2]

Whereas the previous endogenous limits are simply imposed throughout the text, Appendix A illustrates that they could arise as an equilibrium outcome if competitive financial intermediaries were able to set them. In particular, assuming the same financial intermediation sector as the one in Ábrahám and Cárceles-Poveda [1,2], the appendix shows that the loosest possible limits that prevent default constitute a symmetric Nash equilibrium. Moreover, there is no symmetric equilibrium with looser limits, implying that default cannot arise in a (symmetric) equilibrium.

Here, it is important to note that the previous results are derived under the restriction that intermediaries cannot charge different interest rate on borrowers and savers, which is necessary to avoid default in equilibrium in the present setting.[3] In contrast, other authors, such as Dubey, Geanakoplos and Shubik [15], allow for different borrowing and saving rates, in which case they can sustain equilibrium default. Another example is the work by Chatterjee et al. [11], who study household bankruptcy in a production economy with incomplete markets and observable income. In this case, lenders can internalize the default probabilities by designing individual specific lending contracts.[4] Finally, when income is private information, Sanchez [31] shows that the only incentive compatible borrowing contracts that financial intermediaries might offer are a particular case of the limits we consider, namely, the tightest limits across all income levels that do not allow for default in equilibrium. Hence, such a setting provides micro foundations for a particular case of the endogenous borrowing limits we consider.

Production

At each date, the representative firm uses capital $K_t \in \mathbb{R}_+$ and labor $L_t \in (0,1)$ to produce a single good $y_t \in \mathbb{R}_+$ with the constant returns to scale technology:

$$y_t = Af(K_t, L_t), \tag{3}$$

where A is a technology parameter that represents total factor productivity. The production function $f(\cdot,\cdot) : \mathbb{R}_+^2 \to \mathbb{R}_+$ is assumed to be continuously differentiable on the interior of its domain, strictly increasing, strictly concave in K and homogeneous of degree one in K and L.

Each period, the firm rents capital and labor to maximize period profits. The two factor prices are given by:

$$w_t = Af_L(K_t, L_t), \tag{4}$$

$$r_t = Af_K(K_t, L_t) - \delta, \tag{5}$$

where δ is the depreciation rate of capital.

[2] This punishment for default resembles the bankruptcy procedures under Chapter 7. Under this procedure, households are seized from any positive asset holdings but can keep at least part of their labour income. Whereas they are allowed to borrow after some periods, this becomes considerably more difficult and costly because their credit rating deteriorates significantly.

[3] If there is limited commitment but markets are complete, Ábrahám and Cárceles-Poveda [2] show that the endogenous borrowing limits on Arrow securities that avoid default arise as a symmetric Nash equilibrium without the above restriction on borrowing and lending rates.

[4] The welfare implications of a related model with incomplete markets and equilibrium default are also studied by Mateos-Planas and Seccia [28].

Government and market clearing

At each period t, the government consumes the amount G_t and it taxes labor and interest rate income at the rates τ_l and $\boldsymbol{\tau}_k$ respectively. The government budget constraint is therefore equal to:

$$G_t = w_t \tau_l L_t + r_t \boldsymbol{\tau}_k \widehat{K}_t, \tag{6}$$

where $\widehat{K}_t \geqslant K_t$ is the capital income tax base. This corresponds to the sum of capital holdings of those who hold non-negative assets.

As usual, labor and asset market clearing require that the sum of individual labor income shocks and individual capital holdings are equal to the total labor supply and aggregate capital stock respectively. Further, the good's market clearing condition requires that the sum of investment and aggregate consumption, including household and government consumption, is equal to the aggregate output.

Recursive competitive equilibrium

In the present framework, the aggregate state of the economy is given by the joint distribution Ψ of consumers over individual capital holdings k and idiosyncratic productivity status ϵ. Further, households perceive that Ψ evolves according to:

$$\Psi' = \Gamma[\Psi],$$

where Γ represents the transition function from the current aggregate state into tomorrow's wealth-productivity distribution. Since the individual state vector includes the individual labour productivity and capital holdings (ϵ, k), the relevant state variables for a household are summarized by the vector $(\epsilon, k; \Psi)$.

Using this notation, the outside option or autarky value V of a household with income shock ϵ can be expressed recursively as:

$$V(\epsilon; \Psi) = u\big(w(\Psi)(1-\tau_l)\epsilon(1-\lambda)\big) + \beta \sum_{\epsilon'} \Pi\big(\epsilon' \bigm| \epsilon\big) V\big(\epsilon'; \Gamma[\Psi]\big). \tag{7}$$

Eq. (7) reflects that the autarky value is a function of the wealth-productivity distribution. Note that this is in contrast with some of the literature with complete markets and no commitment, where V is exogenous (see e.g. Alvarez and Jermann [6,7]). As we will see later, this is due to the fact that the distribution determines aggregate capital accumulation, which in turn determines future wages and therefore the future value of financial autarky. However, since individual liabilities are forgiven upon default, the autarky value is not a function of the individual capital holdings. Note also that the expression in (7) implicitly assumes that the aggregate state of the economy follows the same law of motion $\Gamma[\Psi]$ if one of the agents defaults. This is correct in the presence of a continuum of agents, since an individual deviation does not influence the aggregate variables and no one defaults in equilibrium.

We are now ready to define the recursive competitive equilibrium. Since the aggregate labor supply is constant due to the law of large numbers, factor prices only depend on aggregate capital and we therefore write $w(\Psi) = w(K)$ and $r(\Psi) = r(K)$ in what follows.

Definition 2.1. Given a transition matrix Π and some initial distribution of shocks $\epsilon_0 \equiv (\epsilon_{i0})_{i\in I}$ and asset holdings $k_0 \equiv (k_{i0})_{i\in I}$, a *recursive competitive equilibrium* relative to the vector of taxes $(\boldsymbol{\tau}_k, \tau_l)$ is defined by default thresholds $\underline{k}(\epsilon; \Psi)$, a law of motion Γ, a vector of factor prices $(r, w) = (r(K), w(K))$, a government consumption G, value functions $W = W(\epsilon, k; \Psi)$ and $V = V(\epsilon; \Psi)$, and individual policy functions $(c, k') = (c(\epsilon, k; \Psi), k(\epsilon, k; \Psi))$ such that:

(i) *Utility maximization*: For each $i \in I$, W and (c, k') solve the following problem given k_0, ϵ_0, Π, Γ and (r, w):

$$W(\epsilon, k; \Psi) = \max_{c,k'} \Big\{ u(c) + \beta \sum_{\epsilon'} \Pi\big(\epsilon' \mid \epsilon\big) W\big(\epsilon', k'; \Psi'\big) \Big\}$$

$$\text{s.t.} \quad c + k' = w(K)(1-\tau_l)\epsilon + \big(1 + r(K)\big(1 - \tau_k(k)\big)\big)k,$$

$$\Psi' = \Gamma[\Psi],$$

$$k' \geqslant \underline{k}\big(\epsilon'; \Psi'\big) \quad \text{for all } \epsilon'|\epsilon \quad \text{with } \Pi\big(\epsilon' \mid \epsilon\big) > 0. \tag{8}$$

(ii) *Profit maximization*: Factor prices satisfy the firm's optimality conditions, i.e., $w(K) = Af_L(K, L)$ and $r(K) = Af_K(K, L) - \delta$.

(iii) *Balanced budget*: The government budget constraint is satisfied, i.e., $G = w(K)\tau_l L + r(K)\boldsymbol{\tau}_k \widehat{K}$, where

$$\widehat{K} = \int_{k \geqslant 0} k \, d\Psi(\epsilon, k).$$

(iv) *Market clearing*:

$$\int k(\epsilon, k; \Psi) \, d\Psi(\epsilon, k) = K',$$

$$\int \epsilon \, d\Psi(\epsilon, k) = L,$$

$$\int \big[c(\epsilon, k; \Psi) + k(\epsilon, k; \Psi)\big] \, d\Psi(\epsilon, k) + G = Af(K, L) + (1-\delta)K.$$

(v) *Consistency*: Γ is consistent with the agents' optimal decisions, in the sense that it is generated by the optimal decision rules and by the law of motion of the shock.

(vi) *No default*: $\underline{k}(\epsilon; \Psi)$ is such that individuals are indifferent between trading and going into autarky, i.e.,

$$\underline{k}(\epsilon; \Psi) = \big\{\underline{k} : W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)\big\}. \tag{9}$$

Several remarks are worth noting. First, as reflected in condition (i), households are only allowed to hold levels of individual capital that are above a state-dependent lower bound for each continuation state with positive probability next period. This implies that the effective limit on capital holdings $\kappa(\epsilon; \Psi)$ faced by a household is the tightest among these state-dependent lower bounds. Using the recursive notation, the effective borrowing constraints can therefore be expressed as[5]:

$$k' \geqslant \kappa(\epsilon; \Psi) \equiv \sup_{\epsilon' : \Pi(\epsilon'|\epsilon) > 0} \big\{\underline{k}\big(\epsilon'; \Gamma[\Psi]\big)\big\}. \tag{10}$$

Second, the definition of the state-dependent lower bounds in (9) implies that we can think about $\underline{k}(\epsilon; \Psi)$ as a state-dependent default threshold, since it represents the level of capital holdings such that households are indifferent between defaulting and paying back their debt. Clearly,

[5] If the probability of all future shock realizations is strictly positive for any given shock, the effective limit faced by the households will not be a function of the current shock, since the trading restriction has to be satisfied for all possible continuation states. This will not be the case, however, in our calibrated example.

condition (vi) implies that we only consider equilibria where the trading limits are such that default is not possible. Whereas there are many borrowing limits that prevent default in equilibrium, we consider the loosest possible ones of such limits.

Finally, it is important to note that the default thresholds are very closely related to the endogenous borrowing limits on Arrow securities that are defined in the literature with complete markets and limited commitment. Among others, Alvarez and Jermann [6] and Ábrahám and Cárceles-Poveda [1] define these limits in endowment and production economies, respectively.

2.1. Characterization of the endogenous default thresholds

This section provides some theoretical results that show the existence of a unique lower bound $\underline{k}(\epsilon;\Psi)$ satisfying Eq. (9). Furthermore, it characterizes the dependence of $\underline{k}(\epsilon;\Psi)$ on the labor income shock. All the proofs are relegated to Appendix B.

Proposition 2.1. *If u is unbounded below, Eq.* (9) *defines a unique, non-positive and finite default threshold $\underline{k}(\epsilon;\Psi)$ for every ϵ and Ψ.*

The proof of this proposition extends Zhang [34,35], who characterizes the default thresholds in exchange economies. In particular, the existence of the default thresholds established by Proposition 2.1 is a consequence of the fact that $V(\epsilon;\Psi)$ is finite whenever ϵ and K is bounded away from zero, while $W(\epsilon,k;\Psi)$ goes to minus infinity as k goes to the natural borrowing limit. In addition, uniqueness simply follows from the fact that $V(\epsilon;\Psi)$ does not depend on k while $W(\epsilon,k;\Psi)$ is strictly increasing in k. An important implication of uniqueness is the fact that the value of staying in the trading arrangement is always higher than the autarky value if the capital holdings are above the default threshold, that is,

$$W(\epsilon,k;\Psi)\geqslant V(\epsilon;\Psi)=W\big(\epsilon,\underline{k}(\epsilon;\Psi);\Psi\big)\quad \text{for } \forall k\geqslant \underline{k}(\epsilon;\Psi).$$

The fact that the thresholds are finite is a consequence of the fact that $V(\epsilon;\Psi)$ is finite. Finally, the equilibrium default thresholds and effective limits have to be clearly non-positive. Intuitively, note that agents would not default with a positive level of asset holdings, since they could then afford a higher current consumption than in autarky and at least as high of a life-time utility as in autarky from next period on by paying back their debt.

To characterize the dependence of $\underline{k}(\epsilon;\Psi)$ on the labor income shock, we assume differentiability of both the trading and autarky values. In addition, to make the exposition easier and to be able to express the differential effect of a change in ϵ on the thresholds, we assume that the idiosyncratic shock follows a continuous $AR(1)$ process that is given by:

$$\log\big(\epsilon'\big)=\mu_\epsilon+\rho_e\log(\epsilon)+\varepsilon'_\epsilon\quad \text{with } \varepsilon'_\epsilon\sim N\big(0,\sigma_\epsilon^2\big).$$

Denote the individual policy functions by $k'=g_k(\epsilon,k;\Psi)$ and $c=g_c(\epsilon,k;\Psi)$. To express the effects of a change in ϵ, we can differentiate Eq. (9), obtaining that:

$$\frac{\partial\underline{k}(\epsilon;\Psi)}{\partial\epsilon}=-\frac{W_\epsilon(\epsilon,\underline{k};\Psi)-V_\epsilon(\epsilon;\Psi)}{W_k(\epsilon,\underline{k};\Psi)}. \tag{11}$$

In the previous equation, $W_\epsilon(\epsilon,\underline{k};\Psi)$ and $V_\epsilon(\epsilon;\Psi)$ represent the derivatives of the two value functions, evaluated at $\underline{k}(\epsilon;\Psi)$, with respect to the income shock ϵ. Similarly, $W_k(\epsilon,\underline{k};\Psi)$ represents the derivative of the trading value, evaluated at $\underline{k}$, with respect to k.

Since more individual capital holdings (ceteris paribus) expand the budget sets, and because the utility function is strictly increasing , it follows that $W_k(\epsilon,\underline{k};\Psi)>0$. Given this, the sign of the previous derivative is determined by wether a change in income increases the trading value W more or less than the autarky value V. If the trading value increases more than the autarky value after an increase in the income shock, the derivative will be negative. In this case, a higher income will lead to looser default thresholds. For our main characterization result, we will use the results of the following lemma.

Lemma 2.1. *At the default threshold, agents consume less in the trading arrangement than they would in autarky, i.e.,* $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi) \leqslant w(K)(1-\tau_l)\epsilon(1-\lambda)$.

The previous lemma implies that agents who are at the default threshold have a higher current consumption in autarky. Note that, if this was not the case, agents would be strictly better off by consuming $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi)$ at the threshold and defaulting next period. However, this would contradict either the definition of the default threshold or the fact that W is the maximal life-time utility that households can achieve. We are now ready to state the proposition that shows the dependence of $\underline{k}(\epsilon; \Psi)$ on the labor income shock.

Proposition 2.2. *Assume that* ϵ *is i.i.d. for each agent. Then, ceteris paribus, the higher is the productivity shock of an agent, the looser are the default thresholds, i.e.* $\frac{\partial \underline{k}(\epsilon; \Psi)}{\partial \epsilon} \leqslant 0$.

Proposition 2.2. shows that the endogenous default thresholds become looser with a higher productivity level. The result of the proposition follows from Lemma 2.1 and from the fact that households are risk averse. To get more intuition, suppose that the level of productivity ϵ increases by $\frac{\Delta\epsilon}{w(1-\tau_l)}$ for a small $\Delta\epsilon$, while the limits are kept at $\underline{k}(\epsilon; \Psi)$. Note that this implies that the after tax labor income $w(1-\tau_l)\epsilon$ increases by $\Delta\epsilon$. It is easy to see that, for small changes in income, the change in the autarky value is approximated well by $\Delta\epsilon u'(c^{au})(1-\lambda)$. Similarly, the change in the value of staying in the contract can be well approximated by $\Delta\epsilon u'(c)$. The concavity of the utility function together with Lemma 2.1 implies that the latter expression is larger than the former. In turn, this implies that for a productivity level of $\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)}$ and an asset position of $\underline{k}(\epsilon; \Psi)$ the agent's participation constraint is satisfied with strict inequality[6]:

$$W\left(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)}, \underline{k}(\epsilon; \Psi); \Psi\right) > V\left(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)}; \Psi\right). \tag{12}$$

The previous condition, together with the fact that W is increasing in assets, imply that $\underline{k}(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)}; \Psi) < \underline{k}(\epsilon; \Psi)$, namely, the level of debt which makes agents indifferent between defaulting and paying back is increasing in income (productivity).

Several remarks are worth noting. First, the ability to borrow is a positive function of income in the data, as we will show in Section 3. Given this, Proposition 2.2 is a desirable property of the present setting.

Second, the above finding may seem somewhat surprising, since it is often argued in the literature on optimal risk sharing with limited enforceability that agents with a higher income have more incentives to default. While this is also true in our model, in the sense that higher income

[6] Another way of seeing this result is to consider two individuals with the same life-time utility. One has debt and access to some intertemporal smoothing technology through financial markets and the other has no access to this technology and no debt. Giving both individuals additional income in a given period will improve welfare more for the agent who can distribute this additional income across periods relative to the one who needs to consume this extra income in that period.

shocks lead to a higher autarky value, Proposition 2.2. shows that this effect does not necessarily translate into tighter borrowing limits, since the value of staying in the trading arrangement increases by even more after an increase in income. Of course, a key aspect for obtaining this result is the fact that there is imperfect risk sharing. Given this, a similar mechanism will operate in environments with a complete set of financial assets and the possibility of default, where risk sharing is imperfect whenever the (endogenous) borrowing constraints on Arrow securities are binding. The key observation is that consumption responds to changes in income under both exogenous and endogenous incomplete market models whenever the agents participation (borrowing) constraint binds.

Third, under i.i.d. shocks, it is also possible to show how the endogenous default thresholds as a proportion of after tax income change as income increases. To do this, note that in the i.i.d. case the relevant state variable in the model is total (disposable) wealth $\omega(k,\epsilon;\Psi) \equiv w(K)(1-\tau_l)\epsilon + (1+r(K)(1-\tau_k(k)))k$, where $\tau_k(\underline{k}(\cdot,\cdot)) = 0$ due to the fact that $\underline{k}(\cdot,\cdot) \leqslant 0$. Since the autarky value is increasing in income, we also know that $V(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi) > V(\epsilon;\Psi)$. This, together with the definition of the endogenous limits in (9), imply that:

$$\tilde{W}\left(\omega\left(\underline{k}\left(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi\right), \epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi\right);\Psi\right) > \tilde{W}\left(\omega\left(\underline{k}(\epsilon;\Psi),\epsilon;\Psi\right);\Psi\right),$$

where $\tilde{W}$ is the value function defined over wealth. Therefore,

$$\omega\left(\underline{k}\left(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi\right), \epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi\right) > \omega\left(\underline{k}(\epsilon;\Psi),\epsilon;\Psi\right).$$

This means that the level of disposable wealth which makes agents indifferent between defaulting or not has to increase with income. Since Proposition 2.2 has shown that $\Delta\underline{k} \equiv \underline{k}(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi) - \underline{k}(\epsilon;\Psi)$ is negative, this can only be the case if disposable income grows more than debt (as long as the interest rate is non-negative). More formally, the previous inequality implies that:

$$\Delta\epsilon + \left(1 + r(K)\right)\Delta\underline{k} > 0 \quad \Longleftrightarrow \quad \frac{\Delta\underline{k}}{\Delta\epsilon} > \frac{-1}{1+r(K)} > -1.$$

If we define $\Phi(\epsilon;\Psi) \equiv \frac{\underline{k}(\epsilon;\Psi)}{w(K)(1-\tau_l)\epsilon}$ as the fraction of after tax income the agent can borrow for a given level of productivity shock, we obtain the following expression:

$$\Phi\left(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi\right) = \frac{\underline{k}\left(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi\right)}{w(K)(1-\tau_l)\epsilon + \Delta\epsilon} = \frac{\underline{k}(\epsilon;\Psi) + \Delta\underline{k}}{w(K)(1-\tau_l)\epsilon + \Delta\epsilon}.$$

When income grows, the previous expression implies that the denominator increases more than the numerator in absolute value. Therefore, if $\Phi(\epsilon;\Psi) \leqslant -1$, we have that $\Phi(\epsilon + \frac{\Delta\epsilon}{w(1-\tau_l)};\Psi) > \Phi(\epsilon;\Psi)$.[7] In other words, the debt limit as a fraction of after-tax labor income gets tighter as labor income increases. As we will see later, this property holds both in the data and in the extended version of the model with persistent income shocks.

Last, we would like to emphasize that the assumption of i.i.d. shocks is crucial to prove Proposition 2.2. The reason is that, if $\rho_\epsilon > 0$, the levels of future consumption also become important in determining the sign of the derivative of the limit in (11), and the relationship between income

[7] If $\Phi(\epsilon;\Psi) > -1$, we cannot obtain analytical results. However, if this ratio is not too far above -1, we should expect the same pattern to hold. We will see that both in the data and in our quantitative model this ratio is less than or around -1.

and limits found above may be weakened or reversed. As we will show numerically in Section 4, however, the results of the proposition are robust to persistent shocks.

3. Credit limits and income in the data

This section documents several facts about the relationship between credit limits and individual income in the data. These facts will help us to evaluate the empirical relevance of Proposition 2.2 as well as the predictions of a calibrated version of the model regarding the borrowing constraints. Here, we would like to stress that, on the one hand, our model does not have default in equilibrium and, for this reason, agents face the same interest rate independently of their income and debt levels. In the data, lenders use both the interest rate and the credit constraints to discriminate between borrowers, since agents may have different (non-zero) default probabilities. Nevertheless, as we will see below, our model is well in line with the data in terms of its predictions regarding how the limits and (labor) income are related. Note that this should not be surprising, since we expect that our results regarding the characterization of the limits remain valid in a model with equilibrium default. Our conjecture is based upon the fact that, in those models, the limits are also determined by some indifference condition between paying back the household's debt or defaulting on it.

Our data source is the 2004 Survey of Consumer Finances. We only consider heads of households that are working full time and report a positive labour income and credit card limit. Our income measure is the annual labor income of the heads of households. Our income data is constructed using survey questions regarding earnings and labor supply (number of weeks worked per year).[8] As to the borrowing limits, the best available information is based on a question that asks the heads of households how much they can borrow in total on all their credit card accounts.

The left panel of Fig. 1 depicts the borrowing limits as a function of labor income. Further, the right panel plots the borrowing limits as a proportion of labor income against labor income. The solid lines display data using deciles of the income distribution, taking averages within a decile. The dashed lines are the predicted borrowing limits from a regression where a third order polynomial of income, together with age, gender and education, are used to explain the limit. The figures show the predicted limits for men with the average age and educational level of the sample.

As we see on the left panel of the figure, there is clearly a negative relationship between the level of income and the credit limits in the data. In other words, higher income people have a higher ability to obtain unsecured credit. While this is consistent with the findings of Proposition 2.2, note that the latter relied on the fact that income shocks are i.i.d. Since it is well documented in the literature that income shocks are persistent, we solve numerically a calibrated version of the model with persistent shocks to see if the results of the proposition are robust to this extension.

Moreover, the right panel of the figure shows that the negative relationship between credit limits and income is reversed when we plot the limits as a proportion of labor income. Essentially, this implies that people with a higher income can borrow only up to lower proportion of their income. This confirms that our characterization from the previous section based upon i.i.d. shocks

[8] Using alternative definitions of labor income based upon W2 forms and total household income, we obtained very similar results.

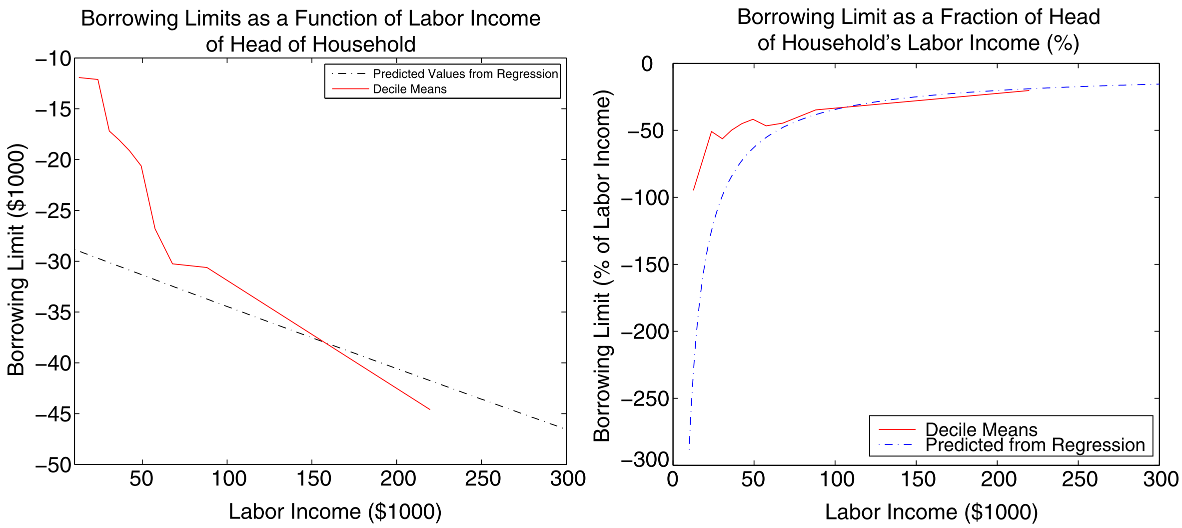

Fig. 1. Limits as a function of labour income in the data.

holds in the data. In the next section, we will also test the predictions of the model with persistent shocks against this fact.

4. Quantitative results

This section studies the stationary distribution of a calibrated version of the model described above. Note that, in the steady state, all aggregate variables, including the asset distribution, government consumption, taxes, the aggregate capital and factor prices are constant. First, we discuss the calibration and solution method for the benchmark economy. Next, we study the properties of the endogenous borrowing limits, particularly the relationship between these limits and income.

4.1. Calibration and solution method

One of the main objectives of the calibration is that the model steady state matches the earnings and wealth distribution in the US. In addition, we target several aggregate statistics, such as the labor share and the investment or capital to output ratios in the US data.

The time period is assumed to be one year. Preferences are of the CRRA class, $u(c) = \frac{[c^{1-\mu}-1]}{1-\mu}$, with a risk aversion of $\mu = 2$. The production function is Cobb Douglas, $f(K, L) = AK^{\alpha}L^{1-\alpha}$, where $\alpha = 0.36$ is chosen to match the labor share of 0.64 in the US data and the technology parameter A is normalized so that output is equal to one in the steady state of the deterministic economy. The depreciation rate is set to $\delta = 0.08$ to match the annual investment to capital ratio in the US and the discount factor $\beta = 0.93$ is set to match a capital to output ratio of 3.32, which is the value reported for the US in Cooley and Prescott [13]. This generates an interest rate of 2.8%.

As to the tax rates, we choose $\tau_k = 0.40$ and $\tau_l = 0.277$, which are very close to the tax rates found by Domeij and Heathcote [16] using the method of Mendoza et al. [29]. With these taxes, the government to output ratio is equal to 21% in the benchmark economy, which is very close to the government to output ratio of 19% in the US data.

Table 1
Earnings process.

$$\epsilon = [0.1018 \quad 0.2192 \quad 0.5817 \quad 1.3045 \quad 2.9057 \quad 8.9879 \quad 16.0170]$$

$$\boldsymbol{\Pi}^* = [0.0996 \quad 0.2256 \quad 0.4775 \quad 0.1092 \quad 0.0545 \quad 0.0294 \quad 0.0062]$$

$$\boldsymbol{\Pi}(\epsilon' \mid \epsilon) = \begin{bmatrix} 0.9400 & 0.0213 & 0.0387 & 0 & 0 & 0 & 0 \\ 0.0265 & 0.8500 & 0.1235 & 0 & 0 & 0 & 0 \\ 0 & 0.0667 & 0.9180 & 0.0153 & 0 & 0 & 0 \\ 0 & 0 & 0.0666 & 0.8669 & 0.0665 & 0 & 0 \\ 0 & 0 & 0 & 0.1334 & 0.8000 & 0.0666 & 0 \\ 0 & 0 & 0 & 0 & 0.1235 & 0.8320 & 0.0445 \\ 0 & 0 & 0 & 0 & 0 & 0.2113 & 0.7887 \end{bmatrix}$$

Table 2
The wealth distribution in the benchmark model and in the data.

Economy	Quintiles					% in debt
	Q1	Q2	Q3	Q4	Q5	
Benchmark (pre-reform)	−1.50	0.39	2.07	7.24	91.7	24.49
USA (net financial assets)	−1.55	0.09	1.61	8.66	91.2	24.31
USA (net worth)	−0.18	1.13	4.37	17.10	82.90	7.14

Table 1 describes the earnings process, which is a seven state Markov chain. The table displays the shock values, the stationary distribution and the transition matrix. Finally, the default penalty is set to $\lambda = 0.178$.[9] The income process and the default penalty are calibrated to match the proportion of people in debt as well as a realistic income and wealth distribution in the benchmark steady state. In particular, the Gini coefficient for earnings is equal to 0.58, which is very close to the targeted Gini of 0.6 in the US data. We also target the percentage of people in debt and the total financial assets held by the lowest and highest quintiles of the US wealth distribution.

Table 2 contains information about the wealth distribution in our benchmark model and in the 2004 Survey of Consumer Finances. Since the present paper is about unsecured credit, we have tried to match some key moments of the distribution of net financial assets. In contrast, most of the macroeconomic literature focuses on the wealth distribution based on net worth, defined as the difference between total assets and total liabilities. When calculating net financial assets, we exclude the value of residential property, vehicles and direct business ownership from the assets, and the value of secured debt due to mortgages and vehicle loans from the liabilities. This level of assets represents better the amount of liquid assets that households can use to smooth out income shocks. Moreover, both residential properties and vehicles can be seen as durable consumption as much as investment.

As we see in Table 2, according to the 2004 Survey of Consumer Finances, the lowest quintile of the wealth distribution, as measured by net financial assets, holds −1.55% of total financial wealth, whereas 91.2 percent is held by the highest quintile. The asset holdings of these two

[9] As discussed in Livshits, MacGee and Tertilt [27], bankruptcy filers face several types of punishment. Apart from the fact that filers cannot save or borrow, a fraction of earnings may be garnished by creditors in the period of filing. In addition, there are utility (stigma) and fixed monetary costs of filing. To match key observations regarding the evolution of bankruptcy filings in the last decades, the authors choose a garnishment rate of 0.319 and set the other costs to zero. Given this $\lambda = 0.178$ does not seem to be excessively high.

quintiles are among our calibration targets and are thus matched very well. In addition, we also match relatively well the asset holdings of the three medium quintiles, although they were not targeted. We also target and match well the proportion of the population in debt in the data: 24.31% (including the individuals with zero net financial assets). In this respect, our model is more reasonable than alternative models studying tax reforms in a similar framework, such as Aiyagari [4] and Domeij and Heathcote [16], who assume no borrowing and thus cannot capture the effect of a reform on the substantial percentage of people in debt.

Solution method

To find the solution, we use a policy function iteration algorithm that is described in detail in Appendix C. Solving the stationary distribution of the model with endogenous trading limits involves several computational difficulties. First, our state space is endogenous, a problem that we address by incorporating an additional fixed point problem to find the state-dependent limits on the individual capital holdings. This also implies that our policy functions have to be calculated over a non-rectangular grid. Further, given that the limits in our model are endogenously determined at the level where the value function from staying in the contract is at least as large as the autarky value, it becomes clear that a good approximation of the value functions close to the limits is needed to obtain reliable results. To address this issue, we use a relatively high number of grid points, we interpolate the policy and value functions over this grid and we allow the limits to take values between grid points as well. In order to speed up the solution procedure, we update the interest rate and the borrowing limits simultaneously.

In order to evaluate the welfare effect of tax reforms, we have also computed the transition of our economy between stationary distributions due to changes in the tax code. The extra difficulty of this exercise is that not only factor prices (due to the accumulation of aggregate capital), the distribution of individuals over asset holdings and labor income change during the transition, but also the endogenous borrowing constraints. We have performed this exercise in two steps. First, we assume that the limits jump immediately to the levels of the second steady state, and compute the transition dynamics for all the other aggregate variables (factor prices and the distribution) using requirements (i)–(v) of the recursive competitive equilibrium. Then, using the solution of the first step, we adjusted the limits and the rest of the aggregate variables such that all the requirements of the competitive equilibrium (including the definition of the endogenous limits in (vi)) are satisfied. The rationale behind this two-step procedure is that, as we will see later, the limits do not affect the transition of the aggregate variables to a large extent. So, in the second step, there are only small adjustments to be made with respect to the time path of the prices.

4.2. Endogenous limits in the benchmark economy

The endogenous limits in the benchmark economy are displayed in Fig. 2. The left panel of the figure shows the level of the endogenous borrowing limits as a function of income, while the right panel plots the limits as a fraction of income against income. The first pattern we observe is that the endogenous limits exhibit a similar behavior to the one in the data. In particular, they get looser with income. This finding confirms that the results of Proposition 2.2 are robust to the presence of persistent shocks. Further, the limits as a proportion of income get tighter with higher income, as in the US data. Again, this is consistent with the analytical derivations we performed for the i.i.d. case in Section 2.1.

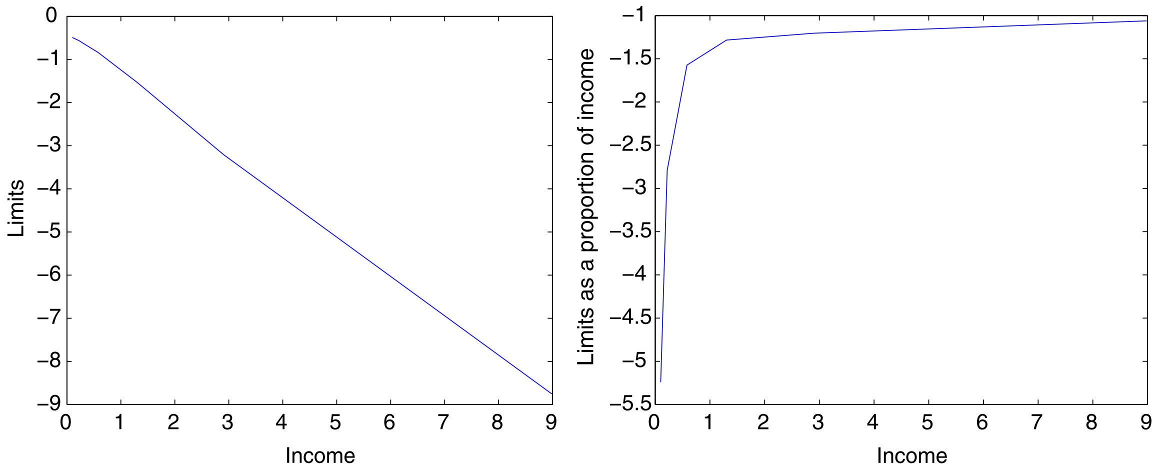

Fig. 2. Limits as a function of labour income in the model.

5. Welfare effects of tax reforms

5.1. Long run

This section analyzes the long run welfare implications of a revenue neutral tax reform that eliminates capital income taxes at the expense of a higher labor income tax. We study two different reforms. In the first, the government replaces the loss of capital income tax revenue by simply increasing (linear) labor income taxes. We label this reform as linear. In the second, the elimination of capital income taxes is accompanied by a transformation of the linear tax system into a progressive tax system. In particular, the government keeps the tax rate of the lowest two income groups the same and increases labor taxes for the rest. We label this reform as progressive. In what follows, τ_p and τ_r denote the tax rates faced by the two lowest groups and by the rest, respectively. Since high income agents tend to have higher asset income, the second reform has the advantage that the beneficiaries of the reform pay the costs. The steady state results for the two cases are displayed in Table 3.

The first four rows of the table display the results for the linear reform, while the last four rows display the results for the progressive reform. Further, the rows initial and full correspond to the pre-reform and post-reform steady states respectively. Note that an important difference between our framework and a fixed limit economy is that the endogenous borrowing limits are affected by the tax reform, since a change in tax policy influences the relative value of default. To disentangle the effect on the limits from the rest, we have also solved for the interim cases in which the reforms are implemented but the endogenous limits are kept the same as in the pre-reform economy. This exercise can also answer the question of what we would conclude about the welfare effects of the tax reform if we had ignored the effect of the reform on the endogenous borrowing constraints. Finally, the different columns of the table display the tax rates $(\boldsymbol{\tau}_k, \tau_p, \tau_r)$, the aggregate capital K, the two factor prices (r, w), the after tax wage rates $w_p^\tau = w(1-\tau_p)$ and $w_r^\tau = w(1-\tau_r)$ for the (income) poor and rich, the after tax (saving) interest rate $r^\tau = r(1-\tau_k)$ (recall that the borrowing interest rate is always equal to r) and the aggregate welfare in consumption equivalent terms W.

We first consider the linear interim reform in which the limits are kept at the level of the pre-reform steady state. The reform, as expected, leads to a higher level of aggregate capital. However, it leads to a small decrease in aggregate welfare. We can get some insight about this

Table 3
Steady state results for the linear and progressive reforms.

	$(\boldsymbol{\tau}_k, \tau_p, \tau_r)$	K	r	w	w_p^τ	w_r^τ	r^τ	W
Linear								
Initial	(0.4, 0.277, 0.277)	4.775	2.815	0.917	0.663	0.663	1.689	100
Interim	(0.0, 0.326, 0.326)	5.181	2.265	0.945	0.636	0.636	2.265	99.76
Full	(0.0, 0.327, 0.327)	5.156	2.297	0.943	0.634	0.634	2.297	98.66
Progressive								
Initial	(0.4, 0.277, 0.277)	4.775	2.815	0.917	0.664	0.664	1.689	100
Interim	(0.0, 0.277, 0.331)	5.112	2.353	0.940	0.680	0.629	2.353	101.62
Full	(0.0, 0.277, 0.332)	5.080	2.394	0.938	0.678	0.627	2.394	100.29

Table 4
Effective limits in the linear and progressive reforms.

	$\kappa(\epsilon_1)$	$\kappa(\epsilon_2)$	$\kappa(\epsilon_3)$	$\kappa(\epsilon_4)$	$\kappa(\epsilon_5)$	$\kappa(\epsilon_6)$	$\kappa(\epsilon_7)$
Linear							
$\tau_k = 0.4$	−0.4899	−0.4899	−0.5619	−0.8398	−1.5358	−3.2103	−8.7564
$\tau_k = 0$	−0.5731	−0.5731	−0.6419	−0.9085	−1.5803	−3.1987	−8.5491
Progressive							
$\tau_k = 0.4$	−0.4899	−0.4899	−0.5619	−0.8398	−1.5358	−3.2103	−8.7564
$\tau_k = 0$	−0.5853	−0.5853	−0.6584	−0.9126	−1.5658	−3.1524	−8.4201

welfare reduction by comparing interest rates and wages before and after the reform. On the one hand, as aggregate capital increases, r and consequently the borrowing interest rate drop. At the same time, the after tax saving interest rate r^τ increases. Both of these effects improve welfare. On the other hand, labor income taxes increase significantly (from 0.277 to 0.326), which will reduce welfare for all agents (note that after tax wages w^τ drop in spite of higher gross wages). The decrease in after-tax wages has a small effect on relatively asset-rich agents, for whom capital is more important, while these agents benefit from the higher r^τ. However, asset-poor agents may benefit from a lower borrowing rate, but they are particularly hurt by the lower after-tax wage rate, since they rely primarily on labor income. Overall the welfare losses of the poor seem to dominate slightly the gains, leading to a relatively small aggregate welfare loss in the long run when the interim reform is implemented.

Consider now the situation where the endogenous limits are allowed to change. The changes in the limits for the two reforms are displayed in Table 4. First of all note that the limits get looser for most income groups. (They become tighter only for the two highest income groups, but these are the least relevant groups because they are not expected to borrow.) The reason for this is that borrowing becomes cheaper after the reform and this decreases the incentives to default and allows for more borrowing. As we can see from Table 4, the full linear reform has very similar effects on factor prices as the interim reform, that is, the aggregate statistics of this economy do not depend much on the level of the limits. However, there is a significant welfare reduction compared to the interim case. This is because the looser limits imply that the poorest individuals in terms of assets get more indebted and end up being worse off in the long run after the reform. We should expect that this effect is (at least) partially offset by the fact that, along the transition, looser limits allow for better consumption smoothing. As we will see in the next subsection, this is indeed the case.

Finally, we analyze the progressive tax reform. As conjectured earlier, this second reform leads to (aggregate) welfare improvements both in the interim and in the full reform cases. First of all, the effect on the factor prices and the aggregate capital is very similar to the linear reform. The main difference is that relatively poor agents (both in assets and labor income) now benefit considerably, since they face a lower borrowing rate and a higher after-tax wage rate. We also see that labor taxes for the higher income groups τ_r increase by more than in the linear reform, but overall the loses induced by this are more than offset by the gains from the poor. Finally, Table 4 reflects that the borrowing limits move in the same direction as in the linear reform for the reasons stated above. As a result, the welfare improvement of the full reform is much lower than in the interim case.

To summarize, the linear tax reform is welfare decreasing in the long run, whereas the progressive reform is welfare enhancing. However, in both cases, welfare decreases considerably when we take into account the effects of taxes on the borrowing limits. We expect this latter effect to be mitigated when we take into account the transition for the full reform (in which limits get looser), but we should not expect things to change much when we consider the transition for the interim case.

Several remarks are worth noting. First, the welfare outcome in the linear reform is consistent with the findings of Aiyagari [4], who first studied the optimal capital income taxation in a model with incomplete markets and no borrowing. In contrast to the seminal papers of Chamley [10] and Judd [19], who show that the optimal capital income tax is zero in the long run for a wide class of infinite horizon models, Aiyagari [4] shows that the optimal capital income tax is always strictly positive in models with incomplete asset markets. In particular, the author argues that the precautionary savings motive that arises in the presence of uncertainty implies that capital is over accumulated above the optimal level, calling for a positive tax rate. Note that, in Aiyagari's model, agents are not allowed to borrow. In contrast to these results, a recent paper by Davila et al. [14] shows that when the income of the poor is mostly composed of labor earnings (as in our case), the aggregate capital achieved by the market allocation is actually too low. This result would therefore suggest that capital needs to be subsidized rather than taxed. A key difference between the models is that Aiyagari [4] assumes distortionary labor taxes, while in Davila et al. [14] allow for optimally designed and person-specific non-distortionary and non-linear labor taxes. Note that this latter property is more in line with our progressive reform.

Second, regardless of the reform we study, the previous results clearly illustrate that it is very important to take into account the effects on the borrowing constraints for the optimal design of government policy. In particular, one would obtain the wrong conclusion regarding the quantitative long run welfare effects of a revenue neutral tax reform if one does not take into account the effects of the reform on the borrowing constraints.

Last, the analysis in the present section does not provide us with a final answer regarding the desirability of the tax reform we consider because as we pointed out it does not take into account the potential welfare costs and benefits during transition towards the new steady state. This is discussed in what follows.

Fig. 3. Aggregate paths in the linear and progressive interim reforms.

5.2. *Transition and welfare*

We start by discussing the results of the interim reforms, which maintain the borrowing limits at the pre-reform level. The transitional path for some of the key aggregate variables are displayed in Fig. 3.[10]

As we see, capital increases smoothly throughout the transition in the two reforms. This is due to the elimination of capital income taxes, which leads to an increase in the after-tax interest rate r^{τ} (the figure shows the borrowing interest rate, but the after tax savings rate follows the same pattern shifted upwars by 67%). The figure also reflects a sharp decrease in after tax wages w_r^{τ} due to the initial increase in labor income taxes. Note that this increase is higher than in the long run due to the fact that capital is still low during the initial periods. In turn, this implies that aggregate welfare decreases on impact in both the linear and the progressive reforms, but the decrease is considerably lower in the latter. In particular, whereas there is a long run welfare loss of -0.24 percent under the linear interim reform and a gain of 1.62 percent under the progressive reform, the welfare loss at impact is of -4.51 percent under the linear interim reform and of -0.66 percent under the progressive reform.

As explained before, the progressive reform does not increase labor income taxes for the two lowest income groups, whereas the rich experience a higher increase in taxes. This mitigates the negative impact on aggregate welfare, which is still negative. Finally, the bigger reduction in disposable income of the rich in the progressive tax reform decreases their savings and therefore the aggregate capital with respect to the linear reform. Hence, our conclusion, on the basis of the interim case, would be that neither welfare reform is desirable after the effect of the transition is taken into account. Recall that when only the long-run effects were taken into account, the progressive case is welfare improving and the linear case has small welfare costs. However,

[10] In order for the government's budget constraint to hold during the transition, we have adjusted the labor income tax rate (τ_r in the case of the progressive reform) in the reforms we have considered.

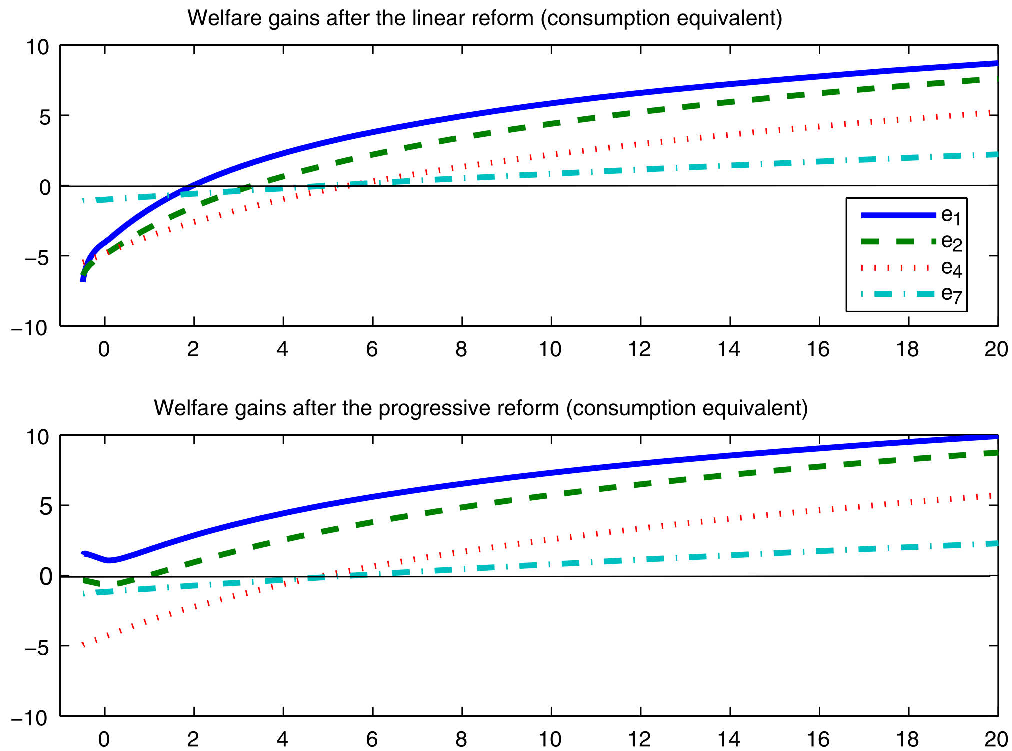

Fig. 4. Welfare gains in the linear and progressive interim reforms.

when the transition is taken into account, there are large welfare costs under the linear reform, while there is a small welfare loss instead of a welfare gain under the progressive reform.

Fig. 4 displays the welfare gains due to the interim reforms in consumption equivalent terms for individuals with different income shocks and asset levels. The upper panel of the figure displays the welfare gains in the linear interim reform and the lower panel displays the gains in the progressive interim reform. This figure is important for two reasons. It shows who are the agents who would be in favour and against the reforms. Also, it indicates whether these reforms could have public support or not.

Several important observations emerge from the figure. First, we see that, typically, the higher is the asset wealth of a given individual, the more he/she prefers any of the two reforms. This is not surprising, as agents with a higher asset wealth benefit from a higher saving interest rate in the two reforms. Second, for agents with high asset levels, the lower is the labor income of a given individual, the more he/she favours the reform. This is because among agents with the same asset level, agents with lower income levels rely less on labor income in relative terms and therefore the increase in labor income taxes hurts them the least. Now let's focus on the difference between the two reforms.

In the linear reform, for agents with low level of assets or with debt, the lower is labor income the less they support the reform. This is due to the fact that for these agents labor income is practically the main or only source of income and hence the increased labor income taxes will hurt them significantly. When we aggregate over the population across asset levels and income levels using the stationary distribution of the first steady state, we see that the overall political support for the linear interim reform is 17 percent.

A different picture emerges, however, when we look at the results for the progressive interim reform, which obtains a support of 30%. As expected, this increase is mostly coming from the

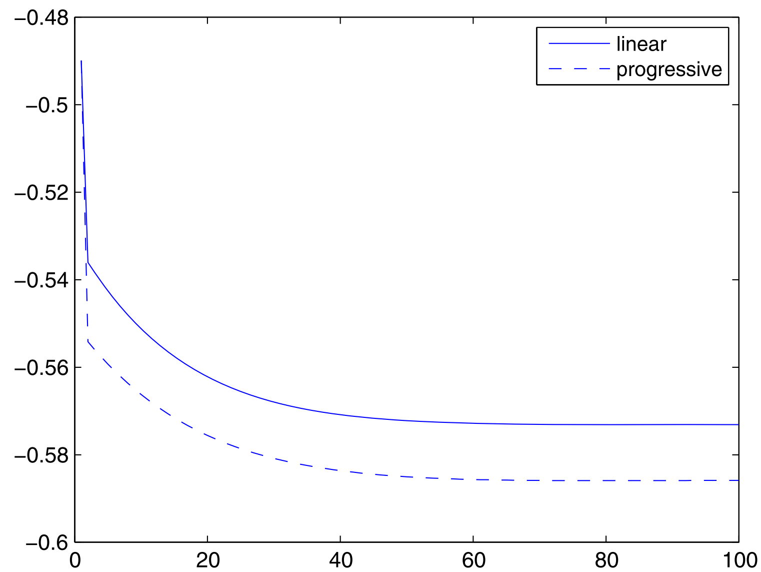

Fig. 5. Borrowing limit for the lowest two income groups.

higher gains of the lowest two income groups. In particular, the lowest income group supports the reform for any asset level, while the second lowest income group supports it for almost all positive asset levels. Since the labor taxes are kept the same for both these groups, their after tax wages do not drop upon impact and they face lower borrowing interest rates from the first period on. Moreover, since shocks are persistent, they expect to see increasing after tax wages during the transition because aggregate capital will increase. These facts explain why they are doing better than in the linear reform. Households from the second lowest income group with low assets are not in favour because they have higher probability of moving to higher income levels in the future and, in that case, they have to pay higher labor income tax rates. Finally, the figure reflects that welfare changes for these two low income groups are non-monotone for debt and low asset levels. The reason is that both agents with negative and positive assets benefit instantaneously from better borrowing and saving interest rates, while agents with no assets do not benefit directly from the interest rate effects.

In what follows, we analyze the two reforms for the case in which the endogenous limits react to policy changes. The evolution of the limits for the first two income groups (note that $\kappa(\epsilon_1) = \kappa(\epsilon_2)$) is displayed in Fig. 5 (since typically agents from these income groups are borrowing constrained, this is the relevant limit for the policy exercise), while Figs. 6 and 7 display the same variables as in the interim reforms. As we see, the behaviour of the borrowing limits is very similar across the two reforms, they become considerably looser at impact and then move smoothly to their long run value, which is also very similar across reforms. Clearly, this will lead to a possibility of more consumption smoothing at impact and will benefit particularly the poor.

It is clear from Fig. 6 that the path of the aggregate variables is only slightly adjusted due to the fact that the limits are adjusted during the transition. However, we see a very different picture for aggregate welfare. As we conjectured before, aggregate welfare is considerably higher for the two reforms when the effect of the limits is taken into account. More importantly, the main conclusion about the aggregate desirability of the progressive reform dramatically changes when the effect of the limit is internalized. In the interim case, we obtain a welfare loss of -0.66 percent, while there is a welfare gain of 1.26 percent when we consider the full effect of the

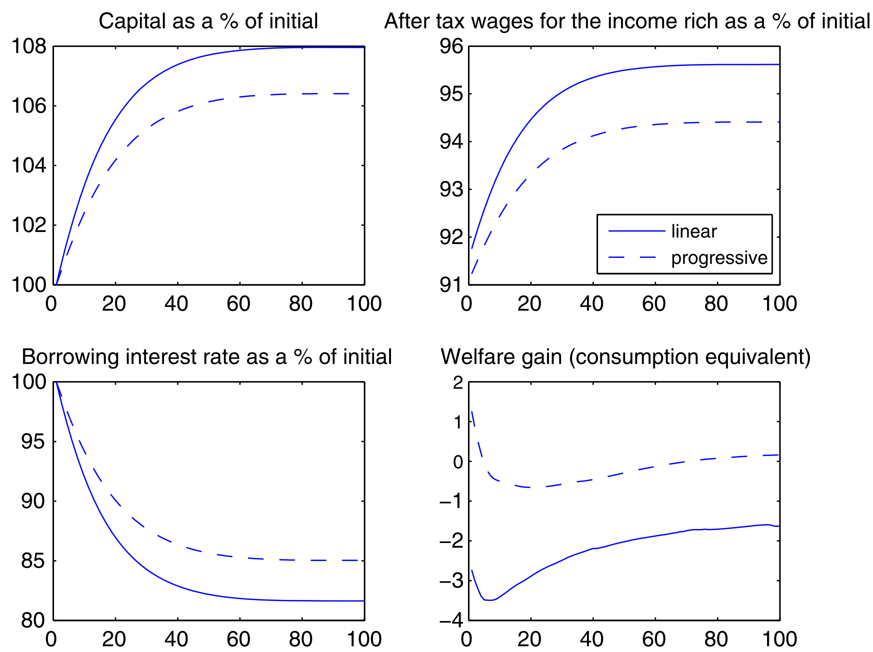

Fig. 6. Aggregate paths in the linear and progressive full reforms.

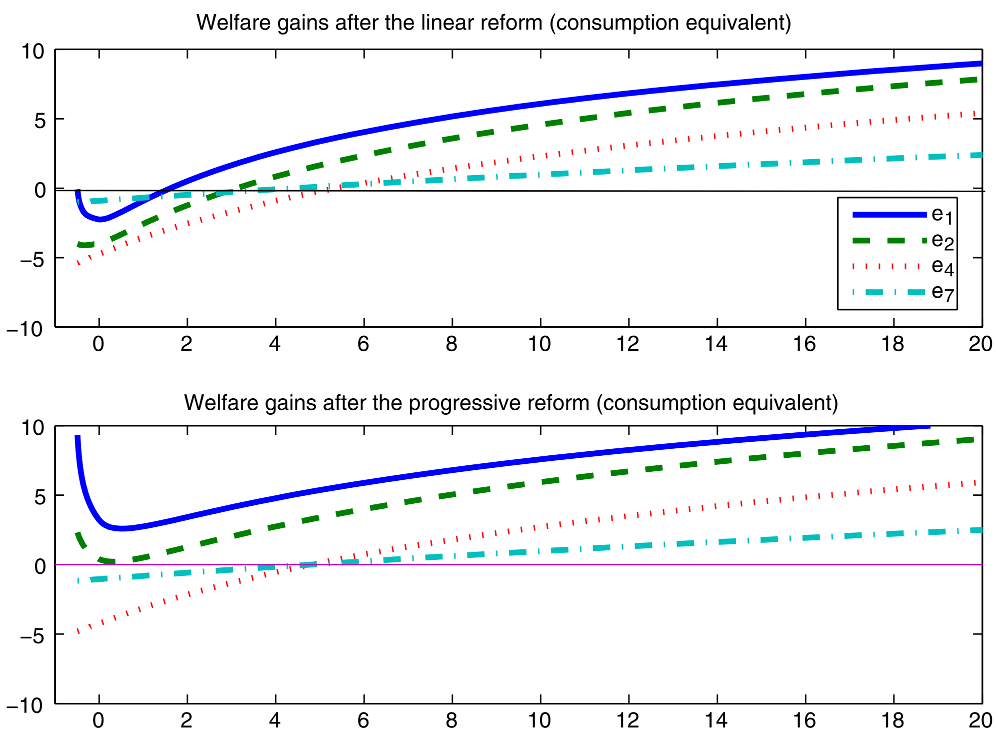

Fig. 7. Welfare gains in the linear and progressive full reforms.

reform. In the linear case, we still obtain a welfare loss of −2.72 percent but this is about half of the loss of −4.51 percent when the effect of the limits is taken into consideration. Also, the time path of welfare gains becomes non-monotone in this case. This is due to the fact that, initially, the increase in consumption smoothing due to the looser limits dominates but as this effect fades away and some agents get unlucky draws of income shocks the poor will become poorer. Then, as capital accumulates, aggregate welfare increases again. To confirm the intuition about the initial welfare gains, Fig. 7 shows the gains for different income and asset groups.

As we see, asset wealth and labor income determine the preference for the reform in a similar way as in the interim reforms. The main difference now is that welfare is considerably higher for the lowest two income groups with low asset levels, since being borrowing constrained or close to the limits they benefit the most from the looser limits. We see that this effect does not dominate the negative effect from higher labor income taxes in the linear reform. However, in the progressive reform, it dominates for the two lowest income groups, who are now in favour of the reform for every asset level, raising the political support from 30% to 49.73%. (The political support increased only marginally from 17% to 18% percent in the linear case.) Finally, it is important to note that we again see a non-monotone behavior of welfare around zero assets, which has a similar interpretation as before. Agents with close to zero asset holdings cannot benefit instantaneously (directly) from lower borrowing interest rates or higher saving interest rates and thus have lower welfare gains than agents with significant debt or significant levels of assets.

Note that Domeij and Heathcote [16] study a tax reform that is similar to ours, but they assume no borrowing. When they take into account the transition, they also find that eliminating capital income taxes and increasing linear labor taxes would not get political support and decrease aggregate welfare. We show that this is not necessarily the case if one allows for a change in labor taxes which make the tax system more progressive in a model when borrowing is allowed.

To conclude, these results illustrate that it is crucial to take into account the effect of changes in the tax system on the endogenous borrowing limits, even though these changes do not seem to have significant effects on aggregate capital accumulation and on factor prices. As we have seen, one could obtain the wrong conclusion regarding the quantitative or qualitative welfare effects of policy reforms if this latter effect is ignored.

6. Conclusions

The present work studies an economy with incomplete markets, capital accumulation and the possibility of default on financial liabilities. In particular, we study competitive equilibria where the loosest possible limits that prevent default are imposed. Further, we characterize how these endogenous limits depend on labour income.

In the presence of i.i.d. shocks, we show analytically that the endogenous limits become looser with a higher individual labour income, while the limits as a fraction of income become tighter. We show that these facts are consistent with the data. Then, we calibrate the model to match the distribution of financial assets (and unsecured) debt in the US economy. Using this calibration, we show that the above results are robust to the presence of persistent shocks.

Then, we analyze the welfare implications of two revenue neutral tax reforms that eliminate capital income taxes at the expense of higher labor income taxes. In the second reform the tax code becomes more progressive because labor taxes are only increased for higher income groups, while the first reform is linear, in the sense that it increases labor taxes for every income group. We show that it is very important to model the borrowing limits appropriately and to take into account that they can change when the reform is implemented. With these effects, welfare in the two reforms is considerably higher when the transition is taken into account because limits become looser and allow for more risk sharing. Moreover, this effect dominates in the progressive reform, implying that taking into account the effects on the limits would reverse the conclusion regarding the welfare effects of the reform.

We believe that our findings can be applied to a variety of other interesting contexts with incomplete markets and endogenous limits. As an example, Krueger and Perri [23] analyze consumption and wealth inequality in a context with complete markets and enforcement constraints, providing a possible explanation as of why the increase in earnings inequality has not been accompanied by an increase in consumption inequality in recent decades. As argued by the authors, borrowing limits might have become looser due to a change in the exogenous earning process, whereas our results suggest that growth (capital accumulation) and changes in tax policy could have the same effect. Moreover, this would be consistent with the decline in savings rates that we have observed during the same period. In addition, Aiyagari et al. [5] study the optimal level of government debt and fiscal policy. In their context, the government optimally accumulates assets to smooth taxes and insure against future expenditure shocks. This implies that households will be in debt in the long-run. However, the possibility of default and the implied endogenous borrowing limits could impose constraints on how much assets the government can accumulate. More generally, our results suggests that fiscal policy and social insurance programs can have significant effects on the level of the endogenous trading constraints, and the welfare analysis of any policy reform should therefore take this into account. This is particularly relevant for the study of social security reforms, where the level of the endogenous limits could considerably affect the financial viability and impact of the reform.[11]

Appendix A. The financial intermediation sector

This section discusses the simplest possible intermediation structure under which the endogenous limits that are not too tight would arise in equilibrium.

Throughout the section, we assume that households can trade through risk neutral financial intermediaries in a risky asset with an endogenous return that we denote by R. In particular, if a household invests in the asset ($a_{it} > 0$), for each unit of consumption he or she gives up at t, the intermediary promises to pay back R_{t+1} units of the consumption good at $t+1$. Further, if the household borrows ($a_{it} < 0$), for each unit of consumption he or she receives, he or she promises to pay back R_{t+1} units to the intermediary at $t+1$. We assume that the intermediaries and the households take R as given. Further, the intermediaries cannot price discriminate, in the sense that they have to pay the same return to investors as they charge to borrowers. For simplicity, we introduce the intermediation sector in a context with no taxes, but the results can be easily extended to the presence of positive income taxation.

Under these assumptions, the budget constraint of household $i \in I$ can be written as follows:

$$c_{it} + a_{it} = w_t \epsilon_{it} + R_t a_{it-1}.$$

The intermediaries live for two periods. At the beginning of the first period, they set the borrowing limits (α_{it}) on the agents' asset holdings. Here, we assume that the intermediaries are free to set these limits and we look for a symmetric equilibrium where all intermediaries set the same limits. Households are therefore subject to a trading constraint of the form:

$$a_{it} \geqslant \alpha_{it}.$$

The cash flows of the intermediaries can be described as follows. During the first period, they trade consumption goods with the households and collect a total amount of $A_t \geqslant 0$ goods.

[11] This is studied by Andolfatto and Gervais [8] and Rojas and Urrutia [30].

Further, they transform this (or some portion of it) into physical capital $K_t^I \leqslant A_t$, which is rented to the representative firm. In order to be consistent with the setup in the main text, we assume that this transformation is one-to-one. In the second period, they receive rental income of $r_{t+1}K_t^I$ from the firm and they have to honor the trading contracts with the households by paying back $R_{t+1}A_t$. If $q_{t,t+1}$ denotes the discount factor of the intermediaries between periods t and $t+1$, their profit function can be written as:

$$A_t - K_t^I + q_{t,t+1}\big[r_{t+1}K_t^I - R_{t+1}A_t\big]. \tag{13}$$

The intermediaries can commit to repay back their debt to the households, but they cannot be forced to lend to or borrow from any household. Further, we assume that $A_t > 0$, implying that intermediaries cannot be solely making pure arbitrage profits but have to mediate between the households and the production sector.[12]

We first discuss the restrictions that need to be satisfied in a symmetric equilibrium where all intermediaries hold the same portfolio. First, an equilibrium implies that $K_t^I = K_t$, where K_t is the demand for capital of the representative firm. Second, the following condition has to hold:

$$r_{t+1} \leqslant R_{t+1}. \tag{14}$$

To see why this is the case, note that the intermediaries could otherwise make arbitrarily large profits by demanding arbitrarily large funds A_t and by setting $K_t^I = A_t$. Third, since the intermediary can commit to repay back its debt to the households at each date, solvency requires that $R_{t+1}A_t \leqslant r_{t+1}K_t^I$. This, together with the fact that $K_t^I \leqslant A_t$, implies that $R_{t+1} \leqslant r_{t+1}$. Finally, combining the last condition with (14), it becomes clear that the only possible equilibrium is to have $R_{t+1} = r_{t+1}$ and $K_t^I = A_t = K_t$. Given this, Eq. (13) implies that all intermediaries make zero profits, regardless of the particular value of the discount factor q.

While there are many allocations with different borrowing constraints that satisfy the above restrictions, we consider the one with the loosest limits. As before, the limits are defined in two steps. First, we define default thresholds that are *not too tight*, in the sense that some agent would default under some continuation state if they were loosened by some $\varepsilon > 0$. Note that we have just shown that the return on a_{it} is the same as the return on physical capital. Given this, we can use the value functions W and V to define the thresholds if we replace the individual capital holdings k with the asset holdings a. Second, we can obtain the effective limits by defining the tightest thresholds among the ones that households could face depending on the idiosyncratic shock realizations tomorrow.

In order to show that these limits actually arise in equilibrium, we have to prove that no intermediary can make positive profits by loosening the trading limits relative to the limits that are not too tight. This is established by the following proposition, also showing that there does not exist any symmetric equilibrium with limits that allow for default.

Proposition A.1. (i) *The allocation with default thresholds that are not too tight is a* (*symmetric*) *equilibrium. Further,* (ii) *a symmetric equilibrium with default does not exist.*

Proposition A.1 implies that the limits set by a competitive intermediation sector are such that no household has an incentive to default in a symmetric equilibrium. The proof of this proposition is provided in Appendix B.

[12] This assumption guarantees the existence of symmetric equilibria where all intermediaries hold the same portfolio.

Appendix B. Proofs

Proof of Proposition 2.1. First, since the periodic utility function $u(\cdot)$ is continuous in consumption, $W(\epsilon, k; \Psi)$ satisfies the same property in k. It is also clear that the value function has to be increasing in k, since everything that is feasible under a given $(\epsilon, k; \Psi)$ has to be feasible under $(\epsilon, \widetilde{k}; \Psi)$ for all $\widetilde{k} \geqslant k$. This implies that $W(\epsilon, \widetilde{k}; \Psi) \geqslant W(\epsilon, k; \Psi)$ for $\widetilde{k} \geqslant k$. Let $\underline{k}^N(\epsilon; \Psi) < 0$ be the appropriately defined natural borrowing limit. This limit is defined as the level of debt such that households are just able to repay their debt under every possible contingency and still have non-negative consumption.[13] Further, define the (possibly infinite) supremum of the utility function as follows: $\overline{U} \equiv \sup_{c \in \mathbb{R}_+} u(c) \leqslant \infty$. Then, the strict monotonicity of the period utility function implies that:

$$\lim_{k \to \underline{k}^N(\epsilon, \Psi)} W(\epsilon, k; \Psi) = -\infty \quad \text{and} \quad \lim_{k \to \infty} W(\epsilon, k; \Psi) = \frac{\overline{U}}{1-\beta}.$$

The first equality follows from the fact that u is unbounded below and from the fact that, at the natural borrowing limit, households would end up consuming zero along some history with positive probability. In addition, since our assumptions imply that the shock ϵ and the aggregate capital K are positive and finite, it also follows that $-\infty < V(\epsilon; \Psi) < \frac{\overline{U}.}{1-\beta}$. Given this, the intermediate value theorem implies that there exists a finite $\underline{k} > \underline{k}^N(\epsilon; \Psi) > -\infty$ for $\forall(\epsilon; \Psi)$ such that $W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)$. Further, the uniqueness of $\underline{k}$ follows from the fact that W is strictly increasing in k.

Second, since $W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)$ and $W(\epsilon, 0; \Psi) \geqslant V(\epsilon; \Psi)$, the fact that W is increasing in k implies that $\underline{k}(\epsilon; \Psi) \leqslant 0$ and therefore $\sup_{(\epsilon)}\{\underline{k}(\epsilon; \Psi)\} \leqslant 0$. In other words, the equilibrium default threshold and effective limits are non-positive. □

Proof of Lemma 2.1. Suppose that $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi) > w(K)(1-\tau_l)\epsilon(1-\lambda)$. In this case, it follows that:

$$u\big(g_c\big(\epsilon, \underline{k}(\epsilon; \Psi); \Psi\big)\big) + \beta \sum_{\epsilon'} \Pi\big(\epsilon' \,\big|\, \epsilon\big) V\big(\epsilon'; \Psi'\big) > V(\epsilon; \Psi) = W\big(\epsilon, \underline{k}(\epsilon; \Psi); \Psi\big).$$

The first inequality follows from the definition of the autarky value in (7), whereas the last equality follows from the definition of the lower bounds in (9). However, the previous inequality contradicts the fact that W is the maximal life-time utility that agents can achieve, since it implies that they could improve welfare by consuming $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi)$ at the default threshold and by defaulting at every future contingency. It therefore follows that $g_c(\epsilon, \underline{k}(\epsilon; \Psi); \Psi) \leqslant w(K)(1-\tau_l)\epsilon(1-\lambda)$. □

Proof of Proposition 2.2. To prove the proposition, we can use the definitions of the two value functions to derive the following expression for the nominator of (11):

$$W_\epsilon(\epsilon_t, \underline{k}_t; \Psi_t) - V_\epsilon(\epsilon_t; \Psi_t) = E_t \sum_{\tau=t}^{\infty} (\beta\rho_\epsilon)^{\tau-t} w(K)\big(u'(c_\tau) - u'\big(c_\tau^{au}\big)\big), \tag{15}$$

[13] See Santos and Woodford [32] for a specification of such limits in a general incomplete markets context. As shown by the authors, the natural limits represent an appropriately defined present value of the individual labor income, which has to be finite in equilibrium.

where $c_\tau^{au} = w(K_\tau)(1-\tau_l)\epsilon_\tau(1-\lambda)$ and $c_\tau = w(K_\tau)(1-\tau_l)\epsilon_\tau + r(K_\tau)(1-\tau_k)k_\tau - k_{\tau+1}$ represent the consumptions in autarky and in the trading arrangement respectively. If $\rho_\epsilon = 0$, the sign of the left-hand side of the previous equation is equal to the sign of $u'(c_t) - u'(c_t^{au})$. Further, the concavity of u and Lemma 2.1 imply that $c_t \leqslant c_t^{au}$ and $u'(c_t) - u'(c_t^{au}) \geqslant 0$. It therefore follows that $\frac{\partial \underline{k}(\epsilon;\Psi)}{\partial \epsilon} \leqslant 0$. □

Proof of Proposition A.1. (i) We first prove that the allocation with limits that are not too tight is a symmetric equilibrium. In particular, we first consider a symmetric equilibrium with the loosest possible limits that avoid default in equilibrium and then show that no intermediary can achieve positive or zero profits by loosening these limits. To do this, assume, without loss of generality, that the participation constraint is binding for agents of a certain type i for some possible realization of the shock tomorrow $\widehat{\epsilon}_{it+1} \mid \epsilon_{it}$. We define type i as an agent with shock and assets of $(\epsilon_{it}, a_{it-1})$. This implies that:

$$W(\widehat{\epsilon}_{it+1}, a_{it}; \Psi_{t+1}) = V(\widehat{\epsilon}_{it+1}; \Psi_{t+1}), \tag{16}$$

or that $a_{it} = \alpha_{it}$.

We now show that no intermediary can build a portfolio $\overline{A}_t > 0$ at date t which gives him at least zero profits and which involves more lending $\bar{a}_{it} < a_{it}$ to the type i agents. To see this, note first that the intermediaries need to satisfy the following condition to be solvent at $t+1$:

$$\overline{R}_{t+1}\overline{A}_t \leqslant r_{t+1}\overline{K}_t^I,$$

where $\overline{R}_{t+1}$ is the return offered by the intermediary at $t+1$ and $\overline{K}_t^I$ is the total capital rented to the firm at t. Second, since $\overline{K}_t^I \leqslant \overline{A}_t$, it has to be the case that $\overline{R}_{t+1} \leqslant r_{t+1} = R_{t+1}$, where the last equality follows from the fact that R and r satisfy condition (14) for a symmetric equilibrium with no default. Finally, note that (16) implies the agents of type i with the shock $\widehat{\epsilon}_{it+1} \mid \epsilon_{it}$ will default under the new portfolio due to the fact that W is decreasing in a and $\bar{a}_{it} < a_{it}$. It therefore follows that $\overline{R}_{t+1} < r_{t+1} = R_{t+1}$. Since this is a strictly worse deal for other agent types who do not default, it follows that the intermediary will not be able to build this portfolio.

To see that the last condition $\overline{R}_{t+1} < r_{t+1} = R_{t+1}$ has to hold, note that the intermediary has at most $r_{t+1}\overline{A}_t$ goods available at $t+1$. On the other hand, he has to pay out $\overline{R}_{t+1}(\overline{A}_t - A_t^d)$ to the other agent types, where A_t^d is the part of debt that is defaulted, namely, the sum of asset holdings $\bar{a}_{it}$ for households of type i that have a shock realization of $\widehat{\epsilon}_{it+1} \mid \epsilon_{it}$ tomorrow. Since the type i agents will only default if $\bar{a}_{it} < 0$, however, this can only be done if $\overline{R}_{t+1} < r_{t+1} = R_{t+1}$. Alternatively, the intermediary could decrease $\overline{R}_{t+1}$ so as to make the type i agents not want to default any more, but this would directly imply that $\overline{R}_{t+1} < r_{t+1} = R_{t+1}$. The other agent types will therefore not be willing to accept the deal in either case.

(ii) We now prove that symmetric equilibria with default cannot exist. To do this, assume first that the limits are such that, at a given date t, there exists at least a possible realization of the shock tomorrow $\widehat{\epsilon}_{it+1} \mid \epsilon_{it}$ (or more generally a set of realizations) where agents of type i will default. We now show that this equilibrium cannot exist, since the intermediaries will be able to make positive profits by not "lending" to the types with positive default probabilities.

We prove it by contradiction, assuming that such an equilibrium exists. First, note that the following condition has to hold in such an equilibrium:

$$r_{t+1} \leqslant R_{t+1} - \frac{1}{A_t} R_{t+1} A_t^d, \tag{17}$$

where A_t^d are the asset holdings of the type i agents that have a shock realization of $\widehat{\epsilon}_{it+1} \mid \epsilon_{it}$ tomorrow and A_t are the total funds collected by the intermediaries. Further, r and R represent the prices in the symmetric equilibrium with default. Note that, if the previous condition was not satisfied, the intermediaries could set $K_t^I = A_t$ and make arbitrarily large profits.

Second, since $K_t^I \leqslant A_t$ and the intermediaries have to be solvent, the only equilibrium with $A_t > 0$ is given when $K_t^I = A_t$ and (17) satisfied with equality. In this case, the intermediaries make zero profits. On the other hand, since the agents of type i do not want to default unless $a_{it+1} < 0$, it also follows that:

$$r_{t+1} > R_{t+1}. \tag{18}$$

Condition (18) implies that an intermediary could make positive profits under the current prices by only accepting deposits from agents that do not default and by not lending to the type i agents. Under this profitable deviation, any intermediary offering the original contract would be driven out of the market. Further, there would not be any lending to the type i agents, contradicting the existence of an equilibrium with default of the agents of type i. □

Appendix C. Numerical algorithm

C.1. Computing the stationary competitive equilibrium

The general algorithm used to solve for the steady state given $\boldsymbol{\tau}_k$ is an extension of the one in Aiyagari (1994) to endogenous limits. We use a generalized policy function iteration which relies on the first-order conditions (mainly the Euler equation) of the model. Further, we approximate all the relevant policy and value functions with linear interpolation over a finite but endogenous grid on assets. To solve the individual problem with policy iterations, we proceed as follows. Given a set of default thresholds $\underline{k}$, an interest rate r and a tax vector $(\boldsymbol{\tau}_k, \tau_l)$, we let h be the vector consisting of the policy functions of interest, i.e., $h = [c, k']$. Let T be a non-linear operator such that $T[h, W, V; \underline{k}, r, \tau_l]$ satisfies the individual optimality conditions given τ_k. To approximate the fixed point, we follow the steps below.

Step 1: Guess an initial vector $[h^0, W^0, V^0; \underline{k}^0, r^0, \tau_l^0]$, where $h^0 = [c^0, k'^0]$.
Step 2: For each iteration $n \geqslant 1$, use the previous guess $[h^{n-1}, W^{n-1}, V^{n-1}]$ and $[\underline{k}^{n-1}, r^{n-1}, \tau_l^{n-1}]$ to compute the new vector $[h^n, W^n, V^n]$ that satisfies the individual equilibrium conditions.
Step 3: Use the value functions $[W^n, V^n]$ to find the new lower bound $\underline{k}^n$ such that $W^n(\epsilon, \underline{k}^n(\epsilon)) \approx V^n(\epsilon)$, and update the grid accordingly.
Step 4: Using h^n and the distribution for the idiosyncratic shock Π, calculate Λ^*, the joint (stationary) distribution of assets and income. Next, use Λ^* to calculate the supply of capital, which is compared to the aggregate capital demanded by the firm to get r^n.
Step 5: The new tax rate on labor τ_l^n is calculated given Λ^* and h^n to satisfy the government's budget constraint.
Step 6: Repeat Steps 2–5 until convergence.

Note that our setting requires the introduction of some notable differences with respect to the standard procedure to solve models with uninsurable income shocks.

First, the key first-order condition is the Euler equation of the agent:

$$u'(c) \geqslant \beta E u'(c')\bigl(1 + r(1 - \tau_k)\bigr).$$

Recall that $\tau_k > 0$ when agents save and $\tau_k = 0$ when agents borrow. In this case, the agent's optimal saving policy may not be continuous (the agents optimal consumption and value function is continuous nevertheless). We first check whether there exists a strictly positive asset level satisfying the Euler equation above with equality and $\tau_k = \boldsymbol{\tau}_k$ and whether there is $k' \in (\kappa, 0]$ satisfying $u'(c) \geqslant \beta E u'(c')(1 + r)$. If there is a solution in the two cases, we choose the one which yields higher life-time utility. If at $k' = \kappa$, we have $u'(c) > \beta E u'(c)(1 + r)$, then we know that the agent chooses to be at the limit. Using the equilibrium policy functions, the value functions $W = W(\epsilon, k)$ and $V = V(\epsilon)$ are calculated then recursively. As usual, in the above inequality for c', we use the consumption policy functions from the previous guess.

Second, in Step 3, we need to update the endogenous default thresholds $\underline{k}$ for every level of income during every iteration. Whenever $W^n(\epsilon, \underline{k}^{n-1}(\epsilon)) > V_n(\epsilon)$, we choose $\underline{k}^n(\epsilon) < \underline{k}^{n-1}(\epsilon)$. This means that we loosen the limit and we do this proportionally to $W^n(\epsilon, \underline{k}^{n-1}(\epsilon)) - V^n(\epsilon)$. Whenever, $W^n(\epsilon, \underline{k}^{n-1}(\epsilon)) < V^n(\epsilon)$, we define $\underline{k}^n(\epsilon)$ such that $W^n(\epsilon, \underline{k}^n(\epsilon)) = V^n(\epsilon)$, that is, we tighten the limit.

Third, in Step 5, we use the labor taxes to guarantee that the government's budget constraint is satisfied with equality period by period. This needs to be done because aggregate capital is endogenous and for any given tax rate the revenue of the government depends on the wage rate, which in turns depends on aggregate capital.

Fourth, as opposed to the general procedure used to solve these type of economies, in which case one iterates only on r (or equivalently K), in our procedure we have to iterate simultaneously on τ_l, r and on the default thresholds $\underline{k}_n(\epsilon)$. In principle, this could make the solution procedure less stable and slower. However, if one first solves the model with fixed and exogenous limits and then use the solution of such model as the initial guess for our procedure, then it converges relatively fast and without major problems.

C.2. Computing the transition between steady states

When we calculate the transition between steady states we need to adjust the above procedure in the following way. First, for the sake of the exposition, assume that convergence to the new steady state takes place in T periods. Then we follow the steps below.

Step 1: Guess a time series for the variables $\{h_t^0, W_t^0, V_t^0; \underline{k}_t^0, r_t^0, \tau_{l,t}^0\}_{t=1}^T$, together with the time series for the distribution of individuals $\{\Lambda_t^0\}_{t=1}^T$. We then initialize the first period with stationary distribution of the first steady state ($\Lambda_1^0 = \Lambda_{SS1}^*$ and $r_1^0 = r_{SS1}$) and we assume that at time T we are already in the second steady state ($\Lambda_T^0 = \Lambda_{SS2}^*$ and $r_T^0 = r_{SS2}$).

Step 2: For each iteration $n \geqslant 1$ and for each time period $1 \leqslant t \leqslant T - 1$, we use the previous guess for the next period $[h_{t+1}^{n-1}, W_{t+1}^{n-1}, V_{t+1}^{n-1}]$ and $[\underline{k}_{t+1}^{n-1}, r_{t+1}^{n-1}, \tau_{l,t}^{n-1}]$ to compute the new vector $[h_t^n, W_t^n, V_t^n]$ that satisfies the individual equilibrium conditions.

Step 3: Further, use the value functions $[W_t^n, V_t^n]$ to find the new lower bound $\underline{k}^n$ for all $2 \leqslant t \leqslant T - 1$ such that $W_t^n(\epsilon, \underline{k}_t^n(\epsilon)) \approx V_t^n(\epsilon)$, and update the grid accordingly.

Step 4: Using h^n and Π, we calculate Λ_{t+1}^n, the joint distribution of assets and income and then use Λ_{t+1}^n to calculate the supply of capital K_{t+1}^n. These two variables are compared the initial guesses Λ_{t+1}^{n-1} and K_{t+1}^{n-1} for all $1 \leqslant t \leqslant T - 1$.

Step 5: The new tax rate on labor for each time period $1 \leqslant t \leqslant T - 1$ is calculated given Λ_t^{n-1} and h_t^n to satisfy the government's budget constraint.

Step 6: Repeat Steps 2–5 until convergence in $\{\underline{k}_{t+1}, K_{t+1}, \tau_{l,t}\}_{t=1}^{T-1}$.

This procedure is implemented in two steps. First, we apply the procedure without Step 3, assuming that the limits are set for every period by the limits of the second steady state. This provides a very good first guess for the time path of aggregate capital, factor prices and taxes (see the interim case and the full reforms on Figs. 3 and 6). This part of the procedure is also used to find T, the endogenous length of the transition using an iterative procedure starting from $T = 3$. Then using the solution of this first step as the initial guess, we implement the whole procedure, which involves adjusting the limits for every period.

Supplementary material

The online version of this article contains additional supplementary material.
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