OPtimal Asset Division Rules for Partnerships

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Motivation

- Partnerships are often formed to share risk across partners.
- They also accumulate assets at the level of the partnership.
- Partnesrhips may dissolve (e.g. when one of the partners receive an attractive outside option).
- Applications: business partnerships, marriage, economic unions.
- We study how to design asset division rules upon break-up in this environment.

What we do in this paper

- Use a simple framework to understand the key trade-offs.
  - Focus on the separation margin.
  - Differentiate between private and public information regarding outside offers.
- Main Questions
  - What is the optimal asset division rule upon dissolution?
  - What is optimal level of separations.
  - How (ex ante) asset accumulation is affected? (not today)
- Key trade-off: Production Efficiency vs. Risk Sharing.

Main take-aways

- Generous asset provisions for "defaulters" improve on output efficiency but hurt risk sharing.
- For log utility:
  - Under public information, equal asset distribution is always optimal.
  - Under private information, asset provisions should be reduced to mitigate inefficient separations.
  - Under private information, there are excessive separations, still.
- For CRRA ($\sigma > 1$) utility:
  - Under public information, defaulters receive less than half of the assets. The share is decreasing in the attractiveness of the outside offer.
  - Under private information, asset provisions have to be constant.
  - Depending on the distribution of outside opportunities there can be too little or too much separations.

Literature

Setup

- We consider a partnership of two infinitely lived partners with capital equal to K that runs a linear technology with return $R \geq 1$.
- The two partners have equal share in the partnership, have CRRA utility functions and a discount factor β.
- In normal times: they make a consumption saving decision (capital depreciates fully between periods).
- In any period, with probability ρ one partner receives an outside production option with productivity $A \geq 1$.
- Two things can happen then:
  - The offer is not taken then they will stay together forever.
  - The offer is taken and the defaulting partner receive $\eta(A)$ fraction of the capital, while the remaining partner will use the remaining capital with productivity $1 \leq L \leq R$. No ex post transfers are enforceable.

Public Information

- The life-time value of an agent running a technology A with initial capital K is given by $V(A, K)$.
- This implies that staying together would deliver utility $W^{nos}(K) = 2V(R, K/2)$.
- This implies that the optimal separation rule under public information $0 \leq \eta(A) \leq 1$ solves the following problem:

$$W^*(A, K) = \max\left\{W^{nos}(K), \max_{\eta(A)} V(A, \eta(A)K) + V(L, (1-\eta(A))K)\right\}.$$

- The separation threshold A^* is given by

$$W^{nos}(K) = V(A^*, \eta(A^*)K) + V(L, (1-\eta(A^*))K).$$

- For log utility it is given by $A^* = R^2/L \geq R$.

Optimal Asset Division Rule (Public Information)

- The optimal asset division rule can be characterised as

$$\frac{\eta(A)}{1-\eta(A)} = \frac{L^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}}{A^{\frac{\sigma-1}{\sigma}} - \beta^{1/\sigma}}.$$

Implications ($A > L$):

- $\sigma > 1$: $\eta(A) < 1/2$ and $\eta'(A) < 0$.
- $\sigma = 1$: $\eta(A) = 1/2$
- $\sigma < 1$: $\eta(A) > 1/2$ and and $\eta'(A) > 0$.

Intuition:

- There is a trade-off between investment efficiency and risk sharing:

$$\frac{c_0^A}{c_0^L} = \left(\frac{A}{L}\right)^{1/\sigma}.$$

- The departing partner has higher initial consumption and a higher consumption growth ($V(A, \eta(A)K) > V(L, (1-\eta(A))K)$).

Private Information: Preliminaries

- Given our assumptions, there cannot be a sharing rule that depends on the the outside option A.
- Given η, agents choose to separate if $V(A, \eta K) > V(R, K/2)$.
- Private separation threshold $\hat{A}(\eta)$ given by $V(\hat{A}(\eta), \eta K) = V(R, K/2)$.
- Hence the optimal separation rule under private information solves ($F(\cdot)$ is the cumulative distribution of A):

$$\max_{\eta} \left\{ F(\hat{A}(\eta)) W^{nos}(K) + \int_{\hat{A}(\eta)}^{\bar{A}} \left(V(A, \eta K) + V(L, (1-\eta)K) \right) dF(A) \right\}.$$

Private Information: Separation Thresholds

- For log utility the threshold is $\hat{A}(\eta) = R\left(\frac{0.5}{\eta}\right)^{1-\beta}$.
  - The lower is η the less separation we have.
  - There is more separation under private information with $\eta = 0.5$: $\hat{A}(1/2) = R < A^*$.
- These results generalize for the case of $\sigma > 1$, $(\hat{A}(\eta(A^*)) < A^*)$.

Implication: At the public information sharing rule, we experience more separations.

Private Information: Asset Division Rule

Asset division rule is adjusted downwards for $\sigma \geq 1$:

- For $\sigma = 1$, $\eta^{PI} < 1/2$.
- For $\sigma > 1$, $\eta^{PI} < \eta(A^*)$.

Intuition (log case)

- Reducing η at $1/2$ has no marginal cost in terms of efficient risk sharing by the envelope condition.
- There is positive marginal gain by reducing inefficient separation.
- In the $\sigma > 1$ case, these effects are amplified by the fact for all $A > A^*$, the optimal asset division rule is decreasing in A.

Private Information: Excessive Separations (log utility)

- The optimal value of η depends on the distribution of A.
- We define $A^*(\eta)$ as the socially optimal separation threshold for a given η.
- It solves $V(A^*(\eta), \eta K) + V(L, (1-\eta)K) = 2V(R, K/2)$.
- $A^*(\eta) = \frac{R^2}{L}\left(\frac{0.25}{\eta(1-\eta)}\right)^{1-\beta} = A^*\left(\frac{0.25}{\eta(1-\eta)}\right)^{1-\beta}$.
- Result: $A^*(\eta^{PI}) > \hat{A}(\eta^{PI})$. Private information tends to generate too much separation in this sense.

Intuition

- Increasing η at $A^*(\eta) = \hat{A}(\eta)$ has no marginal cost in terms of increasing separations but bring η closer to its optimal value $1/2$ for all sates when separations happen.

Private Information: What determines η^{PI}?(log case)

- The optimization problem regarding ηPI maximizes (assume that $A^* > \hat{A}(\eta^{PI})$

$$\int_{\hat{A}(\eta)}^{\bar{A}} \left[V(A,\eta K) + V(L,(1-\eta)K) - V(A,0.5K) - V(L,0.5K)\right] dF(A) \\ + \int_{\hat{A}(\eta)}^{A^*} \left[V(A,0.5K) + V(L,0.5K) - W^{nos}(K)\right] dF(A)$$

Trade-off

- The first term measures the efficiency loss due to inefficient risk sharing during separations.
- The second term measures the loss due to inefficient separations.
- The first term is more important, whenever we have a larger mass of high (above A^*) realisations.

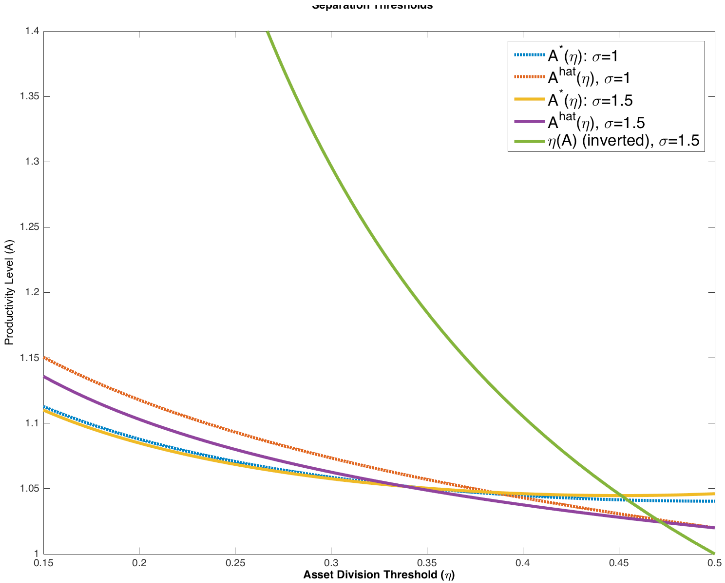
Private Information: What is different for $\sigma > 1$?

- Remember that the optimal asset division rule, $\eta(A)$ is decreasing in A.
- Hence, there is an extra incentive to decrease η starting from $\eta(A^*)$: it increases 'average' risk sharing.
- We can both have $A^*(\eta^{PI}) > \hat{A}(\eta^{PI})$ or $A^*(\eta^{PI}) < \hat{A}(\eta^{PI})$ depending on the distribution of A.
- In the latter case, there is 'too little' separation under private information.
- We give up some efficient separations to provide better risk sharing for those states where separations eventually happen.
- The remaining question: Do separations increase due to private information in absolute sense?

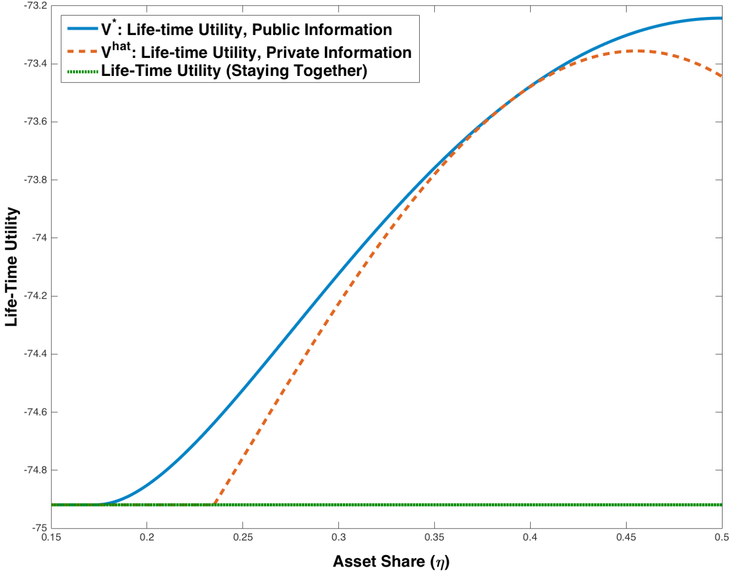
A Numerical Example

- We take values of $\beta = 0.9$, $R = 1.02$, $L = 1$.
- A is distributed uniformly on the interval $[L, \bar{A}]$
- We vary $\bar{A} \in [1.02, 1.6]$ to study how the attractiveness of the outside option affects the optimal asset division rule and the efficiency of separations.

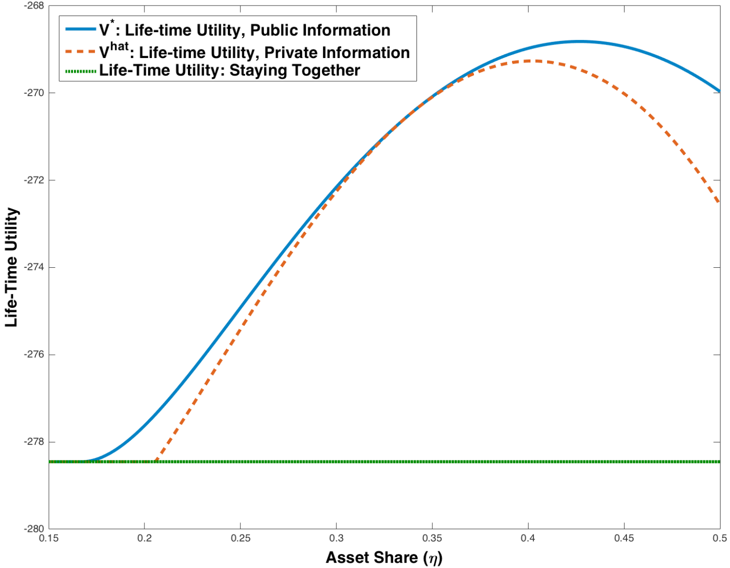
Separation Thresholds and Optimal Asset Division Rule



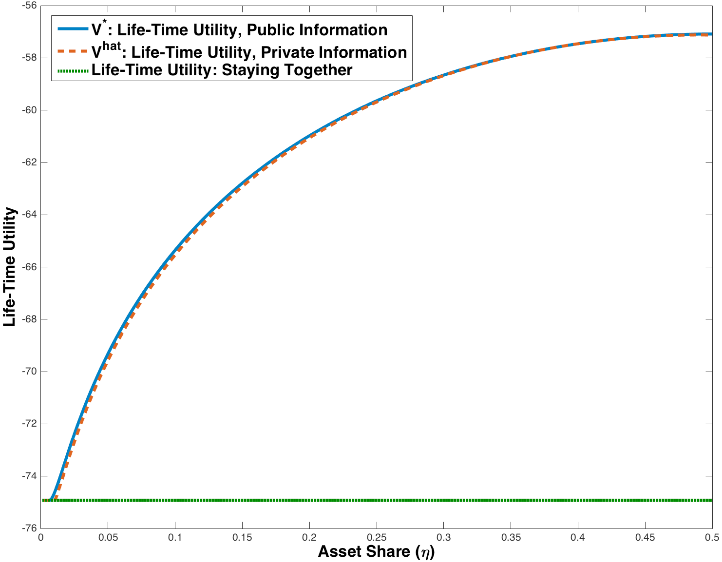
Life-Time Utilities: $\sigma = 1$, intermediate $\bar{A}$



Life-Time Utilities: $\sigma = 1.5$, intermediate $\bar{A}$



Life-Time Utilities: $\sigma = 1$, high $\bar{A}$



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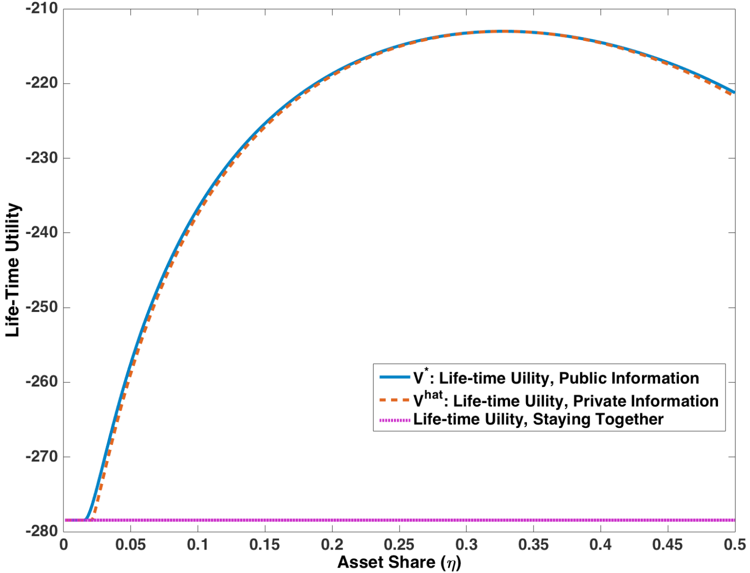
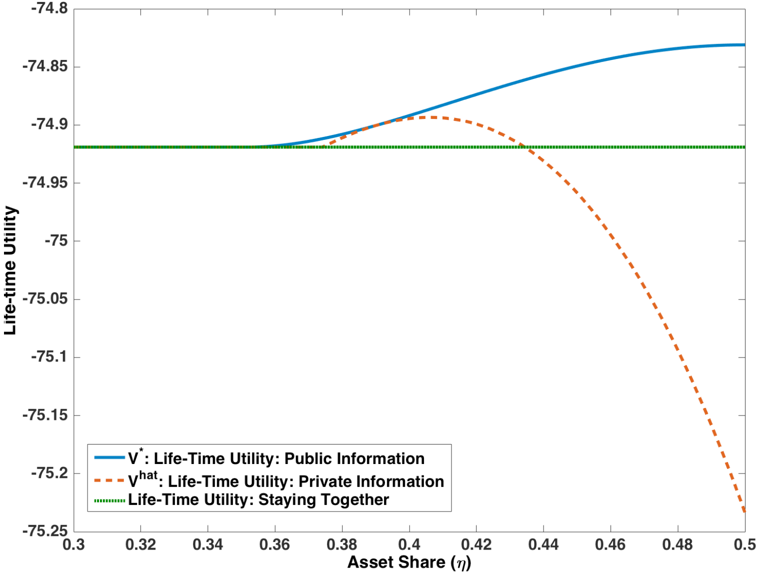


Life-Time Utilities: $\sigma = 1$, low $\bar{A}$



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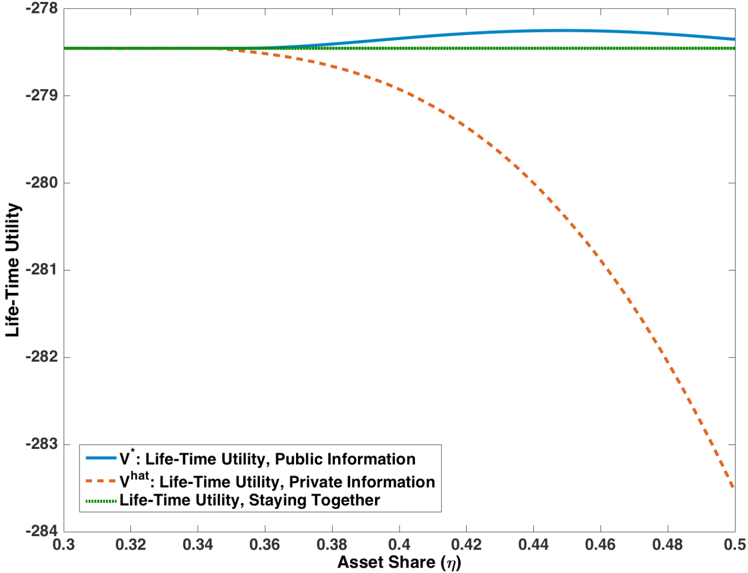
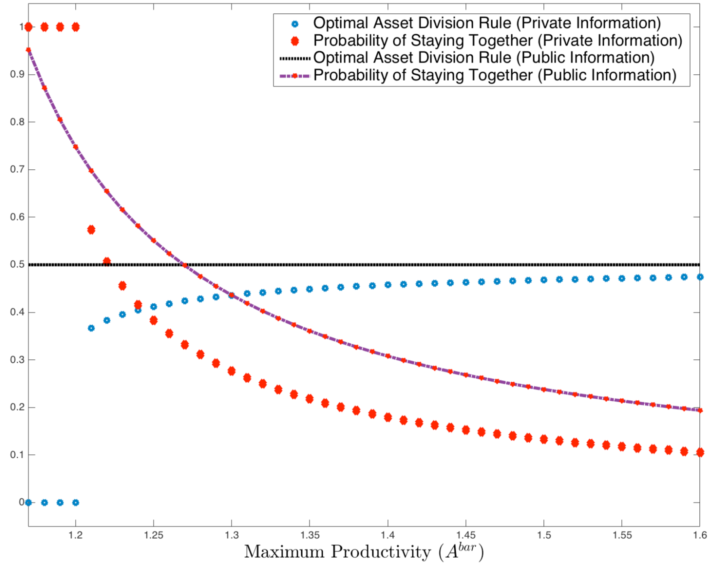
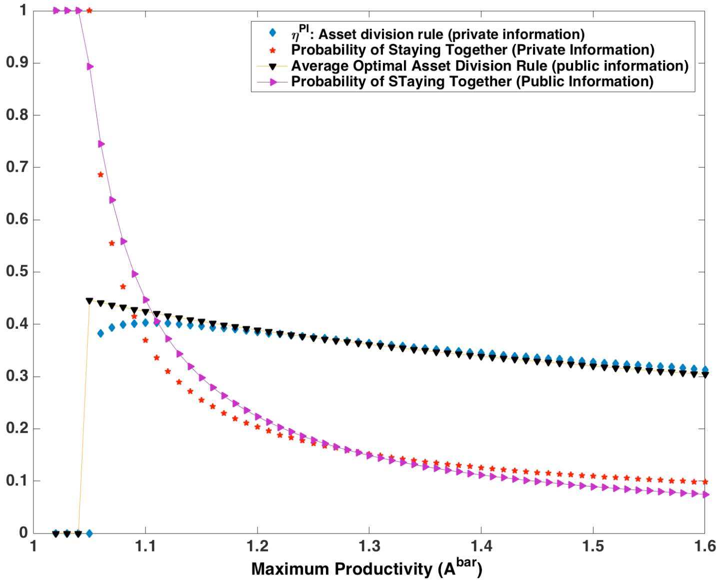


Separations as a function of $\bar{A}$: $\sigma = 1$,



Separations as a function of $\bar{A}$: $\sigma = 1.5$,



Summary and Future Research

Summary:

- Optimal asset division rules under public and private information.
- Trade-off between investment efficiency and risk sharing.
- Private information typically increase separations at the optimum.
- However, it may reduce it if the outside opportunities are very lucrative ($\sigma > 1$) or if they are very limited ($\sigma \geq 1$).

Future research:

- Study the effect of separations and private information on asset accumulation.
- Work out the case of $\sigma < 1$ with private information: new incentive problems.
- Contrast implications to the data on (self)-regulation on partnership exit.
- Compare with 'no-competition clauses', where the 'assets' cannot be taken away.