

Predicting excess returns in financial markets

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Abstract

This paper attempts to reproduce the time series properties of nominal excess returns in a variety of financial markets using a representative agent cash-in-advance model, modified to allow for time variation in the conditional variances of the exogenous processes. The exogenous fundamental processes of the model are estimated from the data and the remaining free parameters are estimated with a simulated method of moments technique. Simulations demonstrate that the model can replicate some of the predictability features of observed excess returns for the period 1978–1991, but that it fails to account for the serial correlation and for the joint properties of one and three months excess returns.

Keywords: Excess returns; Financial markets; Cash-in-advance model

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1. Introduction

There is considerable evidence in the literature that conditional expected returns in a variety of financial markets move over time. This fact has been documented in

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at least two ways: by showing that returns are mean reverting (see e.g. Huizinga, 1987; Poterba and Summers, 1988; Fama and French, 1988) and by showing that there are instruments belonging to the information set of agents which predict returns in excess of the risk free rate (e.g. Fama and Schwert, 1977; Keim and Stambaugh, 1986; Campbell, 1987; Stambaugh, 1988; Cochrane, 1990; among others).

Recently there has been interest in investigating and characterizing the joint time series behavior of excess returns in different markets (see e.g. Campbell and Clarida, 1987; Giovannini and Jorion, 1989; Cumby, 1990; Cutler et al., 1990; Solnick, 1991; Ferson and Harvey, 1991; Campbell and Hamao 1992; Bekaert and Hodrick, 1992). These studies document three important regularities of excess returns across markets and countries. First, there is a small set of instruments (the dividend yield and the forward premium and yield spread in the term structure of interest rates) which are significant in predicting their joint movements. Second, the models used to forecast excess returns across countries are similar in the sense that coefficient estimates do not differ substantially across countries. Third, excess returns for bond portfolios appear to have different statistical properties than those for stock portfolios.

We take these observations as the starting point of our analysis and ask the following question: Is the behavior of excess returns consistent with the predictions obtained from an intertemporal consumption based capital asset pricing (ICCAP) model?

There are several reasons to ask this 'structural' question. First, if excess returns display common predictable movements, aggregate risk may account for these fluctuations. In this case all excess returns would be proportional to one or more factors describing aggregate risk and a limited set of instruments proxying for these factors will be sufficient to characterize their reduced form properties. The ICCAP model implies a specific multivariate factor structure where the conditional variances and covariances of the exogenous processes of the economy determine the cyclical behavior of excess returns.

Failures of the standard version of an ICCAP model where conditional variances are constant and only conditional covariances drive the behavior of excess returns are well documented in several simulation studies. To the now well known equity premium puzzle (see Merha and Prescott, 1985), the literature has added the interest rate–inflation puzzle, the term premium puzzle in the term structure of interest rates and the risk premium puzzle in foreign exchange markets (see Macklem, 1991; Backus et al., 1989; Benninga and Protopapadakis, 1991; Donaldson et al., 1990; Backus et al., 1991). However, recent work by Labadie (1989), Kandel and Stambaugh (1990), Bonomo and Garcia (1991), Canova and Marrinan (1991, 1993) show that slight modifications in the auxiliary statistical assumptions of the model may give rise to significant changes in the outcomes of the simulations.

Our first task here is to examine whether modifying the standard ICCAP

specification to allow for time variation in the conditional variances of the exogenous processes helps in reproducing the time series properties of actual excess returns. Since previous versions of the model in which only conditional covariances drive the cyclical behavior of excess returns fail, our analysis concentrates on the contribution of conditional variances. The presence of heteroskedasticity in variables which may affect financial markets was recognized at least a decade ago (see e.g. Engle, 1982). Reduced form (ARCH-M) models have attempted to account for this feature in estimating statistical models of excess returns. Yet it is surprising that simulation exercises based on the ICCAP model have largely neglected to take this feature into account as a crucial factor in explaining their movements over time.¹

Second, for the particular version of the ICCAP model we use the factors driving excess returns represent real, monetary and fiscal sources of risk. It is therefore of interest to know which of these sources, if any, is important in bringing the time series generated from the ICCAP model close to actual data.

Third, most of the empirical evidence reported so far deals with US financial markets and except for a few recent examples (see e.g. Ferson and Harvey, 1991; Solnick, 1991; Korajczyk and Viallet, 1992), financial markets of other countries are neglected. By presenting empirical evidence and examining the performance of an ICCAP model for financial markets of countries other than the US, we can provide evidence on the question: ‘Are the shortcomings of the ICCAP model intrinsic to US markets or does it do better for other countries?’

The paper is organized as follows: the next section reports reduced form evidence on the properties of several excess returns. We restrict our attention to two countries (US and UK) with well developed financial markets which are relatively free of government intervention. We consider excess returns involving Eurodeposit, foreign exchange and bond markets and provide evidence for their predictability by reporting the Sharpe ratio and the AR(1) coefficient, a test for the significance of the first few terms of the autocorrelation function and the results of a regression of excess returns on a common set of instruments. All excess returns appear to be forecastable using information available to agents at the time the investment decision was made but no one instrument is jointly significant in predicting all excess returns in both countries. In addition, we find no evidence of predictability based on lagged excess returns. Finally, we also find some differences in the behavior of excess returns across holding periods. We take the compiled reduced form evidence as the benchmark for our structural analysis. We are interested in generating excess returns from our ICCAP model and in examining if they display similar reduced form properties.

¹ Notable exceptions here are Kandel and Stambaugh (1990), Bonomo and Garcia (1991), Macklem (1991) and Canova and Marrinan (1991, 1992).

The third section briefly describes the model and the auxiliary assumptions used to compute closed form solutions for the variables of interest and discusses the factor structure implied by the model. Section 4 conducts specification tests to check the reasonableness of the auxiliary assumptions made, describes the technique used to select the parameters and the evaluation procedure to assess the performance of the model. Our strategy is the following. We estimate as many of the model's parameters as possible directly from the data by standard method of moment techniques. For those parameters for which appropriate data does not exist, or existing evidence is unreliable, we estimate them so as to match a vector of simulated and actual statistics of the data. To maintain compatibility with our previous study (Canova and Marrinan, 1993) these parameter estimates are chosen to match the time series properties of excess returns in foreign exchange markets. We then use the estimated parameters to generate time series for all excess returns. We examine the model's ability to reproduce statistics of actual excess returns (such as Sharpe ratios, the regression coefficients on the set of common instruments, etc.) both informally, studying the implication of our parameter selection for excess returns other than those obtained in foreign exchange market, and formally, using the probabilistic approach developed in Canova (1994). In this case we perform a large number of simulations, randomizing over both the parameters and the innovations of the exogenous processes and measure the 'closeness' of simulated and actual data by computing the probability that the model generates statistics which are less than or equal to the ones we observe in the data. Randomization over parameters is done by drawing replications from the joint asymptotic distribution of the estimates.

Section 5 discusses the results and analyzes their robustness by examining a few variants of the model. We find that time variation in the conditional variances of fiscal and monetary variables are crucial in bringing simulated data close to actual data and that although the model matches several qualitative features, it falls short in accounting for several quantitative properties of the data. In addition, we find that the ICCAP model cannot jointly match the time series properties of one month and three month holding returns and that these failures are not restricted to US markets. Conclusions and avenues for future research appear in Section 6.

2. The predictability of excess returns

We use monthly data for two countries: the US and the UK for the sample 1978,5–1991,9. We concentrate on these countries because they possess homogeneous and well developed financial markets where trading volume is substantial. The sources and definitions of all the data we used are in Appendix A.

In studying predictability we face the issue of currency denomination of excess returns. Adler and Dumas (1983) and Solnick (1991) have emphasized that when a nominal CAPM is applied to asset returns hedged against currency risk, it leads to

a pseudo separation theorem where all investors hold a combination of a common portfolio and *their own* country's risk free bill. Therefore, returns should be measured in domestic currency and should be in excess of the domestic risk free rate. For the two countries we are examining we compute excess returns in various ways and, consequently, can examine whether the standard practice of measuring returns in US dollars affects the results. One complication arises because an on-shore risk free rate with comparable characteristics to the US T-bill rate does not exist in all countries for all maturities. In this case, it is typical to use Eurodeposit rates of the same maturities. Moreover, a second complication is that the spread between off-shore and on-shore rates may signal changes in political risks (see e.g. Ferson and Harvey, 1991). For practical considerations we will use the Eurodeposit rate as a measure of the risk free rate. For a sensitivity analysis we also experimented with available on shore rates (US one- and three-month T-bill rates and the UK three-month T-bill rate), without substantial changes in our results.

Throughout this section we study the following five time series for $k = 1, 3$ and $h = 60$ (five years):

$$ER_{1t+k} = \frac{1200}{k} (\ln S_{t+k} - \ln F_{t,k}), \quad (1)$$

$$ER_{2t+k} = \frac{1200}{k} \ln \left(\frac{q_{t+k,us}^{h-k}}{q_{t,us}^h} \right) - ir_{t,us}^k, \quad (2)$$

$$ER_{3t+k} = \frac{1200}{k} \ln \left(\frac{q_{t+k,uk}^{h-k}}{q_{t,uk}^h} \right) - ir_{t,uk}^k, \quad (3)$$

$$ER_{4t+k} = \frac{1200}{k} \ln \left(\frac{S_{t+k} q_{t+k,uk}^{h-k}}{S_t q_{t,uk}^h} \right) - ir_{t,us}^k, \quad (4)$$

$$ER_{5t+k} = \frac{1200}{k} \ln \left(\frac{q_{t+k,uk}^{h-k}}{q_{t,uk}^h} \right) - ir_{t,us}^k + \ln \left(\frac{S_{t+k}}{S_t} \right), \quad (5)$$

where S_t is the exchange rate at time t , $F_{t,k}$ is the forward rate quoted at t for $t+k$, $ir_{t,i}^k$ is the k -period interest rate in country i at time t and $q_{t+k,i}^{h-k}$ is the price at $t+k$ of a bond of country i having $h-k$ periods to maturity. The excess returns computed in (1)–(5) are all obtained from simple buy-and-hold strategies and have straightforward interpretations. (1) measures the nominal dollar denominated excess return from purchasing pounds forward in the foreign exchange market. (2)–(5) measure the excess returns obtained by an investor who *always* invests in one long term bond market. In particular, (2) is the US holding premium, i.e. the dollar excess return one obtains by holding a h period US government bond for k periods, relative to holding a k period dollar denominated eurodeposit to maturity. Similarly, (3) is the UK holding premium return. Follow-

ing Adler and Dumas, (4) measures the excess return from holding a UK bond benchmarked against the US risk free rate and (5) measures the UK holding premium in dollar terms.

For each of these time series we select $k = 1$ and 3 months, and $h = 60$ months (five years). We present results for two different maturities because, as noted by Lewis (1991), the holding period seems to matter both for characterizing the predictable components of returns and for testing structural models. Table 1 reports the Sharpe ratio, i.e., the absolute value of the mean of the series divided by the standard error (Sharpe), the estimated first order autocorrelations of the series (AR1) and Cumby and Huizinga's (1992) test for the presence of serial correlation in the first p autocorrelations (CH(p)). For the case of one-month excess returns, $p = 6$. For the case of three-month excess returns, because the holding period exceeds the sampling frequency of the data, MA components of order 2 may exist and one should expect some serial correlation even if true excess returns are not predictable. In this case the test assessed the significance of the third and fourth order serial correlation when MA components of order 2 may be present.

These three statistics provide us with a rough indication of various forms of predictability of excess returns. The Sharpe ratio provides a semiparametric lower bound to the ratio of the variability of the discount factor relative to its mean of many asset pricing models (see e.g. Hansen and Jagannathan, 1991). In an ICCAP model the discount factor is the intertemporal marginal rate of substitution (IMRS) of consumption between contiguous periods. If excess returns are predictable, Sharpe ratios should be large and in turn the IMRS must be highly variable. The AR1 coefficient and the implied CH test measure the predictability of excess returns on the basis of simple univariate time series prediction equations.

Table 1 also reports the results of regressing the five excess return series for each of the two maturities on a set of seven common instruments belonging to agents' information set. With this regression we hope to provide two types of evidence. First, whether it is possible to use information available to agents at the time the investment decisions were made to predict excess returns. Second, whether there are patterns of predictability that are common to all excess return series and that can be accounted for by the same set of factors. Together with the coefficient estimates we report five diagnostic statistics for the predictive equations: the R^2 of the forecasting regressions, a χ^2 test for the nullity of all coefficients but the constant, a Cumby and Huizinga test for serial correlation, an ARCH test for conditional heteroskedasticity and a Kendall and Stuart test for normality of the residuals of the regression.

The seven instruments used to compute predicted values are: a constant, the forward premium in the foreign exchange market (FP), the dividend yields in the US and the UK (SPDIV and LONDIV), the yield spreads between long and short term government bills in the US and the UK (USSP1 and UKSP) and the yield

spread between low grade corporate bonds and short term government bills in the US (USSP2). A few comments to justify our choice of instruments are worthwhile. As in previous studies we use term spreads, private–public spreads and dividend yields on SP500 and London 500 share indices in the regressions. These variables represent aggregate, world wide information which may influence expectations and are known to have predictive content because of their forward looking nature (see e.g. Keim and Stambaugh, 1986; Campbell, 1987; Harvey and Ferson, 1991). Contrary to Cutler et al., (1990), we use the forward premium in place of the real exchange rate because it appears to be more useful in characterizing the properties of excess returns in foreign exchange markets (see Bekaert and Hodrick, 1992). Due to data limitations and because of consistency problems we were unable to construct a measure of the yield spread between private and public bonds in the UK for the entire sample.

We also considered additional variables such as inflation rates, the price of oil and a January dummy as suggested e.g., by Ferson and Harvey (1991). Although some of these instruments are often statistically significant in predictive regressions (see e.g. Hamao, 1988; Ferson and Harvey, 1991; Campbell and Hamao, 1992), we found that they are highly correlated with measures of interest rate spreads and do not appear to add independent information.

The results contained in Table 1 contain interesting information. First, Sharpe ratios are generally low (the highest are for ER_{1t+k} and ER_{5t+k}) in the range of estimates reported by e.g. Backus et al., (1991) or Breen et al., (1989). They imply that excess returns are highly volatile and that the risk for undertaking a position in the market is of an order of magnitude larger than the ex-post return. Second, there is some positive serial correlation in excess returns for both k 's but the evidence that a simple time series model helps forecast excess returns is weak.² Third, there are variables belonging to the information set of agents which predict future movements in excess returns. However, we find weak evidence of commonality across excess returns or maturities. The variables which have the strongest explanatory power are dividend yields (at least one of the two variables is significant in 9 of the 10 regressions). The US private–public yield spread is significant for excess returns in the foreign exchange market at both maturities and for the 3 month holding premium in the US but not for UK variables. The other instruments are significant in one or more regressions but none appears to enter any prediction equation significantly for both maturities. The explanatory power of the prediction equations is reasonable, ranging from 8 to 17% of total variation. Although the R^2 's of these regressions seem small, they are high in comparison with similar regressions in other markets (see e.g. Campbell, 1987; Solnick, 1991)

² This lack of time series predictability shows up also in the second moments of series (1)–(5). Using ARCH and White tests we found almost no evidence of conditional heteroskedasticity in any series for both maturities.

Table 1
 Statistics of excess returns and regression coefficients on instruments: Sample 78,5-91,9^a

	Sharpe	AR(1)	CH(ρ)	Constant	SPDIV	LONDIV	FP	USSP1	UKSP	USSP2	R ²	$\chi^2(6)$	CH(6)	ARCH(13)	KS
$ER_{1,t+1}$	0.121	0.036	6.44 0.375	-25.06 -1.31	-4.40 -0.58	7.24 0.94	-3.86 -2.20	-8.36 -1.67	-2.26 -0.63	7.21 -1.96	0.11	17.11	5.22	8.63	0.17
$ER_{2,t+1}$	0.014	0.122	7.25 0.297	-1.52 -0.06	14.14 2.03	-14.13 -1.70	-2.42 -1.13	9.04 1.93	-5.42 -1.24	-1.85 -0.88	0.09	14.58	4.72	7.95	1.93
$ER_{3,t+1}$	0.012	0.109	6.33 0.386	-47.25 -2.09	-2.05 -0.30	9.09 1.21	3.19 1.58	6.51 1.15	3.87 1.01	-0.49 -0.11	0.11	14.94	5.92	3.83	1.43
$ER_{4,t+1}$	0.057	0.101	7.13 0.308	-72.30 -2.74	-6.46 -0.68	16.34 1.84	0.33 0.15	-1.85 -0.26	1.60 0.37	6.72 1.52	0.08	15.99	5.68	14.69	1.04
$ER_{5,t+1}$	0.106	0.100	7.11 0.310	-72.30 -2.74	-6.46 -0.68	16.34 1.84	-0.66 -0.31	-1.85 -0.26	1.60 0.37	6.72 1.52	0.08	15.84	5.68	14.69	1.04
$ER_{1,t+3}$	0.181	0.659	1.67 0.433	-21.64 -2.12	-9.66 -1.98	10.77 2.68	0.94 1.22	-3.54 -1.06	-0.71 -0.47	3.88 1.98	0.11	26.25	1.85	15.17	1.65
$ER_{2,t+3}$	0.040	0.661	1.48 0.476	-11.70 -0.91	9.35 2.57	-5.98 -1.53	-0.48 -0.89	13.63 5.19	-4.17 -3.11	-6.51 -3.42	0.17	37.47	3.86	4.14	0.14

ER_{3t+3}	0.007	0.657	1.82	-33.76	-3.82	9.75	-0.52	4.29	-0.11	-0.29	0.09	21.64	1.88	48.58	2.54
			0.401	-2.89	-0.93	3.06	-1.00	1.32	-0.09	-0.13		0.001	0.38	0.00	0.00
ER_{4t+3}	0.082	0.699	1.02	-55.41	-13.48	20.52	1.41	0.74	-0.83	3.59	0.14	30.55	1.71	43.65	1.34
			0.598	-3.93	-1.92	3.68	1.51	0.18	-0.46	1.41		0.000	0.42	0.00	0.35
ER_{5t+3}	0.131	0.690	0.82	-55.41	-13.48	20.52	0.41	0.74	-0.83	3.59	0.12	38.47	1.71	43.65	1.34
			0.658	-3.93	-1.92	3.68	0.44	0.18	-0.46	1.41		0.000	0.42	0.00	0.35

^a ER_{1t+k} measures the nominal dollar denominated excess return from purchasing pounds forward in foreign exchange market, ER_{2t+k} the US holding premium, ER_{3t+k} the UK holding premium, ER_{4t+k} the excess return from holding a UK bond benchmarked against the US risk free rate and ER_{5t+k} the UK holding premium in dollar terms. The first three columns report measures of time series predictability: Sharpe measures the absolute value of the mean of the series divided by the standard error, $AR(1)$ the first-order autocorrelations of the series and $CH(p)$ the Cumby and Huizinga's (1992) test for the presence of serial correlation in the first p autocorrelations. For one-month holding period the test checks whether the first $p = 6$ AR coefficients are identically equal to zero. For three-month holding period $p = 2$ and the test checks whether the 3rd-4th AR coefficients are equal to zero, given that the first two AR coefficients may not be equal to zero. The next seven columns report the coefficients and the t -statistics of the regression of ER_{jt+k} on seven instruments available at time t . SPDIV is the SP500 dividend yield and LONDIV the Financial Times Index dividend yield. FP is the forward premium in the £ exchange market, UPSP1 and UKSP are the spreads in the US and in the UK between the yield on a long term Government bond and a short term instrument and USP2 is the spread in the US between low grade corporate bonds yield and a Government short term instrument. The last five columns report diagnostic statistics for the predictive regression: $\chi^2(6)$ is a test for the joint hypothesis of nullity of all regression coefficients but the constant, $CH(6)$ is the Cumby-Huizinga test for the significance of the first sixth serial correlations, $ARCH(13)$ is a test for conditional heteroskedasticity with 13 degrees of freedom and KS is a Kendall and Stuart test for normality of the residuals.

and we find almost no evidence against the idea that the forecasting model is sufficiently well specified. Fourth, three-month excess returns appear to be ‘more’ predictable than one-month excess returns according to all the statistics we used. While time aggregation issues may be important here, the variability of three months excess returns is smaller than what is predicted by simple aggregation accounting. This implies, for example, that Sharpe ratios and AR(1) coefficients are more significant than those obtained by aggregating the corresponding one month statistics. Finally, Adler and Dumas’s objection seems to be empirically relevant as, e.g. ER_{3t+k} displays different time series characteristics than ER_{4t+k} or ER_{5t+k} even though in terms of predictability they do not differ very much.

We take these facts as the starting point of our structural analysis. The rest of the paper is devoted to examining whether the time series for excess returns generated by an ICCAP model display, both qualitatively and quantitatively, the features reported in Table 1.

3. The model

The theoretical framework we employ is a version of the cash-in-advance monetary model developed by Lucas (1982) and modified by Hodrick (1989). Since the model is well known in the literature, we only briefly describe its features and directly compute the equilibrium values of the variables of interest.

The economy is characterized by two countries. Every period, each country i is endowed with Y_{it} , $i = 1, 2$ units of a nonstorable consumption good. There are two governments which consume G_{it} units of their own country’s good. To finance these consumption requirements each government issues a country specific money, M_{it} , collects real lump sum taxes, T_{it} , levied equally on agents from both countries, and issues debt to finance any purchases in excess of money creation and tax collections. This debt is in the form of state contingent nominal bonds of maturity k , $k = 1, 2, \dots, K$, denominated in their own country’s currency. Endowments, government consumption requirements and money supplies are exogenous and follow a first order Markov process with a stationary and ergodic transition function.

The countries are each populated by a representative household maximizing a time separable utility function defined over the two goods. Households are subject to both a wealth constraint and a liquidity constraint which compels them to purchase goods with cash. The timing of the model follows Lucas with asset markets open first and goods markets following. At the beginning of each period the consumer enters the asset market and decides how to allocate her wealth among the productive assets of the two countries, currencies, and the state contingent nominal bonds issued by the two governments. After the asset market closes, the consumer enters the goods market and makes her consumption purchases with previously accumulated currency.

Equilibrium requires that households optimize and all markets clear. Since capital markets are complete, this permits an unconstrained Pareto optimal allocation of the time–state contingent nominal bonds.

Let $e_{it,k}^{-r}(v)$ denote the discount price at t of a bond paying one unit of currency i at time $t+k$ if event v occurs and $r_{it,k}(v)$ denote the associated continuously compounded interest rate.

In equilibrium nominal interest rates reflect optimal consumption–saving decisions by equating bond prices to individuals’ expected marginal rate of substitution of future nominal expenditure for current nominal expenditure, i.e.,

$$e^{-r_{it,k}} = E_t \frac{\beta^k P_{it} U_{c_i}(c_{1t+k}, c_{2t+k})}{P_{it+k} U_{c_i}(c_{1t}, c_{2t})}. \tag{6}$$

Because all uncertainty is resolved prior to the household’s money holding decisions, they hold just enough currency to finance their current consumption purchases. This implies that the quantity theory holds so that $P_{it} = M_{it}/Y_{it}$ and ³

$$e^{-r_{it,k}} = \frac{\beta^k E_t Y_{it+k} (M_{it+k})^{-1} U_{it+k}}{Y_{it} (M_{it})^{-1} U_{it}}. \tag{7}$$

In equilibrium, the nominal spot rate is equal to the marginal rate of substitution of domestic currency for the foreign currency:

$$S_t = \frac{U_{2t} p_{1t}}{U_{1t} p_{2t}} = \frac{Y_{2t} M_{1t} U_{2t}}{Y_{1t} M_{2t} U_{1t}}. \tag{8}$$

Therefore, the k -period ahead conditional future log spot rate is

$$E_t \ln S_{t+k} = E_t \ln \left[\frac{Y_{2t+k} M_{1t+k} U_{2t+k}}{Y_{1t+k} M_{2t+k} U_{1t+k}} \right]. \tag{9}$$

From (7) and (8) and using covered interest parity we can price a k -period forward exchange rate as

$$F_{t,k} = S_t e^{r_{1t,k} - r_{2t,k}} = \frac{E_t Y_{2t+k} M_{1t+k} U_{2t+k}}{E_t Y_{1t+k} M_{2t+k} U_{1t+k}}. \tag{10}$$

If we let the time interval of the model be a month, the approximate annualized percentage expected nominal profits from purchasing foreign exchange forward

³ Hodrick et al. (1991) show that when a one country version of the above model is calibrated to the US economy the cash-in-advance constraint almost always binds. Bekaert (1992) shows that the same occurs in a two country setting. Therefore, there appears to be little practical gain in specifying models with more complicated nonbinding constraints.

(corresponding to strategy (1)), defined as $EP_{1t+k} = (1200/k) * (E_t \ln(S_{t+k}) - \ln(F_{t,k}))$, can be computed from (9) and (10) as

$$EP_{1t+k} = \frac{1200}{k} * \left(E_t \ln \left[\frac{Y_{2t+k} M_{1t+k} U_{2t+k}}{Y_{1t+k} M_{2t+k} U_{1t+k}} \right] \right) - \ln \left(\frac{E_t [Y_{2t+k} M_{1t+k} U_{2t+k}]}{E_t [Y_{1t+k} M_{2t+k} U_{1t+k}]} \right) \quad (11)$$

The holding premium in the two countries corresponding to strategies (2) and (3) is

$$EP_{(i+1)t+k} = \left\{ - \ln \left[\frac{\beta E_t Y_{it+k} (M_{it+k})^{-1} U_{it+k}}{Y_{it} (M_{it})^{-1} U_{it}} \right] + E_t \ln \left[\frac{\beta E_{t+h} Y_{it+k} (M_{it+k})^{-1} U_{it+k}}{Y_{it+h} (M_{it+h})^{-1} U_{it+h}} \right] + \ln \left[\frac{\beta E_t Y_{it+h} (M_{it+h})^{-1} U_{it+h}}{Y_{it} (M_{it})^{-1} U_{it}} \right] \right\} \quad i = 1, 2. \quad (12)$$

The excess return from holding an h period bond of country 2 for k periods relative to the k period risk free rate of country 1 (strategy (4)) is

$$EP_{4t+k} = EP_{3t+k} + EP_{1t+k}. \quad (13)$$

Finally, the holding premium in country 2, measured in terms of currency 1 (strategy (5)) is

$$EP_{5t+k} = EP_{3t+k} + E_t \Delta_k \ln S_{t+k}. \quad (14)$$

where $E_t \Delta_k \ln S_{t+k} = E_t \ln S_{t+k} - \ln S_t$. Inspection of (11)–(14) reveals some interesting features. First, expected nominal excess returns will be different from zero even when agents are risk neutral. Note, however, that expected returns will be zero when all the exogenous processes are constant or deterministically fluctuating. Second, all excess returns depend on expectations about future outputs, future money supplies and future terms of trade. Since in equilibrium expectations about future terms of trade depend on expectations about future government purchases of goods, both supply and demand factors affect expected excess returns. Finally, uncertainty about regime shifts or regime persistence influence the expectation formation and therefore the statistical properties of the expected excess returns. In other words, if a ‘peso problem’ exists, it will appear in (11)–(14) as well as in the forecast error in predicting these excess returns.

To obtain closed form expressions for (11)–(14) the instantaneous utility function is specialized to be of a constant relative risk aversion (CRRA) type as

$$U(c_{1t}, c_{2t}) = \frac{(c_{1t}^\delta c_{2t}^{1-\delta})^{1-\gamma}}{1-\gamma}, \tag{15}$$

where δ is the share of domestic goods in total consumption expenditure and γ is the parameter of risk aversion. The CRRA specification has attractive features: it is easy to manipulate and allows the construction of a risk neutral utility function in multigood settings (see Engel, 1992). Its major drawback is that it restricts the spot rate to be independent of supply factors (see e.g. Bekaert, 1992).

Let Φ_{it} be the proportion of government i 's consumption in total output of good i at time t . In an equilibrium where agents pool aggregate risk $c_{it} = 0.5(Y_{it} - G_{it}) = 0.5Y_{it}(1 - \Phi_{it})$ (see e.g. Hodrick, 1989). Evaluating the marginal utilities in (11)–(14) at these equilibrium consumption levels gives expressions for expected excess returns entirely in terms of the distributions of the exogenous variables. The complete solution requires substituting in the specific processes governing the exogenous variables.

We assume that all exogenous processes are conditionally independent. As we will show later on this is a reasonably good approximation to the processes for money supply, government expenditure and output in the US and the UK. The processes for the growth rate of outputs and money supplies are assumed to be conditionally lognormally distributed. The process governing the fraction of each country's output purchased by the governments is assumed to be conditionally uniformly distributed. Let $z_t = [\Delta \ln(Y_{1t}), \Delta \ln(Y_{2t}), \Delta \ln(M_{1t}), \Delta \ln(M_{2t}), \Phi_{1t}, \Phi_{2t}]$ where $\Delta \ln(x_t) = \ln(x_t) - \ln(x_{t-1})$. All six processes are assumed to follow a AR(1)-GARCH(1,1) specification

$$z_{jt} = A_{0j} + A_{1j}z_{jt-1} + \epsilon_{jt}, \quad \epsilon_{jt} \sim (0, \sigma_{jt}^2), \tag{16}$$

$$\sigma_{jt}^2 = a_{0j} + a_{1j}\sigma_{jt-1}^2 + a_{2j}\epsilon_{jt-1}^2, \quad j = 1, \dots, 6. \tag{17}$$

With these assumptions (11)–(14) reduce to the five expressions reported in Appendix B. They involve the risk aversion parameter, the share of domestic goods in total consumption, the conditional variances of all exogenous processes and the level and the conditional means of the money processes. While the distributional assumptions we made allow us to derive exact closed form solutions, one could alternatively follow Breeden (1986) and take a second order Taylor expansion of (11)–(14) around z_t . The expressions in the appendix would still hold, apart from an approximation error reflecting conditional covariances and higher order terms. We prefer the first approach here because to simulate and formally evaluate the model we will have to make distributional assumptions on the exogenous processes anyway.

It is easy to verify that (i) the unconditional variances of the exogenous variables influence the average size of each EP_{it+k} , (ii) deviations of their conditional variances relative to the unconditional variances affect the unconditional variances of EP_{it+k} , (iii) the parameter of risk aversion γ affects both the unconditional means and the unconditional variability of EP_{it+k} , (iv) the serial correlation properties of the conditional variances of the exogenous processes are responsible for the serial correlation properties of expected excess returns.

The closed form expressions for expected excess returns have a peculiar factor structure: ‘fiscal’, ‘real’ and ‘monetary’ uncertainty of both countries are reflected in excess returns in a variety of financial markets. For example, changes over time in the conditional variances of both countries’ government expenditure shares affect all five excess returns. Changes in the variance of outputs affect all but the expected profits from forward foreign exchange speculation. Finally, changes in the conditional variance of US money affect all but the UK holding premium. Likewise, the conditional variance of the UK money supply does not affect the US holding premium. The hope is that in the reduced form analysis, the seven instruments we used in Table 1 proxy for those factors which determine variations in actual realized excess returns.

It is also clear from these expressions that it is the relative riskiness of domestic versus foreign factors that determines the magnitudes of excess returns. For example, an expected increase in the variance of the US money supply decreases the purchasing power of the dollar. Therefore, traders require higher nominal expected profits to engage in speculative transactions involving a currency which is expected to depreciate in the future (see also Black, 1990). On the other hand, in an economy where both countries have identical conditional moments of fiscal, monetary and real variables, excess returns will be negligible and entirely determined by the convexity term arising from Jensen’s Inequality (see Canova and Marrinan (1993) for an account of the importance of the convexity term in determining the properties of predictable profits from forward foreign exchange speculation).

4. Specification tests, parameter selection and model evaluation procedures

To generate time series for excess returns from (11)–(14) it is necessary to select both the auxiliary parameters characterizing the exogenous stochastic processes (A_{0j} , A_{1j} , a_{0j} , a_{1j} , a_{2j} , $j = 1, \dots, 6$) and the economic parameters (γ , δ). In choosing values one could follow a calibration approach and pick them so as to match relevant long run averages of the actual data. For those parameters for which data do not exist, one could try a few settings and check the sensitivity of the results. We do not follow this approach here because calibration, although widely used in the profession, does not allow a formal evaluation of the properties of the model. Instead, to provide discipline in the simulation, we estimate as many

Table 2

Panel A: Diagnostic tests for the exogenous processes. Sample 75,1–91,9, *P*-values^a

Series	AR(1)	CH(6)	ARCH(12)	W(24)	BD
M1US	0.35 (0.00)	2.00 (0.91)	22.89 (0.03)	44.72 (0.04)	1.78
M1UK	-0.19 (0.01)	3.02 (0.84)	21.76 (0.04)	31.17 (0.14)	2.11
IPUS	0.32 (0.00)	3.39 (0.05)	34.03 (0.00)	78.24 (0.00)	1.85
IPUK	-0.28 (0.00)	0.33 (0.99)	27.71 (0.00)	58.98 (0.00)	2.65

Panel B: Sample cross correlations of univariate residuals. Sample 75,1–91,9

	M1UK			IPUS			IPUK		
	-1	0	1	-1	0	1	-1	0	1
M1US	0.07	0.02	0.07	-0.05	0.19(*)	0.13	0.01	0.17	-0.14
M1UK				-0.04	0.04	0.02	0.13	0.06	-0.16
IPUS							0.12	0.02	-0.04

Panel C: Granger causality tests for squares of univariate residuals. Sample 75,1–91,9, *P*-values

Equation Variable	M1US	M1UK	IPUS	IPUK
M1US	0.03	0.29	0.02	0.62
M1UK	0.32	0.04	0.16	0.15
IPUS	0.04	0.21	0.00	0.36
IPUK	0.76	0.07	0.19	0.00

^a M1US and M1UK are the growth rates of M1 in US and UK, IPUS and IPUK are the growth rates of industrial production indices in US and UK. Residuals are obtained using a first order autoregression. AR(1) is the first AR coefficients of the series, CH refers to Cumby–Huizinga's test for serial correlation, ARCH and W to ARCH and White tests for conditional heteroskedasticity and BD to the Brock–Dechert test for whiteness of the residuals. The number next to each test refers to the number of degrees of freedom.

* indicates correlations which are significantly different from zero at the 5% level.

parameters as possible from observed data using time series methods. The rest we estimate by simulation. Once point estimates and standard errors are available, we can statistically evaluate the ability of the model to reproduce actual data using Monte Carlo techniques.

Since the model describes the US and UK economies, we estimate the time series properties of the two outputs and of the two money processes from comparable US and UK aggregates. Table 2 contains diagnostic tests for our chosen AR(1)-GARCH(1,1) specification for the four exogenous processes. In

each case a first order univariate autoregression on the difference of the log of the series was used to construct residuals. For each residual series we apply the Cumby and Huizinga test for serial correlation, the ARCH and White tests for conditional heteroskedasticity and the Brock and Dechert test for nonlinearities in the normalized residuals and compute a few terms of the cross correlation function of the residuals. The results appear to support our selected time series specification. None of the cross correlations of the residuals was found to be significantly different from zero (except for the contemporaneous correlation of the US money and output index), no leftover serial correlation is evident and the Brock and Dechert test does not reject the hypothesis that the normalized residuals are different from white noises, providing support for our univariate specification. We also find a smooth decay of the autoregression coefficient of the squared residuals, suggesting that a GARCH(1,1) is a reasonable characterization of the conditional variances. Table 3, panel A reports the results of estimating an AR(1)-GARCH(1, 1) specification for the four series. This pins down 20 parameters (A_{0j} , A_{1j} , a_{0j} , a_{1j} , a_{2j} , $j = 1, \dots, 4$).

Since data on the share of government spending in total output is not available at the monthly frequency, we choose the parameters regulating the conditional means and variances of government expenditure shares by simulation. Since quarterly data on government spending is available, we further impose the consistency requirement that if the simulated series for government expenditure shares are aggregated at a quarterly frequency, they must have the same unconditional means and variances as the actual data. For any set of values for A_{15} , a_{15} , a_{25} and A_{16} , a_{16} , a_{26} , this restriction pins down the values of A_{05} , a_{05} and A_{06} , a_{06} and imposes cross equation restrictions which limit the range of parameter values allowed in the simulations. We also choose the two economic parameters by simulation. We do so because standard ways of estimating γ are downward biased (see e.g. Kocherlakota, 1990), while δ cannot be directly estimated from the data for the UK since the consumption series available in the national accounts do not distinguish among locations where the goods are produced.

To select these parameters we employ the ‘estimation by simulation’ technique proposed by Lee and Ingram (1991). The method computes optimal parameter estimates by minimizing the distance between a vector of statistics of the actual and the simulated data in the metric given by the covariance matrix of the statistics. There are several ways to proceed because there is a large number of possible statistics available to estimate the remaining six parameters. To maintain comparability with our previous work (see Canova and Marrinan, 1993), we select parameters to match some of the time series properties of excess returns in foreign exchange markets. Let $\theta = (\delta, \gamma, a_{15}, a_{16}, a_{25}, a_{26})$ be the vector of free parameters, x_t , $t = 1, \dots, T$ be a vector of time series of actual data and let $y_\tau(\theta)$, $\tau = 1, \dots, N$, $N = nT$ be a vector of simulated time series obtained from the model. Define $H_x(T)$ to be a $m \times 1$ vector of statistics of x_t , computed using a sample of size T and define $H_y(N, \theta)$ to be the corresponding $m \times 1$ vector of

statistics for $y_t(\theta)$ computed using a sample of size N . A simulated estimator $\hat{\theta}(T, N)$ is obtained by minimizing

$$Q(\theta) = (H_x(T) - H_y(N, \theta))' W(T, N) (H_x(T) - H_y(N, \theta)) \quad (18)$$

for a given random weighting matrix $W(T, N)$ with rank $\{W(T, N)\} \geq \dim(\theta)$. The matrix $W(T, N)$ defines the metric for the problem and it is assumed to converge almost surely to a nonstochastic matrix $W(0)$. Following Lee and Ingram, an optimal choice for $W(0)$ is given by

$$W(0) = ((1 + n^{-1})S)^{-1}, \quad (19)$$

$$S = \text{diag}\left(\sum_j R_{x_i}(j)\right) = \text{diag}\left(\sum_k R_{y_i}(j)\right), \quad (20)$$

where the last equality holds under the null hypothesis that the θ are chosen correctly and where $R_{x_i}(j)$ and $R_{y_i}(j)$ are the autocovariance functions of the statistics of the actual and of the simulated data, $i = 1, \dots, 6$. Duffie and Singleton (1990) show that under fairly general mixing conditions $\hat{\theta}(T, N)$ is consistent and asymptotically normal.⁴ In our case an estimate for S is computed by smoothing 12 sample autocovariances with a set of Parzen weights. Following Newey and West (1987) it is immediate to show that \hat{S}_T is a consistent estimator of S .

Minimization of (18) is undertaken numerically. Details on the minimization routine appear in Appendix C. Initially, we attempted to jointly fit the time series properties of actual one and three month excess returns from forward speculation. The vector of statistics was constructed by stacking the unconditional mean, the unconditional variance and the first five autocovariances of the nominal excess returns on the dollar for $k = 1$ and $k = 3$ (for a total of $m = 14$ statistics). However, the minimized value of the objective function was very large and the fit of the model was very poor. Essentially, the model is not rich enough to account for the substantial differences in the autocovariance function of one and three month excess returns with the same set of parameters. This outcome is not peculiar to foreign exchange markets. When we try to jointly match one and three month holding premiums to the actual data the same outcome emerges. This result mirrors conclusions obtained by Lewis (1992), who showed using other techniques that the holding period of the investment matters for latent variable tests of CAPM models, and by Canova and De Nicolo' (1993), who demonstrated that the economic relevance of the equity premium puzzle changes with the holding period of the investment. All these results suggest, on one hand, the possible segmenta-

⁴ Since in our model EP_{1t+k} is a GARCH process, there is no insurance that the mixing conditions necessary to prove asymptotic normality hold in our case. However, Hansen (1991) shows that under certain conditions GARCH processes are near epoch dependent so we expect them to satisfy other types of mixing requirements.

Table 3

Panel A: Estimated GARCH specification for the exogenous processes ^aModel: $\Delta \log y_t = A_0 + A_1 \Delta \log y_{t-1} + \epsilon_t$, $\epsilon_t \sim (0, h_t)$

$$h_t = a_0 + a_1 h_{t-1} + a_2 \epsilon_{t-1}^2$$

Sample	Variable	a_0	a_1	a_2	A_0	A_1
75–91	M1US	0.00002 (6.45)	0.000002 (0.0003)	0.316 (2.70)	0.003 (5.66)	0.382 (4.83)
	M1UK	0.0003 (8.63)	0.099 (1.39)	-0.100 (-1.39)	0.001 (5.11)	-0.168 (-1.85)
	IPUS	0.00003 (8.11)	0.0003 (0.01)	0.219 (1.96)	0.001 (2.76)	0.339 (3.79)
	IPUK	0.0001 (2.38)	0.003 (0.01)	0.284 (2.37)	0.0009 (0.80)	-0.180 (-1.74)
75–82	M1US	0.00002 (3.41)	0.00004 (0.01)	0.207 (0.81)	0.004 (2.04)	0.293 (1.41)
	M1UK	0.0006 (4.88)	0.10 (0.91)	-0.10 (-0.91)	0.001 (2.64)	-0.328 (2.08)
	IPUS	0.00006 (3.92)	0.0003 (0.01)	0.220 (1.07)	0.0009 (0.65)	0.303 (1.71)
	IPUK	0.0001 (2.34)	0.003 (0.57)	0.534 (1.98)	-0.002 (-1.05)	0.111 (0.60)
82–91	M1US	0.00002 (3.87)	-0.00002 (-0.001)	0.206 (1.25)	0.003 (4.27)	0.438 (4.47)
	M1UK	0.0003 (6.38)	0.022 (0.17)	-0.022 (-0.18)	0.001 (3.90)	-0.107 (-0.65)
	IPUS	0.00003 (5.80)	-0.000002 (0.001)	0.137 (0.86)	0.001 (2.86)	0.352 (3.30)
	IPUK	0.0001 (4.37)	-0.00006 (-0.0001)	0.179 (1.01)	0.001 (1.37)	-0.307 (-2.80)

Panel B: Estimated bivariate GARCH specification for the exogenous processes

Model: $\Delta \log y_{1t} = A_{10} + A_{11} \Delta \log y_{1t-1} + \epsilon_{1t}$, $\epsilon_{1t} \sim (0, h_{1t})$,

$$\Delta \log y_{2t} = A_{20} + A_{21} \Delta \log y_{2t-1} + \epsilon_{2t}$$
, $\epsilon_{2t} \sim (0, h_{2t})$,

$$h_{1t} = a_{01} + a_{11} h_{1t-1} + c_{11} \epsilon_{1t-1}^2 + c_{12} \epsilon_{2t-1}^2$$
,

$$h_{2t} = a_{02} + a_{12} h_{2t-1} + c_{21} \epsilon_{1t-1}^2 + c_{22} \epsilon_{2t-1}^2$$
.

Sample	Variable	a_0	a_1	c_{11}	c_{12}	c_{21}	c_{22}	A_0	A_1
75–91	M1US	0.00002 (5.43)	0.0001 (0.86)	0.283 (2.17)	0.127 (2.40)			0.003 (5.66)	0.382 (4.83)
	IPUS	0.00003 (6.21)	0.0002 (0.98)			0.212 (1.98)	0.198 (2.12)	0.001 (2.76)	0.335 (3.79)

^a M1US and M1UK are the growth rates of M1 in US and UK, IPUS and IPUK are the growth rates of industrial production indices in US and UK. *t*-statistics are in parentheses.

tion of the market and, on the other, the inability of standard asset pricing models to handle the heterogeneity due to holding period segmentation.

Because of this, we present results obtained by matching parameters to each maturity separately. The estimated values for θ and the minimized value of Q are as follows (asymptotic standard errors are in parentheses):

Maturity	δ	γ	a_{15}	a_{25}	a_{16}	a_{26}	$Q(\hat{\beta})$
1 month	0.5183 (0.0122)	0.0001 (0.1661)	0.1206 (0.1562)	0.1150 (0.1913)	0.0972 (0.1223)	0.0821 (0.1881)	11.58
3 months	0.5049 (0.0208)	0.997 (0.2108)	0.2105 (0.0956)	0.2150 (0.0713)	0.1508 (0.1248)	0.2012 (0.1003)	5.56

Given these estimates and those contained in Table 3 we can simulate time series for EP_{jt+k} $j = 1, \dots, 4$. To simulate a time series for EP_{5t+k} , we need in addition estimates of A_{05} and A_{06} which are obtained by imposing the aggregation restrictions on the conditional means of z_{5t} and z_{6t} . To compute simulated coefficients from the predictive regressions, we regress the simulated excess returns on the actual values of the six instruments. Alternatively, one could estimate a time series model (say a VAR) on the instruments and randomly draw time series for these variables from the estimates of the parameters and some hypothesized distribution of the errors. While this approach is reasonable, it produces weaker results since instead of conditioning the joint distribution of excess returns and instruments on the actual value of the instruments, it allows the regression coefficients to be located anywhere in the joint excess returns–instruments space.

We examine the properties of the model in two ways. As a first pass we perform an informal evaluation by examining the properties of excess returns other than those in foreign exchange markets. In this case, we take the point estimates of the parameters, generate time series for EP_{jt+k} , $k = 2, \dots, 5$ and compare the relevant statistics of actual and simulated data. As a second step, we formally evaluate the model by taking into account the sampling variability surrounding estimates of the parameters. The approach we use was recently developed by Canova (1994), incorporates ideas of Monte Carlo testing contained in Marriott (1979) and automatically provides a global sensitivity analysis for reasonable perturbations of the parameters.

In this second case our task is to generate probability statements for statistics of the simulated data. For example, we would like to know what is the probability that the model can generate Sharpe ratios of the same magnitude as those presented in Table 1. Available information on the parameters is summarized by means of a joint density $\pi(\theta | \mathcal{F})$, where \mathcal{F} is the information set available and $\theta \in \Theta \subset R^q$. Let $G(w_t(z_t) | \theta, m)$ be the density for the $q \times 1$ vector of endogenous time series w_t , conditional on the parameter vector θ and the particular

economic model m we have chosen. Here w_t includes excess returns corresponding to (1)–(5). $G(w_t(z_t) | \theta, m)$ describes the likelihood of obtaining a w_t path from our model once a particular θ vector is chosen. For given θ , randomness in w_t is due to the randomness in the exogenous processes z_t .

Let $J(w_t(z_t), \theta | m, \mathcal{F})$ be the joint distribution of w_t and θ given the model specification and the information set. In the analysis we focus on statistics of the simulated data which are functions $h(\theta, z_t)$ of the parameters θ and of the exogenous processes z_t . In our case $h(\theta, z_t)$ includes unconditional Sharpe ratios, the AR(1) coefficient of w_t and the regression coefficients of excess returns (1)–(5) on the set of common instruments. Model based probabilities for $h(\theta, z_t)$ can be obtained for any $\mathcal{A} \subset \Theta$ by evaluating integrals of the form

$$E(h(\theta, z_t) | m, \mathcal{F}, \mathcal{A}) = \int_{\mathcal{A}} h(\theta, z_t) J(w_t(z_t), \theta | m, \mathcal{F}) d\theta dz_t. \quad (21)$$

Although theoretically straightforward, expressions like (21) are generally impossible to compute analytically or using simple numerical rules when Θ is high dimensional. Our approach is to use a Monte Carlo methodology. The main idea is simple. Let θ_i be a $k \times 1$ -dimensional i.i.d. vector of parameters and $\{z_{it}\}_{i=1}^T$ be a path for z_t where the subscript i refers to the draw. If the probability function from which the θ 's and the z 's are drawn is proportional to $J(w_t(z_t) | m, \mathcal{F}_t)$, then, by the law of large numbers, $n^{-1} \sum_{i=1}^n h(\theta_i, z_{it})$ converges almost surely to $E(h(\theta, z_t))$, where n is the number of replications. Therefore, by drawing a large number of replications for θ and z from $J(w_t(z_t), \theta | m, \mathcal{F}_t)$, we can approximate arbitrarily well $E[h(\theta, z_t)]$.

Probability statements for the statistics of interest are easily obtained as a by-product of the Monte Carlo procedure. Suppose we have a vector of statistics H from the actual data. Then we can evaluate the model by computing the number of Monte Carlo replications such that $h(\theta_i, z_{it})$ is less than or equal to H , i.e. take the actual realization of the statistics as a critical value and evaluate the model's likelihood of realizing the vector of statistics we observe in the data. If the model is approximately correct, H should lie around the median of the distribution, i.e. $P(h(\theta, z_t) \leq H) \approx 0.5$. If H lies in the tail of the distribution, the model fails to capture the features of the data we are interested in.

The only question which is application dependent is the choice of $\pi(\theta | \mathcal{F})$. Here we select it to be the asymptotic distribution of the estimated parameters.

5. The results

We first briefly comment on the results of the estimation by simulation. First, the estimated values for the risk aversion parameter are small. In fact, for $k = 1$, the utility function is linear in aggregate consumption, while for $k = 3$ the utility function is approximately logarithmic. Second, the estimated parameters for the

conditional variance of government expenditure shares in the two cases are not significantly different because of the large standard errors. Third, the major difference across maturities is in the estimates of the risk aversion parameter. It appears as if the representative agent investing for three months in foreign exchange markets is more risk averse than the one investing for one month. One explanation for this difference is once again due to the lower variability of excess returns which, to a large extent, determines the properties of the estimated risk aversion. Fourth, for each maturity, the minimized value of the objective function is sufficiently large that the overidentifying restriction is not satisfied (note that there is one overidentifying restriction since for each maturity there are 7 statistics and 6 parameters to be estimated). Next we turn to the basic simulation results. As a benchmark, we first discuss the results obtained when the exogenous driving forces of the economy are conditionally homoskedastic. In this case it is sufficient to examine equations (B.1)–(B.5) in appendix B to note that EP_{jt+k} , $j = 1, \dots, 4$ will be different from zero at each t , but constant over time. Therefore, under this commonly used assumption, the version of the model considered here is unable to account for the time series features of many excess returns. The exception is EP_{5t+k} which will vary over time even when conditional variances are constant. Under the conditional homoskedasticity assumption time variation in dollar denominated UK holding premiums is entirely due to unexpected variation in exchange rates, which are in turn generated by unexpected variation in money supplies and government expenditure shares (see Eqs. (8) and (9)). Table 4 reports statistics for EP_{5t+k} . As the Sharpe ratios show, unexpected variations in money supplies and government expenditure shares are too small to induce enough variability in the series and none of the instruments is significant in the regression. Hence the model with conditionally homoskedastic driving forces is far from being able to explain time series features of excess returns.

Next, we study the situation where the exogenous processes are allowed to be conditionally heteroskedastic and the parameters used in the simulations are those reported in Table 3 and in Section 4. We proceed by allowing heteroskedasticity in one source of uncertainty at a time in order to examine the contribution of real, monetary and fiscal uncertainty to the time series properties of excess returns. First, we let the variances of outputs be time varying, keeping the other conditional variances constant and equal to the unconditional variances. For the selected values of γ and δ the variability in simulated excess returns induced by time variation in the variance of output is very small. In practice, time variations in the simulated time series for the eight excess returns which depend on the variance of output appear only after the fifth decimal. Hence, the risk generated in this artificial economy is too small to be priced and the model cannot explain movements in actual excess returns.

Second, we maintain the assumption that government expenditure shares are constant but now we also allow the conditional variances of the money supplies to be time varying. Although the performance of the model improves in this case, it

Table 4
 Simulated data: Statistics of excess returns and regression coefficients on instruments^a

	Sharpe	AR(1)	CH(<i>p</i>)	Constant	SPDIV	LONDIV	FP	USSPI	UKSP	USSP2	R ²	χ ² (6)	CH(6)	ARCH	KS	
															(13)	
Case of no conditional heteroskedasticity																
EP_{5t+1}	59.1	0.183	9.97	0.299	0.0002	-0.001	0.0001	0.0009	-0.0004	-0.0006	0.13	14.89	5.93	9.54	0.41	
			0.125	116.4	0.26	-1.27	1.41	1.51	-1.70	-1.30		0.02	0.43	0.65	0.79	
EP_{5t+3}	22.4	0.426	12.27	0.117	-0.001	-0.0007	-0.0007	-0.002	0.0003	0.0008	0.26	62.75	4.36	14.46	1.65	
			0.002	36.95	-1.80	-1.04	3.97	-1.74	1.48	1.86		0.00	0.11	0.15		
Case of conditional heteroskedasticity in all variables																
EP_{1t+1}	0.169	-0.010	0.79	-900.3	192.2	311.7	-0.98	.1	7.76	-417.2	0.99	60489	0.52	1.62	3.17	
			0.992	-1.23	0.39	0.70	-181.0	1.77	0.03	-2.45		0.00	0.997	0.999	0.00	
EP_{2t+1}	0.720	-0.040	5.69	26.64	5.42	-0.86	-0.00005	15.98	2.25	8.64	0.04	4.51	6.60	0.23	2.74	
			0.457	1.70	0.35	-0.07	-0.45	-1.16	0.65	1.24		0.607	0.358	1.00	0.00	
EP_{3t+1}	0.167	-0.010	0.79	929.45	-186.7	-312.7	0.98	-442.0	-5.53	425.8	0.99	57881	0.46	1.64	2.43	
			0.992	1.27	-0.38	-0.70	178.5	-1.83	-0.23	2.49		0.000	0.99	0.99	0.00	
EP_{4t+1}	0.761	-0.038	5.59	12.61	9.07	-2.59	0.0002	-4.60	-0.50	3.77	0.04	127.3	5.31	0.20	2.55	
			0.470	0.50	0.54	-0.23	7.64	-0.76	-0.18	0.77		0.000	0.50	1.00	0.00	
EP_{5t+1}	0.167	-0.010	0.774	-8538	4470	-4778	-0.09	-3988	-5944	2877	0.05	4.40	1.00	0.01	3.04	
			0.992	-0.91	0.61	-0.70	-0.91	-1.01	-1.22	1.11		0.569	0.92	1.00	0.00	

EP_{1t+3}	0.088	0.161	9.49	-90.63	4.16	15.44	-0.05	-4.31	-0.35	3.98	0.14	16.41	5.41	2.43	1.98
			0.008	-3.26	0.76	2.58	-0.90	-0.69	-0.14	0.91	0.011	0.06	0.09	0.09	0.03
EP_{2t+3}	0.668	0.089	5.46	7.35	-1.69	3.06	-0.0005	-7.33	-0.19	7.13	0.06	7.97	4.38	0.91	2.91
			0.065	0.38	-0.36	0.62	-0.16	-1.58	-0.10	1.99	0.239	0.111	0.99	0.99	0.00
EP_{3t+3}	0.617	0.314	23.60	113.3	-3.66	-17.19	0.004	-7.40	0.51	5.00	0.25	34.38	12.94	29.83	2.88
			0.000	5.58	-1.11	-3.93	1.79	-1.72	0.35	1.89	0.000	0.001	0.002	0.00	0.00
EP_{4t+3}	0.687	0.301	5.40	3.71	1.48	3.40	-0.007	2.24	-0.23	3.15	0.05	9.39	3.64	0.58	3.34
			0.066	0.18	0.30	0.67	-1.63	-0.41	-0.09	0.83	0.152	0.11	0.99	0.99	0.35
EP_{5t+3}	0.692	0.088	22.86	111.6	-2.95	-16.95	0.004	-7.01	0.66	4.66	0.23	28.11	12.82	29.47	3.34
			0.000	5.44	-0.89	-3.85	1.62	-1.68	0.45	1.74	0.000	0.001	0.003	0.00	0.00

^a EP_{jt+k} measures the nominal dollar denominated excess return from purchasing pounds forward in foreign exchange market, EP_{2t+k} is the US holding premium, EP_{3t+k} the UK holding premium, EP_{4t+k} the excess return from holding a UK bond benchmarked against the US risk free rate and EP_{5t+k} the UK holding premium in dollar terms. The first three columns report measures of time series predictability: Sharpe is the absolute value of the mean of the series divided by the standard error, $AR(1)$ the estimated first-order autocorrelation and $CH(p)$ the Cumby and Huizinga's (1992) test for the presence of serial correlation. For one-month holding period the test checks whether the first $p = 6$ AR coefficients are identically equal to zero. For three-month holding period $p = 2$ and the test checks whether the 3rd-4th AR coefficients are equal to zero, given that the first two AR coefficients may not be equal to zero. The next seven columns report the coefficients and the t -statistics of the regression of ER_{jt+k} on seven instruments available at time t . SPDIV is the SP500 dividend yield and LONDIV the Financial Times Index dividend yield, FP is the forward premium in the £ exchange market, UFPS1 and UKSP are the spreads in the US and in the UK between the yield on a long term Government bond and a short term instrument and USP2 is the spread in the US between low grade corporate bonds yield and a Government short term instrument. The last five columns report diagnostic statistics for the predictive regression: $\chi^2(6)$ is a test for the joint hypothesis of nullity of all regression coefficients but the constant, $CH(6)$ is the Cumby-Huizinga test for the significance of the first six serial correlation, $ARCH(13)$ is a test for conditional heteroskedasticity (with 13 degrees of freedom) and KS is a Kendall and Stuart test for normality of the residuals.

is still far from being adequate. For example, Sharpe ratios in six out of ten cases are still greater than 1, indicating very limited variability in simulated excess returns and SPDIV, LONDIV, FP and USSP are significant in the forecasting regression but they have, in general, the wrong sign.

Third, we allow all processes to be conditionally heteroskedastic. The results of this experiment are reported in table 4. The results with this specification are more encouraging than the previous ones. Sharpe ratios are all less than one so that the variability of the simulated series is of the same order of magnitude as the variability of actual data. Following Hansen and Jagannathan (1991), this finding implies that the volatility of the discount factor in our ICCAP model is in the range required to match the mean–variance features of excess returns. Moreover, as in the actual excess returns, serial correlation is negligible in 1 month returns while three month returns display significant correlation. Finally, LONDIV and the private–public spread have significant predictive power in several regressions but in certain cases they still enter with the wrong sign.

Several conclusions can be drawn at this point. First, time variation in the volatility of real output has little importance in accounting for movements in the actual data. Second, variation in the volatility of monetary and fiscal aggregates is instead crucial to generate variability in simulated excess returns which is comparable to that in actual returns. Third, although the model can replicate several qualitative features of the predictability of excess returns across markets, it is still inadequate. For example, as already mentioned, we need two sets of parameters to match both one and three month excess returns. In addition, Sharpe ratios are often numerically inconsistent with the actual data, in some cases too high and in others too low.

Since the failures we have documented so far may be due to auxiliary assumptions rather than intrinsic to the model, we next undertake a sensitivity analysis to check the robustness of the results to modifications in the model's statistical assumptions and to alternative settings of the parameters within a 'reasonable' range.

5.1. Some sensitivity results

This subsection reports the results of three experiments which modify some of the assumptions of the model. First, we consider the case where there is a structural break in the AR-GARCH specification for the exogenous processes around 1982. Second, we examine the implication of assuming a multivariate GARCH structure for the exogenous processes. Third, we consider adding conditional covariance terms to the expressions for simulated excess returns.

As mentioned in Section 3, we have assumed so far that the unconditional distributions of the stochastic processes driving the economy are stationary. This assumption may be inappropriate for the sample under consideration since there is some evidence that the US money process may have had at least one structural

break in 1982 when the Fed switched back from targeting the money supply to targeting interest rates. In addition, a casual look at the plot of the growth rate of UK IP indicates a possible statistical break around that date, probably as a consequence of Thatcher policies. If a structural break does exist in the data, the parameters of the AR-GARCH specification are biased when estimated over the entire sample and may underestimate the time variation in the variability of the series. To check for this possibility, Table 3 panel A reports the estimated GARCH parameters for the subsamples 75–82 and 82–91 for all four processes. While there are only minor changes in the features of the unconditional distribution of the US money supply, the UK IP index does show a significant structural break around 1982. The break in the properties of the mean of the UK IP index, however, is not dramatic enough to change substantially the major features of the results contained in Table 4. In practice, although the variability of UK IP index increases approximately by 50%, it induces changes in the EP_{5t+k} no larger than 5%. Because, as mentioned, time variation in the conditional variance of the output series induces time variation in excess returns only after the fifth decimal, none of the major results are altered.

Next we allow the stochastic processes to follow a multivariate GARCH structure. The reason for doing so is that by assuming univariate structures on the variances of the processes we possibly neglect some interaction terms which may help to boost the extent of the conditional heteroskedasticity and bring the statistics more in line with the actual data.

We conduct some specification tests by running distributed lag regressions on the square of the residuals of an AR(1) regression of the form

$$\epsilon_{it}^2 = a_0 + \sum_{k=1}^4 \sum_{j=1}^J a_{kj} \epsilon_{kt-j}^2 + u_t \quad (22)$$

and check for the significance of the a_{kj} , $k \neq i$ using a likelihood ratio test. The results of the tests are contained in Table 2, panel C. There is little evidence of interactions in the growth rates of money or output across countries. Also, there appears to be very small spillover effects in the variability of UK variables. We therefore proceed by allowing a bivariate GARCH(1,1) specification on US variables but maintain the univariate structure on UK variables. Because of the small sample we further limit the number of new parameters to two by setting to zero the coefficients on cross country lagged variances.⁵ The estimated specification appears in table 3, panel B. Note that the two new cross equation coefficients c_{12} and c_{21} are significant.

⁵ When we attempted to estimate these parameters they turned out to be insignificantly different from 0.

When we use these estimated parameters to compute agents' forecasts of future conditional variances the fit if the model improves in some dimensions but worsens in others. Sharpe ratios slightly improve but serial correlation completely disappears from simulated excess returns. Moreover, all the other statistics move out of line with the data. For example, LONDIV and the private–public spread lose their predictive power in the regressions, while evidence of misspecification emerges from the residuals of the predictive regression (they are too skewed and highly leptokurtic). All of these results therefore indicate that this modification is not particularly helpful in improving the performance of the model. It increase the variability of simulated excess returns but fails to bring other statistics in line with actual evidence.

As mentioned in the introduction, there is ample evidence that an ICCAP model entirely driven by the conditional covariances of the exogenous processes is unable to account for many features of excess returns in financial markets. Despite these failures, it may be interesting to know whether the risk due to time varying conditional covariances is significant when the risk induced by time varying conditional variances is already taken into account. In other words, we would like to know if the order of magnitude of the risk induced by time varying conditional variances is larger or smaller than that induced by time varying conditional covariances.

To incorporate conditional covariances we proceed as in Breeden (1986) and take a second order Taylor expansion of (11)–(14) around z_t . This allows interaction terms to enter the approximate closed form expressions for EP_{jt+k} . From the discussion in Section 4 and the evidence in Table 3 we know, however, that there is very little evidence of correlation among the exogenous stochastic processes, except perhaps, between US money and the US IP index. But accounting for this US money–IP correlation does not affect the substance of our basic results. Inspection of (11)–(14) indicates that because of the CRRA assumption, the spot exchange rate and EP_{1t+k} are both independent of supply side factors and that the term structure of interest rates in each country depends only on domestic factors. Hence, the conditional covariance of US output and money will enter only the simulated US holding premium return. Noting that $\text{cov}_t(M, IP) = \rho\sqrt{\text{var}_t(IP)\text{var}_t(M)}$, where ρ is the correlation coefficient, and that the magnitude of the conditional variance of US output is small, the additional term appearing in EP_{2t+k} is too small to account for any additional time variation in the actual US holding returns. This analysis therefore suggests that time varying conditional variances are more relevant than conditional covariances in determining the risk characteristics of excess returns.

We conclude this section by formally evaluating the ability of the model to reproduce the data. We would like to know whether the results we presented occurred by chance or if they are intrinsic to the model. For this we use the Monte Carlo methodology described in Section 4, randomizing 1000 times over both the parameters and the exogenous processes and report in Table 5 the probability that

Table 5

Simulated data: Case of conditional heteroskedasticity in all variables, P -values^a

	Sharpe	AR(1)	Constant	SPDIV	LONDIV	FP	USSP1	UKSP	USSP2
EP_{1t+1}	0.12	0.69	0.40	0.01	0.28	0.98	0.01	0.12	0.86
EP_{2t+1}	0.01	1.00	0.12	0.06	0.32	0.92	0.82	0.01	0.07
EP_{3t+1}	0.03	0.99	0.06	0.85	0.04	0.08	0.00	0.00	0.00
EP_{4t+1}	0.08	0.99	0.04	0.04	0.02	0.05	0.42	0.01	0.26
EP_{5t+1}	0.00	0.98	0.33	0.00	0.01	0.04	0.56	0.02	0.20
EP_{1t+3}	0.11	1.00	0.08	0.17	0.72	0.09	0.83	0.77	0.72
EP_{2t+3}	0.00	1.00	0.05	0.08	0.02	0.85	0.06	0.44	0.03
EP_{3t+3}	0.00	0.97	0.14	0.68	0.06	0.08	0.13	0.38	0.14
EP_{4t+3}	0.18	0.98	0.04	0.06	0.73	0.11	0.52	0.59	0.66
EP_{5t+3}	0.00	1.00	0.09	0.10	0.03	0.23	0.38	0.21	0.59

^a EP_{1t+k} measures the nominal dollar denominated excess return from purchasing pounds forward in foreign exchange market, EP_{2t+k} measures the US holding premium, EP_{3t+k} measures the UK holding premium, EP_{4t+k} measures the excess return from holding a UK bond benchmarked against the US risk free rate and EP_{5t+k} measures the UK holding premium in dollar terms. The number in each cell represents the probability that the model generates the statistic observed in the actual data. For regression coefficients the number in the cell represents the probability that the coefficient has the correct sign and the correct significance level.

the model generates the statistics we observe in the actual data. For the regression coefficients we present the probability that the estimated value has the correct sign and the correct significance level (above or below 5%). Figs. 1 and 2 present the simulated densities for the Sharpe ratios for each of the five excess returns for the two maturities. Vertical bars indicate the location of the actual value of the Sharpe ratio.

The results of Table 5 strengthen results previously obtained. Although there are parameter configurations which may generate the quantitative features described in Table 1, statistics obtained from the actual data are low probability events. In many cases, the model generates values for the statistics which are consistently higher than those we see in the data (such as Sharpe ratios) or consistently lower (such as the values of AR(1) coefficients). Also, while the model appears to be appropriate in accounting for features of the UK holding premium benchmarked against the US risk free rate, it is weaker in explaining the properties of excess returns in all US markets. Finally, the model does somewhat better for three month excess returns than for one month returns.

6. Conclusions

In this paper we attempted to account for the predictability features of a variety of excess returns using a representative agent ICCAP model with complete markets. We show that although allowing for time variations in the second

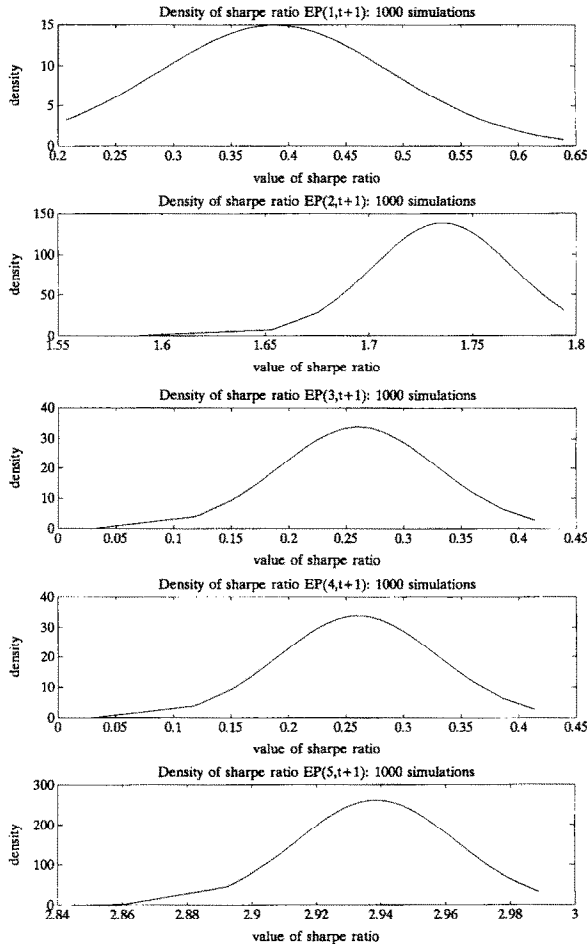


Fig. 1. Densities of Sharpe ratios.

moments of monetary and fiscal variables improves the theory's performance dramatically the model fails in at least three dimensions. First, it cannot jointly account for the time series properties of one and three month excess returns. Second, it fails to quantitatively replicate the serial correlation properties of excess returns. Third, it cannot replicate the significance of dividend yields and spreads in predictive regressions with excess returns as dependent variables. Since these failures are robust to model specification and, to a certain extent, to parameter choices in a reasonable range, they lead us to conclude that the shortcomings are

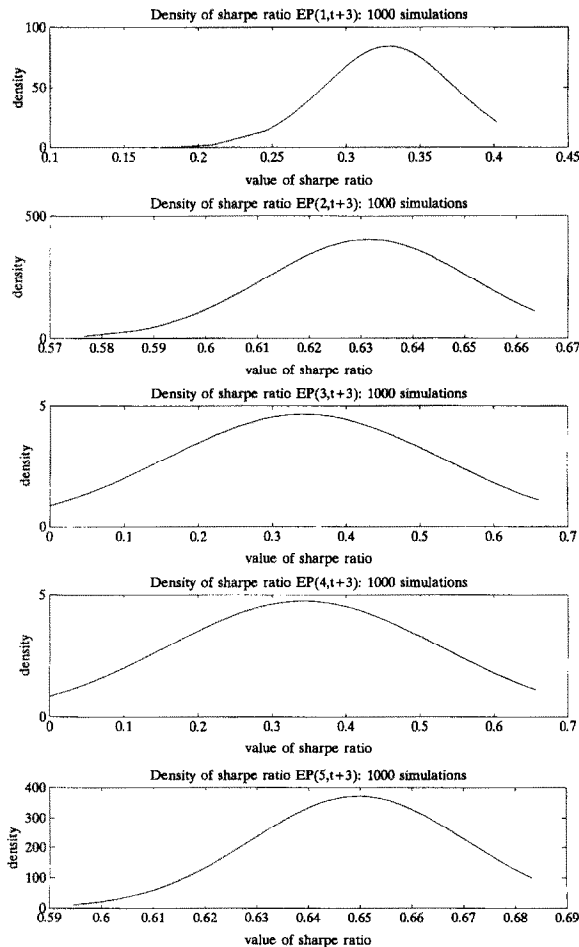


Fig. 2. Densities of Sharpe ratios.

intrinsic to the model and not simply specific to the particular specification of the model used.

Numerous modifications of the basic ICCAP model have been suggested in the literature to overcome its failures. These include modifications of the preference structure (see e.g. Constantinides, 1990; Epstein and Zin, 1991), the introduction of some form of liquidity constraint or market incompleteness (see e.g. Lucas, 1991; Telmer, 1991) or the use of production based ICCAP models (see Cochrane, 1991). We doubt that these modifications will be useful to improve the results presented here. The reason is that all these modifications are designed to increase

the volatility of the discount factor in the asset pricing equation. The paper has shown that the amount of conditional heteroskedasticity in the exogenous stochastic processes is already sufficient to boost the variability of the discount factor in the range needed to approximately account for the variability of actual data. Adding one or more of these features will probably add more variability to the simulated time series but will not help to improve in those dimensions where the model does poorly.

A more promising line of research seems to be one where some form of market segmentation is introduced or where the representative agent assumption is abandoned. Market segmentation can, in principle, account for different statistical properties in one and three month excess returns. Moreover, the fact that the utility function obtained by fitting the model to one month excess returns is close to being linear while that obtained by fitting the model to three month returns is approximately logarithmic, suggests nontrivial differences in the risk characteristics in agents investing at different maturities. Hence a model where the representative agent assumption is abandoned may help to explain these differences. We leave the evaluation of the usefulness of these extensions of the basic ICCAP model for future research.

Appendix A: Data description

All data is taken from Datastream data set and records end of the month values. The Datastream code names of the series used are in parentheses. The spot exchange rate is measured as dollars per pound and reports the value at the London market (UKUSEX). The forward rate is constructed from Eurodeposit rates using the covered interest parity relation. Eurodeposit rates are those quoted in the London market for one- and three-month \$ and £ denominated deposits (USEURO1, USEURO3, UKEURO1, UKEURO3). Long term government bond yields are taken from the National Government Series for the UK and the US. Observations are averages of yields on various five-year government securities (USBGOV5, UKBGOV5). All interest rate data is in annualized percentage terms. Low grade private corporate bonds are available only for the US and represent averages of yields on four- to seven-year securities (USBNDY3P). Dividend yields are calculated by averaging the dividend paid over the past twelve months on SP500 and London 500 share indices and are calculated using local currencies (SPCOMP(DY) and FTA500(DY)).

The money supply data for the US and the UK is taken from IFS tapes. The industrial production series, which we use as proxies for outputs, are taken from Datastream (USINDPROD and UKINPROD) and are seasonally adjusted indices. The data covers the 1975,1–1991,8 period except for the UK money supply which ends at 1989,12.

Appendix B: Closed form solutions

The closed form expressions for EP_{it+k} , $i = 1, \dots, 5$ are given by:

$$\begin{aligned}
 EP_{1t,k} = & \frac{1200}{k} \left\{ 0.5\sigma_{3t,k}^2 - 0.5\sigma_{4t,k}^2 - \frac{1-h_{5t,k}}{h_{5t,k}} \ln(1-h_{5t,k}) \right. \\
 & + \frac{1-h_{6t,k}}{h_{6t,k}} \ln(1-h_{6t,k}) \\
 & - \ln[1 - (1-h_{5t,k})^{1+\delta(1-\gamma)}] + \ln[1 - (1-h_{5t,k})^{\delta(1-\gamma)}] \\
 & - \ln[1 - (1-h_{6t,k})^{(1-\delta)(1-\gamma)}] + \ln[1 - (1-h_{6t,k})^{1+(1-\delta)(1-\gamma)}] \\
 & + \ln[1 + \delta(1-\gamma)] - \ln[\delta(1-\gamma)] + \ln[(1-\delta)(1-\gamma)] \\
 & \left. - \ln[1 + (1-\delta)(1-\gamma)] \right\}, \tag{B.1}
 \end{aligned}$$

$$\begin{aligned}
 EP_{it+k} = & \frac{1200}{k} \left\{ 0.5\sigma_{(i+1)t,k}^2 + \frac{\delta^2(1-\gamma)^2}{2} \sigma_{1t,k}^2 + \frac{(1-\delta)^2(1-\gamma)^2}{2} \sigma_{2t,k}^2 \right. \\
 & - (1-\delta(1-\gamma)) \frac{1-h_{5t,k}}{h_{5t,k}} \ln(1-h_{5t,k}) \\
 & + (1-\delta)(1-\gamma) \frac{1-h_{6t,k}}{h_{6t,k}} \ln(1-h_{6t,k}) \\
 & + \ln[1 - (1-h_{5t,k})^{\delta(1-\gamma)}] + \ln[1 - (1-h_{6t,k})^{1+(1-\delta)(1-\gamma)}] \\
 & - \ln[\delta(1-\gamma)] - \ln[1 + (1-\delta)(1-\gamma)] - \ln(h_{5t,k}) \\
 & \left. - \ln(h_{6t,k}) - \gamma \right\}, \quad i = 2, 3, \tag{B.2}
 \end{aligned}$$

$$EP_{4t+k} = EP_{3t+k} + EP_{1t+k}, \tag{B.3}$$

$$\begin{aligned}
 EP_{5t+k} = & EP_{3t+k} + \left[z_{3t,k} - z_{3t} - z_{4t,k} + z_{4t} - \ln(1-z_{5t}) + \ln(1-z_{6t}) \right. \\
 & \left. - \frac{1-h_{5t,k}}{h_{5t,k}} \ln(1-h_{5t,k}) + \frac{1-h_{6t,k}}{h_{6t,k}} \ln(1-h_{6t,k}) \right], \tag{B.4}
 \end{aligned}$$

where $h_{it,k} = \sqrt{12\sigma_{it,k}^2}$, $i = 5, 6$; $z_{jt,k}$ is the conditional mean at time t of z_{jt+k} and $\sigma_{jt,k}$ is the conditional variance at time t of z_{jt+k} .

Appendix C: Minimization routine

The minimization routine we use to compute SMM estimates of the parameters is numerical because the function Q is not well behaved and a standard hill climbing routine produces values for the gradient which are too small to move away from initial conditions. The procedure we employ is as follows. First, we evaluate Q at five different points in each of the five dimensions and use an interpolation procedure to reconstruct the shape of Q and to obtain a guess for the gradient and for the most likely direction where the minimum is located. Second, we grid the space around this first minimum using the guessed gradient to select the ranges in the five dimensions and then repeat the function evaluation and the interpolation procedure to obtain a new guess for the minimum of Q and for the gradient. We repeat this procedure five times and we report the minimum of Q and the values of β obtained at the last iteration. To confirm that the value of Q obtained in the fifth iteration is really the minimum we perform a sensitivity analysis in two ways: first we arbitrarily perturb one parameter at a time in a neighborhood of its optimal value to see if another minimum is achieved. Second, we restart the minimization procedure from different initial conditions to check if the algorithm converges to a new minimum. Because the function is ill-behaved, this second step of the sensitivity routine is often crucial to avoid getting stuck in a local minimum.

Since each grid requires $5^5 = 3125$ evaluations of Q and because we start the procedure three times from different initial conditions, the total number of function evaluations is 9375. On a 25-mhz 486 machine using the RATS random number generator and the seed command set equal to 2, the total computation time for the grid search was about 80 minutes. Given simulation results contained in Gourieroux and Monfort (1991), we set $n = 10$ in estimating the free parameters.

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