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Profits, risk, and uncertainty in foreign exchange markets*

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This paper examines the properties of nominal profits from speculation in dollar-dominated forward contracts using a representative agent cash-in-advance model, modified to allow for heteroscedasticity in the exogenous processes. The model is simulated by estimating exogenous processes from the data and the remaining free parameters with a simulated method-of-moments technique. Simulated expected profits are variable, heteroskedastic, and serially correlated, but the magnitude of these second moments fall short of those of the predictable component of observed profits on the U.S. dollar. As in the actual data simulated forward rates display biasedness in predicting simulated future spot rates.

Key words: Exchange rates; Time series models; Statistical simulation methods
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1. Introduction

A large body of empirical work indicates that throughout the 1980's nominal profits from speculation in forward contracts on the U.S. dollar were highly

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volatile but also displayed a predictable component, which was itself volatile and serially correlated [see, e.g., Frankel and Meese (1987), Hodrick (1987)]. Whether this behavior reflected required profits arising from traders' assessment of future fundamentals or was simply the result of forecast errors is still unresolved. A central issue in the debate has been determining whether the predictable component of profits resulted from changes in the perceived risk of engaging in a forward contract.

Frankel (1986) uses a mean–variance optimization framework to derive theoretical restrictions on the size of the risk premium in foreign exchange markets. Since these theoretical bounds imply small and constant expected profits, he finds it difficult to attribute the time series properties of realized nominal profits on the dollar experienced in the 80's to risk [see also Giovannini and Jorion (1988)]. Instead, based on survey data measuring expectations of future spot rates, Frankel and Froot (1987) and Froot and Frankel (1989) provide empirical support for the view that the statistical properties of realized nominal profits on the dollar are more closely related to those of the forecast error than the risk premium. These results led some authors [see, e.g., Lewis (1989), Kaminsky (1989)] to construct theoretical models where the risk premium is negligible and the properties of observed profits are due entirely to expectations of unrealized policy regime shifts, which generate volatile and serially correlated forecast errors.

Another branch of literature [see, e.g., Hodrick and Srivastava (1984), Cumby (1988), Flood (1988), Macklem (1991)] has examined theoretical models where the risk premium can vary over time and thus, in principle, account for the statistical properties of nominal profits. This literature, which is based on the intertemporal consumption-based capital asset pricing model (ICCAPM) with time-separable preferences, has concluded that the framework is unable to replicate the variability and serial correlation properties of predictable profits under a relatively wide range of parameterizations. This result is consistent with the evidence in other asset markets [see, e.g., Mehra and Prescott (1985), Backus, Gregory, and Zin (1989), Giovannini and Labadie (1991)]. In general, the paradigm fails to reconcile the small variability in aggregate consumption with the relatively large, volatile, and serially correlated profits in excess of the risk-free rate observed for many risky assets. In response to these failures, many authors have recently modified the standard ICCAPM to account for habit persistence. For foreign exchange markets Backus, Gregory, and Telmer (1990) found the modification helpful in reproducing the variability of expected profits from forward speculation.

In a standard international ICCAPM, expected profits depend on the conditional covariance between the nominal intertemporal marginal rate of substitution and the change in the nominal exchange rate. The conditional variances of these two quantities, typically assumed to be constant, may affect the level but do not account for the time series properties of expected profits. This paper explicitly recognizes that the conditional volatility of fundamentals may be an important determinant of expected profits. We attempt to determine whether variation over time in the variability of fundamentals of the economy is useful in providing a quantitative account of the time series behavior of the predictable component of actual dollar-denominated profits.

In this exercise we employ a standard two-country, two-good, cash-in-advance (CIA) model. Exogenous stochastic processes governing the behavior of output, monetary, and fiscal variables determine the endogenous variables of the model. Following Hodrick (1989), we introduce distributional assumptions that imply that the population properties of equilibrium expected profits from forward speculation depend on the time series features of the conditional moments of the exogenous processes. In the closed form solution we derive, the theoretical time series properties of expected profits depend on three factors: the parameter of risk aversion, the share of foreign goods in household consumption, and the conditional variances of the money supplies and government expenditure shares.

The model is simulated by estimating the exogenous processes from actual data and choosing the remaining parameters using a simulated method-of-moments approach [see Lee and Ingram (1990), Duffie and Singleton (1990)]. We use this approach, as opposed to more standard estimation techniques, because it uses the complete representation of the stochastic equilibrium model. Also, it is preferable to simple calibration exercises since it allows us to both formally select free parameters and undertake sensitivity analysis by examining the statistical properties of the time series generated by the model under a wide variety of parameterizations.

The simulations demonstrate that although the model generates expected profits that are variable, heteroskedastic, and serially correlated, the magnitude of these second-order moments falls short of those we observe in the predictable component of actual profits on the dollar. We find that the second-order properties of the simulated data must be attributed primarily to fluctuations in the conditional variability of government spending, rather than money or output growth. We also find that the properties of the consumption risk premium of the model, that is, the component of simulated expected profits arising solely from risk-averse behavior, can be quite different from those of the predictable component of actual profits.[1] This suggests that, although it is common to attribute movements in expected nominal profits to a time-varying risk premium, the error in doing so may be large [see also Engel (1992) for this point]. Finally, we show that the simulated forward rate is a biased predictor of

[1]The expected component of profits, often interpreted as a risk premium, is actually the sum of a risk premium and a convexity term arising from Jensen's inequality. It is, however, common to ignore the convexity term on the grounds that it is small [see, e.g., Frenkel and Razin (1980)] and attribute the statistical features of expected profits to a time-varying risk premium.

the simulated future spot rate. This feature of the actual data has been extremely puzzling from the point of view of the simple expectational hypothesis. Here biasedness occurs because the forward rate forecast error in predicting future spot rates is neither homoscedastic nor uncorrelated with the available information set. When spot and forward rates are generated by heteroskedastic driving processes, regression tests of efficiency miss the dynamics of the data and provide erroneous conclusions regarding efficiency.

The paper is organized as follows. The next section analyzes the statistical properties of expected nominal profits on the dollar in five foreign exchange markets. Section 3 presents the theoretical framework of analysis and identifies the determinants of expected profits in the model. Section 4 describes the estimation of exogenous processes from actual data, introduces the simulated method-of-moments technique, and provides estimates of the free parameters. Section 5 contains a discussion of the results and a sensitivity analysis. Section 6 compares the model's implications for consumption, spot, and forward rates with the actual data. Conclusions appear in section 7.

2. Properties of expected profits from forward speculation

This section examines the statistical properties of the predictable component of dollar-denominated profits for five different exchange markets and for an equally-weighted portfolio of currencies for two holding maturities. The point of view is the one of an investor who takes a long forward position in the foreign currency. The markets considered are German mark/US dollar (DM/\$), French franc/US dollar (FF/\$), UK pound/US dollar (£/\$), Japanese yen/US dollar (Y/\$), and Swiss franc/US dollar (SF/\$) [or Canadian dollar/US dollar (Can\$/\$)]. We employ monthly observations on the closing value of the last business day of the month at the London market. Let S_t be the foreign currency price of a US dollar for immediate delivery and $F_{t,k}$ the foreign currency price of a k-month contract for delivery of a dollar at $t+k$. Then, the (approximate) annualized percentage realized nominal profits in market i is computed as $RP^i_{t,k} = (1200/k) * [\ln(F^i_{t,k}) - \ln(S^i_{t+k})]$. To construct a measure of expected profits, $EP^i_{t,k}$, we follow Cumby (1988) and regress $RP^i_{t,k}$ on a set of variables belonging to the information set of agents at time t. We present summary statistics for $k = 1$ for the sample period 1974.7–1986.10 and $k = 3$ for the sample period 1975.1–1991.9 when the available information set includes a constant and the forward premium (defined as $FP^i_{t,k} = (1200/k) * [\ln(F^i_{t,k} - \ln(S^i_t)])$.[2]

[2]We tried different specifications which also included the forward premium squared, dividend yields, a measure of interest rate spread, and a set of seasonal dummies to capture deterministic monthly patterns that might be present in the data, with no appreciable change in the results.

There are two reasons for selecting these two holding maturities. We chose $k = 1$ to maintain comparability with existing work [see, e.g., Backus, Gregory, and Telmer (1990), Macklem (1991)]. We would like to know if our modification of the basic model helps to understand their pattern of results. In addition, we select a longer holding period because there is some evidence [see, e.g., Lewis (1991)] that the holding period matters for both the statistical properties of profits and for tests of the ICCAP model. For example, while practically all the literature using a weekly or monthly holding period rejects the model [see, e.g., Hodrick and Srivastava (1984)], for $k = 3$ Campbell and Clarida (1987) fail to reject it. Table 1, columns 1–5 report the statistical properties of $EP^i_{t,3}$ in the first five exchange markets, while table 2, columns 1–5 report those of $EP^i_{t,1}$. For one-month profits we use the Canadian dollar/US dollar market in place of the Swiss franc/US dollar market to maintain the same set of currencies used by Backus, Gregory, and Telmer (1990). In addition we construct the expected component of the cross-sectional average nominal profits on the dollar over the five different exchange markets. In this case the information set used to compute expected profits contains a constant and the forward premium in each of the five markets. This series corresponds to the expected profits obtainable at each t by a US trader who purchases an equally-weighted portfolio of forward contracts on the five currencies and sells the contracts at maturity. The statistical features of expected returns from this portfolio are presented in columns 6 of each of the two tables. The two time series are plotted along with their estimated moving average representations in figs. 1 and 2, panel A.

Several features of the results deserve comment. First, for both maturities the unconditional means of the portfolio expected returns are small and insignificantly different from zero, while for some of the individual markets taken separately the mean of expected profits differs from zero. Second, both for the individual markets and for the portfolios, the unconditional variability is large relative to the unconditional mean and variability is larger for three-month expected profits than for those of one month. Moreover, this variability constitutes a nonnegligible fraction of the variability of realized profits despite the large unanticipated movements in exchange rates during the sample period; for one-month profits the standard error of the predictable component is 18% of the standard error of the realized profit series, and for three-month profits it is 32%. Third, for both maturities expected profits have very strong persistence. Because the three-month holding period exceeds the sampling frequency of the data, one should expect some serial correlation to appear even though the true profit series is not predictable using time t information. However, even if MA components of order 2 may exist, the third and forth AR coefficients should equal zero under the null of no serial correlation. We use Cumby and Huizinga's (1992) test to examine the significance of these AR coefficients when MA(2) components are present in the data and find strong evidence against the null hypothesis of no serial correlation. Fourth, estimated third and fourth

Table 1

Monthly statistics of expected and of simulated expected three-month profits, 1975.1–1991.9.[a]

	Expected profits					Average expected profits	Simulated expected profits			Simulated risk premium	Simulated expected profits	
	DM/$	SF/$	FF/$	£/$	Yen/$		Joint	Single	$\mathrm{var}_t(\Phi_{hit})$ = const.		$\hat{\beta}$: $k = 1$	DM/$
Mean	−1.26	−1.64	0.13	0.94	−1.05	−0.42	1.64	−0.37	−0.72	0.35	0.05	−0.29
P-value	0.00	0.00	0.57	0.19	0.30	0.41	0.00	0.24	0.00	0.26	0.09	0.36
Std. error	5.16	8.12	3.16	10.04	12.59	7.17	1.57	4.58	0.0003	4.53	0.23	5.46
Skewness	0.88	0.60	0.64	0.20	−0.10	0.27	−0.21	−1.72	3.30	−1.70	−0.19	−1.42
P-value	0.00	0.00	0.00	0.27	0.63	0.14	0.23	0.00	0.00	0.00	0.26	0.00
Kurtosis	4.78	3.42	4.14	3.46	3.39	3.27	0.04	6.43	16.31	6.28	−0.05	7.81
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.90	0.00	0.00	0.00	0.88	0.00
Autocor. 1	0.89	0.94	0.77	0.90	0.91	0.90	0.43	0.30	−0.03	0.31	0.29	0.27
Autocor. 2	0.74	0.86	0.65	0.82	0.79	0.80	0.44	0.39	0.26	0.39	0.35	0.32
Autocor. 3	0.62	0.78	0.52	0.73	0.65	0.69	0.33	0.27	0.05	0.27	0.24	0.26
Autocor. 4	0.54	0.72	0.44	0.65	0.54	0.61	0.20	0.16	0.07	0.16	0.12	0.13
CH(2)	23.62	31.16	20.84	30.30	21.12	22.09	13.34	10.15	1.74	10.23	8.46	10.19
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00	0.01	0.06
ARCH(12)	47.62	21.24	52.98	35.21	45.03	43.38	14.54	5.66	8.64	5.75	16.01	6.66
P-value	0.00	0.05	0.00	0.00	0.00	0.00	0.26	0.93	0.73	0.92	0.19	0.77
BP(12)	37.81	42.04	27.20	27.00	37.14	34.26	19.85	24.26	19.07	24.34	19.46	22.06
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.01	0.08	0.01	0.07	0.02
White(24)	85.37	61.32	56.80	43.20	67.23	44.47	36.23	32.51	19.07	32.70	37.02	30.57
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.11	0.74	0.11	0.04	0.14
BD	1.88	2.06	1.13	1.29	1.76	1.81	2.08	4.92	1.66	1.91	1.38	1.59

[a] CH refers to Cumby–Huizinga *l*-test, BP to Breush–Pagan test, White to White test, and BD to the Brock–Dechert test. The numbers in parentheses refer to the number of degrees of freedom of the χ^2 statistics. The values of the skewness and kurtosis are those obtained by prefiltering the series with twelve lags. 'Joint' indicates simulations where the parameters are selected to joint match one- and three-month profits. 'Single' indicates simulations where the parameters are selected to match three-month profits only. $\hat{\beta}$: $k = 1$ indicates simulations where the parameters are selected to match one-month profits.

Table 2

Monthly statistics of expected and of simulated expected one-month profits, 1974.6–1986.10.[a]

	Expected profits					Average expected profits	Simulated expected profits				
	DM/\$	CNS/\$	FF/\$	£/\$	Yen/\$		Joint	Single	$\text{var}_t(\Phi_{hit}) = \text{const.}$	$\hat{\beta}$: $k = 3$	DM/\$
Mean	1.08	1.39	−0.79	1.22	−2.25	0.66	11.42	−2.34	−2.17	0.38	0.84
P-value	0.00	0.00	0.06	0.00	0.00	0.85	0.16	0.18	0.0	0.04	0.07
Std. error	2.05	2.03	5.10	4.67	9.86	4.97	115.25	15.16	0.01	13.28	6.75
Skewness	1.75	−0.19	−0.61	1.09	−0.78	−0.17	−2.57	−2.89	0.75	0.75	−1.13
P-value	0.00	0.37	0.00	0.00	0.00	0.42	0.00	0.00	0.00	0.00	0.00
Kurtosis	14.08	3.74	4.05	12.25	4.50	0.77	16.04	15.18	4.80	7.41	19.38
P-value	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.00
Autocor. 1	0.57	0.74	0.58	0.73	0.81	0.81	−0.01	0.04	−0.08	0.07	0.02
Autocor. 2	0.40	0.58	0.55	0.63	0.73	0.69	0.27	0.23	0.17	0.13	0.13
Autocor. 3	0.27	0.44	0.39	0.53	0.67	0.57	0.02	0.16	0.10	0.04	0.16
Autocor. 4	0.22	0.33	0.28	0.45	0.53	0.47	0.05	0.03	0.02	0.10	0.02
CH(5)	125.78	239.96	165.30	274.78	361.87	332.58	0.73	4.83	2.51	2.26	5.16
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.08	0.28	0.32	0.07
ARCH(12)	7.10	18.94	41.15	14.19	18.36	22.45	7.41	3.14	5.23	4.84	5.13
P-value	0.85	0.09	0.00	0.29	0.10	0.03	0.82	0.99	0.54	0.96	0.95
BP(12)	4.66	13.02	28.86	9.83	9.75	15.93	4.94	14.50	14.20	7.97	1.43
P-value	0.97	0.37	0.00	0.63	0.63	0.19	0.95	0.26	0.28	0.78	0.99
White(24)	12.45	55.75	51.81	17.64	20.08	27.45	5.17	21.68	21.03	14.58	6.14
P-value	0.97	0.00	0.00	0.82	0.66	0.28	0.99	0.59	0.63	0.93	0.99
BD	1.74	1.93	1.56	1.03	1.42	1.88	1.58	0.87	3.02	0.95	1.26

[a] CH refers to Cumby–Huizinga *l*-test, BP to Breush–Pagan test, White to White test, and BD to the Brock–Dechert test. The numbers in parentheses refer to the number of degrees of freedom of the χ^2 statistics. The values of the skewness and kurtosis are those obtained by prefiltering the series with twelve lags. 'Joint' indicates simulations where the parameters are selected to joint match one- and three-month profits. 'Single' indicates simulations where the parameters are selected to match one-month profits only. $\hat{\beta}$: $k = 3$ indicates simulations where the parameters are selected to match three-month profits.

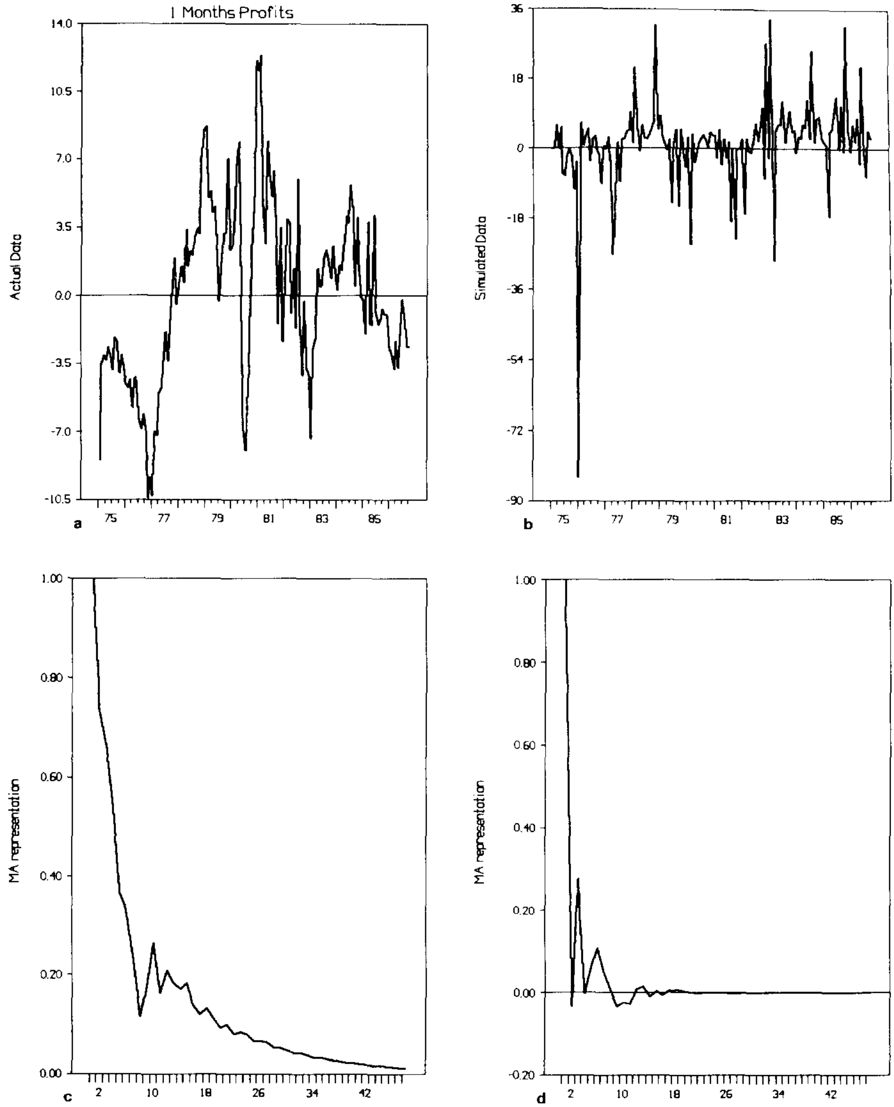

Fig. 1. One-month profits.

unconditional moments suggest that the expected portfolio profits for the two maturities deviates from normality; although not skewed, they are generally leptokurtic. There is also skewness in some of the individual markets. Strong skewness emerges for expected profits in DM/\$, FF/\$, and SF/\$ markets, while in other markets the evidence is mixed and depends on the maturity.

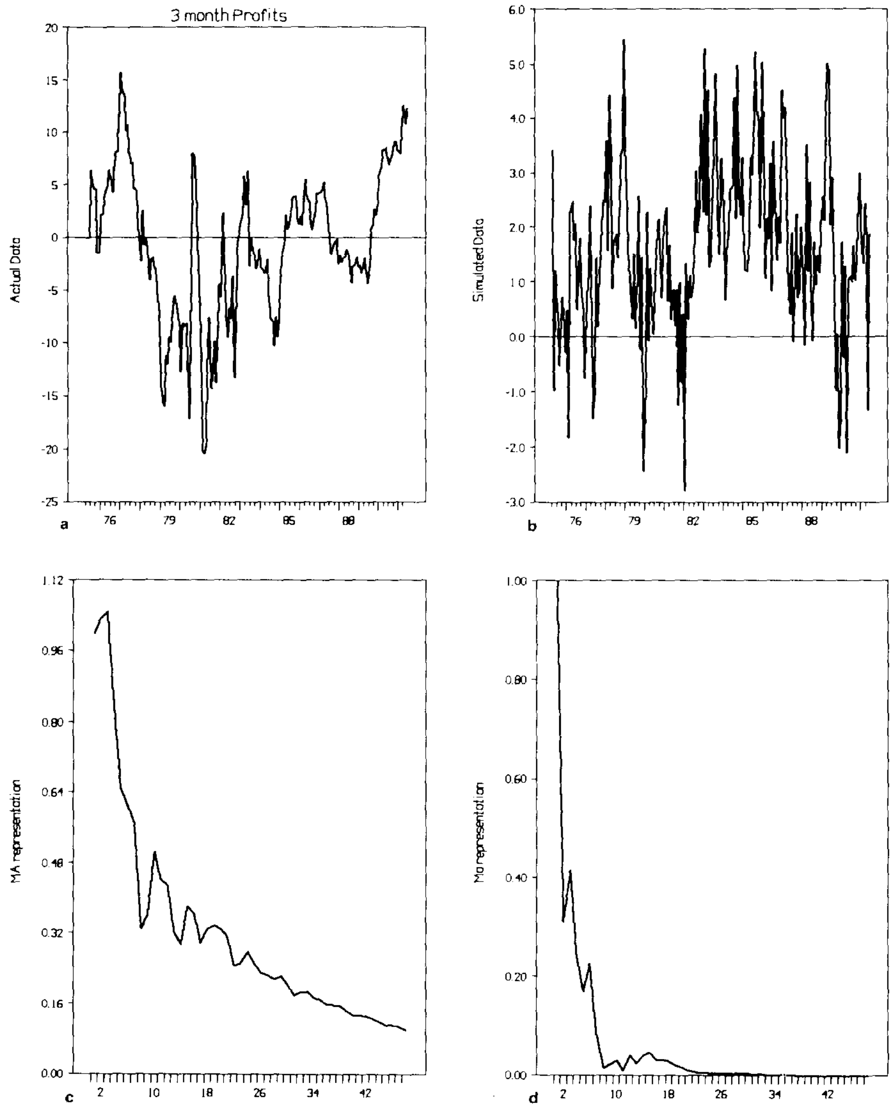

Fig. 2. Three-month profits.

An examination of the conditional distributions of the expected profits series indicates evidence of time variation in the conditional variances. For three months the three tests reject the null hypothesis of no heteroskedasticity, both for the individual markets and for the portfolio of currencies. For expected one-month portfolio profits, the evidence is more mixed. A test for ARCH in the squares of the residuals of a six-lag autoregression of each series rejects the null of no heteroskedasticity for the portfolio expected profits, but the Breush and Pagan (1979) and White (1980) tests do not. For individual markets only for the FF/$ expected profits the three tests agree in rejecting the null hyothesis. Finally, the Brock and Dechert (1988) test for nonlinearities in the recursive residuals $\hat{\varepsilon}_t = (y_t - \hat{a}_t(L)y_{t-1})/\hat{\sigma}_t$, where $\hat{a}_t(L)y_{t-1}$ and $\hat{\sigma}_t$ are estimates at t of the conditional mean and the conditional standard error of nominal profits in each market, does not reject the hypothesis that $\hat{\varepsilon}_t$ is a white noise in all markets.

We take the predictable component of the two portfolios profit series as representative of the conditions existing in foreign exchange markets during the floating regime era and compare their statistical properties with those of expected profits simulated by the model.

3. The theoretical framework of analysis

The theoretical framework we employ is a version of the cash-in-advance monetary model developed by Lucas (1982) and modified by Hodrick (1989). Since the model is well known in the literature, we only briefly describe its features and proceed directly to the computation of the equilibrium values of the variables of interest.

The economy is characterized by two countries: The US and the rest of the world. Every period, each country i is endowed with Y_{it}, $i = 1,2$, units of a nonstorable consumption good. There are two governments which consume G_{it} units of their own country's good. To finance these consumption requirements each government issues a country-specific money, M_{it}, collects real lump sum taxes, T_{it}, levied equally on agents from both countries, and issues debt to finance any purchases in excess of money creation and tax collections. This debt is in the form of state-contingent nominal bonds of maturity k, $k = 1,2,\ldots,K$, denominated in their own country's currency. Endowments, government consumption requirements, and money supplies are exogenous and follow a first-order Markov process with a stationary and ergodic transition function.

The countries are each populated by a representative household maximizing a time-separable utility function defined over the two goods. Households are subject to both a wealth constraint and a liquidity constraint which compels them to purchase goods with cash. The timing of the model follows Lucas with asset markets open first and goods markets following. At the beginning of each period the consumer enters the asset market and decides how to allocate her wealth among the productive assets of the two countries, currencies, and the state-contingent nominal bonds issued by the two governments. After the asset market closes, the consumer enters the goods market and makes her consumption purchases with previously accumulated currency.

Equilibrium requires that households optimize and all markets clear. Since capital markets are complete, this permits an unconstrained Pareto-optimal allocation of the time-state-contingent nominal bonds.

Let $e^{-r_{it,k}(v)}$ denote the discount price at t of a bond paying one unit of currency i at time $t+k$, if event v occurs, and $r_{it,k}(v)$ denote the associated continuously-compounded interest rate.

In equilibrium nominal interest rates reflect optimal consumption–saving decisions by equating bond prices to individuals' expected marginal rate of substitution of future nominal expenditure for current nominal expenditure, i.e.,

$$e^{-r_{it,k}} = E_t \frac{\beta^k P_{it} U_{it+k}(c_{1t+k}, c_{2t+k})}{P_{it+k} U_{it}(c_{1t}, c_{2t})}. \tag{1}$$

Because all uncertainty is resolved prior to the household's money holding decisions, they hold just enough currency to finance their current consumption purchases. This implies that the quantity theory holds so that $P_{it} = M_{it}/Y_{it}$ and

$$e^{-r_{it,k}} = E_t \frac{\beta^k Y_{it+k}(M_{it+k})^{-1} U_{it+k}}{Y_{it}(M_{it})^{-1} U_{it}}. \tag{2}$$

Hodrick, Kocherlakota, and Lucas (1991) show that when a one-country version of the above model is calibrated to the US economy, the cash-in-advance constraint almost always binds. Bekaert (1991) shows that the same occurs in a two-country setting. Therefore, there appears to be little practical gain in specifying models with more complicated nonbinding constraints [as in Hodrick (1989)].

In equilibrium, the nominal spot rate is equal to the marginal rate of substitution of domestic currency for foreign currency:

$$S_t = \frac{U_{1t} P_{2t}}{U_{2t} P_{1t}} = \frac{Y_{1t}(M_{1t})^{-1} U_{1t}}{Y_{2t}(M_{2t})^{-1} U_{2t}}. \tag{3}$$

Therefore, the k-period-ahead conditional future log spot rate is given by

$$E_t \ln S_{t+k} = E_t \ln \left[\frac{Y_{1t+k}(M_{1t+k})^{-1} U_{1t+k}}{Y_{2t+k}(M_{2t+k})^{-1} U_{2t+k}} \right]. \tag{4}$$

Finally, from (2) and (3) and using covered interest parity we can price a k-period forward rate as

$$F_{t,k} = S_t e^{r_{2t,k} - r_{1t,k}} = \frac{E_t Y_{1t+k}(M_{1t+k})^{-1} U_{1t+k}}{E_t Y_{2t+k}(M_{2t+k})^{-1} U_{2t+k}}. \tag{5}$$

If we let the time interval of the model be a month, the approximate annualized percentage expected nominal profits on the \$, defined as $EP_{t,k} = (1200/k * (\ln(F_{t,k}) - E_t \ln(S_{t+k}))$, can be computed from (4) and (5) as

$$EP_{t,k} = \frac{1200}{k} * \left(\ln \left\{ \frac{E_t[Y_{1t+k}(M_{1t+k})^{-1} U_{1t+k}]}{E_t[Y_{2t+k}(M_{2t+k})^{-1} U_{2t+k}]} \right\} - E_t \ln \left\{ \frac{Y_{1t+k}(M_{1t+k})^{-1} U_{1t+k}}{Y_{2t+k}(M_{2t+k})^{-1} U_{2t+k}} \right\} \right). \tag{6}$$

Inspection of (6) reveals some interesting features. First, as Backus and Gregory (1989), Sibert (1989), and others have recently pointed out, expected nominal profits from forward speculation will be different from zero even when agents are risk-neutral. Note, however, that expected profits will be zero when all the exogenous processes are constant or deterministically fluctuating. Second, $EP_{t,k}$ depends on expectations about future outputs, future money supplies, and future terms of trade. Since in equilibrium expectations about future terms of trade depend on expectations about future government purchases of goods, both supply and demand factors affect expected profits. Finally, uncertainty about regime shifts or regime persistence influence the expectation formation and therefore the statistical properties of expected profits. In other words, if a 'peso problem' exists, it will appear in (6) as well as in the forecast error in predicting future spot rates.

To obtain a closed form expression for $EP_{t,k}$ the instantaneous utility function is specialized to be of a constant relative risk aversion (CRRA) type as:

$$U(c_{1t}, c_{2t}) = \frac{(c_{1t}^{\delta} c_{2t}^{1-\delta})^{1-\gamma}}{1-\gamma}, \tag{7}$$

where δ is the share of domestic goods in total consumption expenditure and γ is the parameter of risk aversion. The CRRA specification has attractive features: it is easy to manipulate and allows the construction of a risk-neutral utility function in multigood settings [see Engel (1992)], a feature we will use in our simulations. Its major drawback is that it makes the spot rate depend only on demand factors (monetary and fiscal) while supply factors do not enter [see, e.g., Bekaert (1991)].

Let Φ_{it} be the proportion of government i's consumption in total output of good i at t ime t. In a pooled equilibrium $c_{it} = 0.5(Y_{it} - G_{it}) = 0.5\, Y_{it}(1 - \Phi_{it})$. Evaluating the marginal utilities in (6) at these equilibrium consumption levels gives an expression for expected profits entirely in terms of the distributions of the exogenous variables. The complete solution requires substituting in the specific processes governing the exogenous variables.

We assume that all exogenous processes are conditionally independent. The processes for the growth rates of outputs and money supplies are assumed to be conditionally lognormally distributed. The processes governing the fraction of each country's output purchased by the governments is assumed to be conditionally uniformally distributed. Let $z_t = [\Delta\ln(Y_{1t}), \Delta\ln(Y_{2t}), \Delta\ln(M_{1t}), \Delta\ln(M_{2t}), \Phi_{1t}, \Phi_{2t}]$, where $\Delta\ln(x_t) = \ln(x_t) - \ln(x_{t-1})$. All six processes are assumed to follow a first-order autoregression,

$$z_{jt} = A_{0j} + A_{1j}z_{jt-1} + \varepsilon_{jt}, \qquad j = 1, \ldots, 6, \tag{8}$$

and their conditional variances are assumed to follow a GARCH (1, 1) process,

$$\sigma_{jt}^2 = a_{0j} + a_{1j}\sigma_{jt-1}^2 + a_{2j}\varepsilon_{jt-1}^2, \qquad j = 1, \ldots, 6. \tag{9}$$

With these assumptions (6) reduces to

$$\begin{aligned} EP_{t,k} = \frac{1200}{k}\{&\mathcal{G}_{1t} - \mathcal{G}_{2t} - 0.5\sigma_{4t,k}^2 + 0.5\sigma_{3t,k}^2 - \ln[1 + \delta(1 - \gamma)] \\ &+ \ln[\delta(1 - \gamma)] + \ln[(1 - \delta)(1 - \gamma)] \\ &- \ln[1 + (1 - \delta)(1 - \gamma)]\}, \end{aligned} \tag{10}$$

where $\sigma_{it,k}^2$ is the variance of the process i at $t + k$ conditional on information available at time t and where $\mathcal{G}_{1t}$ and $\mathcal{G}_{2t}$ are given in appendix A and involve the risk aversion parameter, the share of domestic goods in total consumption, and the conditional variances of government consumption shares. While the distributional assumptions we make allow us to derive an exact closed form solution, one could alternatively follow Breeden (1986) and take a second-order Taylor expansion of (6) around z_t. Eq. (10) would still hold, apart from an approximation error reflecting conditional covariances and higher-order terms.

For the version of the model considered here $EP_{t,k}$ depends on the risk aversion parameter, on the share of the domestic good in total private consumption, and on the conditional variances of both countries' money supplies and governments' consumption shares. Therefore, in the closed form solution we derived, expected profits have a peculiar factor structure with the conditional variances of the exogenous processes accounting for their time series properties.

It is easy to verify that (i) the unconditional variance of the exogenous variables influences the average size of $EP_{t,k}$, (ii) deviations of their conditional variances relative to the unconditional variances affect the unconditional variance of $EP_{t,k}$, (iii) the parameter of risk aversion γ affects both the unconditional mean and unconditional variability of $EP_{t,k}$, (iv) the serial correlation properties of the conditional variances of the exogenous processes are entirely responsible for the serial correlation properties of expected profits.

To generate time series for expected profits from (10) it is necessary to select values for 14 parameters (γ, δ, a_{03}, a_{13}, a_{23}, a_{04}, a_{14}, a_{24}, a_{05}, a_{15}, a_{25}, a_{06}, a_{16}, a_{26}). To provide discipline in the simulation, we estimate as many parameters as possible from observed data. Since the model describes the US economy vs. the rest of the world, we estimate the conditional variances of the two money growth processes from comparable US and foreign monetary aggregates. This pins down six parameters (a_{03}, a_{13}, a_{23}, a_{04}, a_{14}, a_{24}). Also, because of the symmetry of the model, and in agreement with previous simulation studies [see Engel (1992)], we fix $\delta = 0.5$.[3]

Since monthly data on the share of government spending in total output is not available, we choose the parameters regulating the variances of government expenditure shares and the risk aversion parameter γ by simulation. That is, we choose these parameters to formally match statistics of the simulated and of the actual data. Since quarterly data on government spending is available, we further impose the consistency requirement that if the simulated series for government expenditure shares are aggregated at a quarterly frequency, they must have the same unconditional means and variances as the actual data. This restriction pins down the values of a_{05} and a_{06} and imposes cross-equation restrictions which effectively limit the range of parameter values allowed in the simulations.

4. Specification tests and estimation

Money supply data is obtained from IFS tapes. Since the raw data still displays some seasonal fluctuations despite being officially seasonally adjusted and because seasonality is not explicitly modelled in the paper [for such an attempt see Ferson and Harvey (1991)], we deseasonalize it by regressing each series on twelve dummies and on the twelfth lag coefficient. This takes care of both deterministic and stochastic seasonals which appear to be present in the data. The measure for the world money supply is constructed by averaging the growth rates of comparable M1 aggregates for UK, West Germany, and Japan (AGGM in the tables). All these series span the 1975.1–1990.12 period.

Table 3 contains diagnostic tests for our chosen AR(1)–GARCH(1,1) specification for the money supply processes. In each case a first-order univariate autoregression on the difference of the log of the series was used to construct residuals. For each residual series we apply the Cumby and Huizinga test for serial correlation, the ARCH, Breush and Pagan, and White tests for

[3]We conducted experiments varying δ in the range [0.5, 1.0] with no appreciable change in the results.

Table 3

Diagnostic tests on the exogenous processes, 1975.1–1990.12.[a]

Series	CH(6)	ARCH(12)	BP(12)	W(24)	BD
USM1	3.09 (0.69)	24.52 (0.01)	20.19 (0.06)	42.03 (0.01)	1.88
AGGM1	0.81 (0.97)	21.79 (0.03)	17.33 (0.13)	31.66 (0.13)	2.02
WGM1	1.58 (0.90)	11.12 (0.51)	28.37 (0.00)	49.05 (0.00)	1.56

Sample cross-correlations of univariate residuals

	Residuals			Squared residuals		
	−1	0	1	−1	0	1
USM1–AGGM1	0.19	0.16	−0.17	−0.23	−0.17	0.04
USM1–WGM1	0.23	0.02	−0.25[b]	−0.15	−0.03	−0.11

[a] CH refers to Cumby–Huizinga l-test, BP refers to Breush–Pagan test, W to White test, and BD to the Brock–Dechert test. For each series a log first-order difference transformation is used and residuals are prewhitened using one lag. The number next to each test refers to the degrees of freedom of the test. The significance levels of the statistics are in parentheses.

[b] Correlation different from zero at the 5% significance level.

conditional heteroskedasticity, and the Brock and Dechert test for nonlinearities to the normalized residuals. The table also reports cross-correlations of the residuals and of the squared residuals.

The results support our time series specification. First, for both money processes none of the cross-correlations of the residuals or squared residuals were found to be significantly different from zero, providing evidence in favour of univariate specifications for both the mean and the variance. No serial correlation appears in the residuals of AR(1) regressions but there is evidence of conditional heteroskedasticity. In general for both money processes we find a smooth decay of the autocorrelation function of the squared residuals, suggesting that a GARCH (1, 1) is a reasonable characterization of their conditional variances. Finally, the Brock and Dechert test does not reject the hypothesis that the normalized residuals of our estimated processes are white noises. Table 4 reports the results of estimating an AR(1)–GARCH(1, 1) specification for the two series.

To estimate the remaining five parameters of the model we employ the 'estimation by simulation' technique proposed by Lee and Ingram (1991). The method computes optimal parameter estimates by minimizing the distance between a vector of statistics of the actual and the simulated data in the metric given by the covariance matrix of the statistics.

Table 4

Estimated GARCH specification for the exogenous processes, 1975.1–1990.12.[a]

$$\Delta \log y_t = A_0 + A_1 \Delta \log y_{t-1} + \varepsilon_t, \quad \varepsilon \sim (0, \sigma_t^2)$$

$$\sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 + a_2 \varepsilon_{t-1}^2$$

	a_0	a_1	a_2	A_0	A_1
USM1	0.00003	− 0.00002	0.26	0.0002	0.28
	(5.59)	(− 3.59)	(1.89)	(0.59)	(2.77)
AGGM1	0.0001	− 0.00007	0.09	− 0.0006	− 0.24
	(6.19)	(− 3.72)	(0.78)	(− 0.64)	(− 2.61)
WGM1	− 0.0002	− 0.00004	0.15	− 0.0005	− 0.35
	(− 4.54)	(− 0.25)	(1.81)	(− 0.27)	(− 3.45)

[a] t-statistics are in parentheses.

Let x_t, $t = 1,\ldots,T$, be a vector of time series of actual data and let $y_\tau(\beta)$, $\tau = 1,\ldots,N$, $N = nT$, be a vector of simulated time series obtained from the model, where β is the 5×1 vector of free parameters. Define $H_x(T)$ to be a $p \times 1$ vector of statistics of x_t, computed using a sample of size T which includes unconditional moments of the predictable component of the cross-sectional average monthly nomial profits on the dollar for each of the maturities. Define $H_y(N,\beta)$ to be the corresponding $p \times 1$ vector of statistics for $y_\tau(\beta)$ computed using a sample of size N. A simulated estimator $\hat{\beta}(T,N)$ is obtained by minimizing

$$Q(\beta) = (H_x(T) - H_y(N,\beta))' W(T,N)(H_x(T) - H_y(N,\beta)), \tag{11}$$

for a given random weighting matrix $W(T, N)$ with rank $\{W(T, N)\} \geq \dim(\beta)$. The matrix $W(T, N)$ defines the metric for the problem and is assumed to converge almost surely to a nonstochastic matrix $W(0)$. Following Lee and Ingram, an optimal choice for $W(0)$ is given by

$$W(0) = ((1 + n^{-1})S)^{-1}, \tag{12}$$

$$S = \operatorname{diag}\left(\sum_j R_{x_i}(j)\right) = \operatorname{diag}\left(\sum_k R_{y_i}(j)\right), \tag{13}$$

where the last equality holds under the null hypothesis that the β are chosen correctly and where $R_{x_i}(j)$ and $R_{y_i}(j)$ are the autocovariance functions of the statistics of the actual and of the simulated data, $i = 1,\ldots,6$. Duffie and Singleton (1990) show that under fairly general mixing conditions $\hat{\beta}(T, N)$ is consistent

Table 5

Simulated method-of-moment estimates of the parameters.[a]

Maturity	γ	a_{15}	a_{25}	a_{16}	a_{26}	$Q(\hat{\beta})$
			Portfolio expected profits			
$k = 3$	0.9932	0.2517	0.1439	0.4510	0.0579	11.08
	(0.2108)	(0.0956)	(0.0713)	(0.1248)	(0.1003)	
$k = 1$	0.0001	0.1699	0.1052	0.4560	0.0476	13.17
	(0.1661)	(0.1562)	(0.1913)	(0.1223)	(0.1881)	
$k = 1, k = 3$	0.9901	0.2478	0.1252	0.4540	0.0479	130.04
	(0.1901)	(0.1133)	(0.1404)	(0.1372)	(0.2153)	
			Expected profits in DM/$ market			
$k = 3$	0.9927	0.2526	0.1397	0.4507	0.0512	12.13
	(0.2096)	(0.0992)	(0.0685)	(0.1209)	(0.0988)	
$k = 1$	0.0001	0.1603	0.0947	0.4552	0.0447	9.99
	(0.1650)	(0.1253)	(0.1993)	(0.1235)	(0.1860)	

[a] Asymptotic standard errors are in parentheses.

and asymptotically normal.[4] Also, when the dimension of β is smaller than the dimension of H, a goodness-of-fit test for the model is $T\{Q[\hat{\beta}(T,N)]\} \sim \chi^2(p-5)$. In our case an estimate for S is computed by smoothing six sample autocovariances with a set of Parzen weights. Following Newey and West (1987) it is immediate to show that $\hat{S}_T$ is a consistent estimator of S.

Minimization of (11) is undertaken numerically. Details on the minimization routine are provided in appendix B. We estimate the free parameters of the model in two ways. First, we match the actual time series for $k = 1$ and $k = 3$ separately using six statistics (unconditional mean, unconditional variance, and the first four unconditional autocorrelations). In this case there is one overidentifying restriction for each maturity. Second, we estimated free parameters by jointly matching the properties of the unconditional means, variances, and autocorrelations of expected portfolio profits for $k = 1$ and $k = 3$. In this second case there are seven overidentifying restrictions. The estimated values for β and the minimized value of Q for the two different specifications are presented in table 5.

We simulate a time series for $EP_{t,k}$ of the same length as the actual data using the estimated $\hat{\beta}$ vectors and the unconditional variances of the exogenous

[4]Since in our model $EP_{t,k}$ is a GARCH process, there is no insurance that the mixing conditions of Duffie and Singleton necessary to prove asymptotic normality hold in our case. However, given the results of Hansen (1991), we expect GARCH processes to satisfy some type of mixing conditions.

processes as initial conditions. Figs. 1 and 2, panel B plot the realizations for $k = 3$ and $k = 1$, respectively, and their estimated MA representations when the parameters of the two specifications are fitted separately. Tables 1 and 2, column 8 present the statistics of these simulated series and diagnostic tests for nonlinearities in their conditional moments, while column 7 presents statistics for the simulated series when β is estimated jointly matching statistics for both $k = 1$ and $k = 3$.

A few features of the results deserve comment. First, the estimated values for the risk aversion parameter are small. In fact, when parameters are fit separately to each maturity we find that for $k = 1$, the utility function is linear in aggregate consumption. Second, the estimated parameters for the conditional variance of government expenditure shares in the two cases are not significantly different because of the large standard errors. Third, when the free parameters are jointly fit to both maturities, the minimized value of the function is very large and the overidentifying restrictions strongly rejected. In the simulated three-month expected profits there is some evidence of heteroskedasticity and the serial correlation is about half of what we see in the data, but the simulated one-month expected profits show no heteroskedasticity or serial correlation. Moreover, the variance is way too high in the latter. The model does a bit better when the parameters are fit to each holding maturity separately. For one-month profits the variance is drastically lowered (now it remains three times too high), and there is more serial correlation but it is not enough. For the simulated three-month expected profits the standard deviation is close to the actual data and the series remain serially correlated with some evidence of heteroskedasticity (with the Breusch–Pagan test only) but the magnitudes are too low. For the five currencies we use in table 2, Backus, Gregory, and Telmer (1990) report experiments where with moderate risk aversion and time-additive preferences the simulated variability of expected profits is less than 5% of the variability in the predictable component of actual profits. With heteroskedastic driving forces we managed to push up the variability of simulated profits in the range of the variability of actual profits for the three-month maturity, while keeping time additive preferences and very low risk aversion.

5. Some explanations and sensitivity analysis

Although our attempt to account for the time series properties of the predictable cross-sectional average nominal profits on the dollar has had limited success, we would like to have a better idea of what allowing for heteroskedasticity in the fundamentals adds to the model and how it affects previous results.

In a standard consumption-based asset pricing model equilibrium, expected profits of a risky asset typically depend on the conditional covariance between the marginal utility of consumption and the real payoff of the risky asset. Any asset that tends to pay a low real return in states where agents are poor (marginal utility is high) will require a positive premium to induce agents to hold it. The real payoff of the risky asset in turns depends on the distributional properties of the underlying exogenous forces of the economy. In general, the conditional variability of the exogenous processes affects both the expected payoff of the risky asset and the expected marginal utility of consumption and therefore matters for the level of expected profits. However, since the conditional variability is generally assumed to be constant, it plays no role in explaining the volatility and serial correlation properties of required profits.

For the case of exchange rate markets, the excess profits required for taking a risky position in one currency is linked to the covariation of the marginal utility of consumption with the purchasing power of the currency [see, e.g., Hodrick and Srivastava (1984), Sibert (1989)]. Attempts by Hansen and Hodrick (1980), Hodrick and Srivastava (1984), Mark (1985), and others to fit the model to exchange rate data using the Euler equations of the model and a Generalized Method of Moments (GMM) procedure were generally unsuccessful. Their failures stem from being unable to reconcile the small variability in aggregate consumption data with the volatile and serially correlated nature of predictable profits from forward speculation. In their models the time series properties of expected profits are determined entirely by time variation in the conditional covariances. The conditional second moments of the exogenous processes are assumed to be constant.

This paper follows Hodrick (1989) and isolates the influence of the conditional second moments of the exogenous processes on the time series properties of expected profits. This is accomplished in two ways. First, since evidence presented in Hansen and Hodrick (1983), Hakkio and Sibert (1990), and Engel (1992) indicates that conditional covariances cannot account for the behavior of excess profits, we abstract from them entirely by assuming that the exogenous processes are conditionally independent. Second, we allow the conditional second moments of money supplies and government expenditures to be time-varying. It is only if there is enough volatility and serial correlation in these conditional moments, that expected profits will also be volatile and serially correlated. From table 4 we know that time variation in the estimated conditional variances of money supplies is significant but small, so the second-order properties we see in the simulated expected three-month profits probably arise primarily from the government expenditure processes.

Next, we proceed to confirm this intuition about what specific features of the model are responsible for the results. In particular, we are interested in assessing the relative contributions of the conditional variances of the money supplies and of government consumption shares in fitting the second-order properties of expected nominal profits. In addition, we would like to determine whether the time series properties of expected profits arise from risk-averse behavior.

These are done by conducting two experiments. First, we restrict the share of government purchases to have constant conditional variances and examine the properties of the resulting expected profits series leaving all other parameters at their optimal values.

Because of the poor fit of the model when β is jointly estimated for $k = 1$ and $k = 3$, we report results when the parameters are estimated for the two maturities separately. From tables 1 and 2, column 9 it is evident that setting $\mathrm{var}_t(\Phi_{it}) = \mathrm{var}(\Phi_{it})$, $\forall t$, significantly affects the entire moment structure of the simulated expected profits. Since changes over time in the conditional variances of the two money supply processes are the only sources of variability and serial correlation in $EP_{t,k}$ one can see that very little of the second-order properties of the original $EP_{t,k}$ come from these series. It is useful to ask how large the parameters of the conditional variance of the money supply would have to be so as to match exactly the serial correlation properties of the data. For simplicity assume that only one money supply is heteroskedastic and consider one-month expected profits. From (10) and using the fact that the autocovariance function of a GARCH(1,1) process is the same as the one of an AR(1) process with the AR coefficient equal to the sum of the two GARCH coefficients [see, e.g., Bollerslev (1986)], we see that the sum of GARCH coefficients should be around 0.8, which is very far from the sum of the estimated GARCH parameters reported in table 4. Thus, heteroskedasticity in government spending is required to replicate the properties of the predictable component of actual profits. To understand intuitively why fluctuations in the conditional variability of government expenditure shares affect expected profits, note that, under the assumptions made, an expected increase in the conditional variance of the domestic government expenditure share decreases the expected price of domestic currency relative to the foreign currency. Therefore, traders require higher nominal expected profits to engage in speculative transactions in a currency that is expected to depreciate in the future [see also Black (1990)].

Second, we consider the question of whether fluctuations in the simulated expected profits arise from risk-averse behavior of agents. By now it is widely recognized that even when agents are risk-neutral, efficiency in the foreign exchange market does not dictate that expected *nominal* profits are zero. Expected profits can be measured in terms of either currency. When the purchasing powers of the currencies are uncertain, Jensen's inequality implies that expected profits must exist, at least in terms of one of the two currencies.

We can decompose the simulated expected nominal profits into a component arising entirely from risk-averse behavior and another due to Jensen's inequality (a 'convexity' term). The convexity term is computed by simulating the model under the assumption that agents are risk-neutral, i.e., by setting $\gamma = 0$. The risk premium is then obtained by subtracting the resulting series from $EP_{t,k}$. Unlike $EP_{t,k}$, the risk premium series has the property that it will identically equal zero when agents are risk-neutral.

Table 1, column 10 reports statistics for the nominal risk premium for $k = 3$. Here, the risk premium is on average significantly negative (i.e., the basket of foreign currencies is on average less risky than the dollar) and close in magnitude to the mean value of simulated expected profits. In addition, we find that the risk premium, like expected profits, has a large variance and a moderate degree of persistence. On the other hand, for $k = 1$ the risk premium is zero everywhere because the estimated value of γ is for all purposes zero, so that the properties of the simulated profit series are entirely due to the convexity term. Therefore, contrary to the case of $k = 3$, the variability and autocorrelation properties of expected profits are entirely due to the convexity term rather than the risk premium.

The results obtained for $k = 1$ make it clear that the error in identifying the risk premium with expected nominal profits could in fact be larger than was previously recognized. Engel (1992) proposes a method for constructing a measure of the risk premium that is related to expected real profits and likely to be more relevant in capturing the response of agents to risk. Hakkio and Sibert (1990) examine the properties of four different measures of expected profits (two real and two nominal) with data simulated from an OG model. Since little empirical work has been done to characterize the behavior of appropriately measured real risk premia in the actual world, further studies are necessary to determine the importance of risk considerations for the dynamics of foreign exchange markets.

Since the SMM estimate of the parameters differ across the two holding periods, we would like to know whether the properties of the time series generated with the alternative specification significantly differ from the other. Column 11 of table 1 reports the results of inputting in the model for $k = 3$ the parameters estimated with $k = 1$, while column 10 of table 2 reports the results of simulating an expected profits series for $k = 1$ using the parameters estimated with $k = 3$. The results indicate that for both holding periods expected profits series are substantially different from the specifications presented in column 8 of both tables, supporting recent speculations of Lewis (1991) that the properties of ICCAP model may depend on the holding period used to calculate expected profits.

Finally, because the statistical properties of the portfolio expected profit series differ in some cases from those of expected profits in individual markets, we would like to know whether the results obtained are due to the choice of a portfolio instead of a particular currency market. For this reason, we repeat both the specification tests and the estimation using the information available in DM/$ market. The results of the specification tests are contained in table 3 and estimates of the parameters of the model appear in tables 4 and 5. The outcome of the simulation exercises are reported in the last column of tables 1 and 2. All the conclusions previously derived also hold in this case. One additional feature that should be mentioned is that, with the chosen parameters, the mean of simulated expected profits has the same sign as the mean of actual predicted profits but, unlike the actual data, it is not significantly different from zero.[5]

6. Properties of consumption, spot and forward rates

The relevance of our findings depends on whether the theoretical implications for other variables are also born out by the data. In particular, since previous failures of the standard asset pricing paradigm stemmed largely from the low variability in aggregate consumption data, we would like to be certain that the simulations do not induce excess variability in the generated consumption series. It is easy to show that this is not the case since we imposed the consistency condition on the quarterly unconditional moments of government expenditure shares.

The variability of equilibrium consumption growth relative to the variability of output growth, $W = \mathrm{var}\,(\Delta \log C_t)/\mathrm{var}\,(\Delta \log Y_{1t})$, is given by

$$W = \{\mathrm{var}[\delta[\log(1 - z_{5t+1}) - \log(1 - z_{5t})] + (1 - \delta)[\log(1 - z_{6t+1}) - \log(1 - z_{6t})] + \delta z_{1t+1} + (1 - \delta) z_{2t+1}]\} / \{\mathrm{var}[z_{1t+1}]\}.$$

For the simulated monthly realization reported in fig. 1, taking the actual values for the growth rate of industrial production in the US and an average growth rate of industrial production in Japan, West Germany, and the UK as a measure of foreign output growth, W is 0.2726. Using monthly data for the 1975–90 period for real U.S. consumption and industrial production this ratio is estimated to be 0.2156.[6] Therefore, the simulations do not induce excess volatility in consumption.

To further examine the implications of the model we check the properties of simulated spot and forward rates. These quantities, unlike expected profits, are observable both in the model and the data. This allows us to abstract from

[5] We also conducted experiments changing some of the assumptions of the model. For example, we allowed for a structural break in the US money supply process in 1979, 1982, and 1985, and we allowed innovations in government expenditure to be correlated with output innovations. None of these modifications appear to be useful in improving the ability of the model to reproduce the data.

[6] Backus and Kehoe (1992), using different quarterly data detrended with the Hodrick and Prescott filter over the entire post-WWII period, set this ratio at a higher 0.65. One reason for this difference is that, over the entire post-WWII period, the volatility of the share of US government expenditure is much larger than over the 1975–90 period. Another explanation is that detrending the data using the Hodrick and Prescott filter induces different time series properties than a log difference filter [see Canova (1991)]. Finally, it should be mentioned that any estimate for monthly consumption should be taken with a grain of salt, because of the dubious statistical properties of available monthly consumption series [see, e.g., Wilcox (1992)].

sampling variability in our comparisons. One way of summarizing the information contained in the simulations is to examine whether the forward rate is an unbiased predictor of future spot rates. This property is typically tested by running one of the following two complementary regressions:

$$f_{t,k} - s_{t,k} = a_1 + b_1(f_{t,k} - s_t) + u_{t+k}, \tag{14}$$

$$s_{t,k} - s_t = a_2 + b_2(f_{t,k} - s_t) + e_{t+k}, \tag{15}$$

where $f_{t,k}$ and s_t are the logs of the forward and spot rates. The unbiasedness hypothesis implies that $a_1 = b_1 = 0.0$ or alternatively $a_2 = -a_1$ and $b_2 = 1 - b_1$. The essence of the test is that when the forward rate exceeds the spot rate, we expect the future spot rate to rise by the same amount. It is well known that the null hypothesis is strongly rejected in the actual data for various currencies, samples, and frequencies [see, e.g., Frankel and Meese (1987) or Hodrick (1987)]. In many cases b_2 turns out to be significantly negative suggesting a failure of the simple expectational theory in both level and sign.

Table 6 reports the regression results for our two available data sets. The general pattern of results is consistent with previous evidence. For three-month profits and except for the FF/$ rate all the b_2 coefficients are significantly negative. For one-month profits they are all negative but insignificantly different from zero.

To determine whether our model can reproduce this biasedness, we generate artificial data for spot and forward rates using the closed form expressions for (3) and (5) when the β vector is fitted separately for $k = 1$ and $k = 3$, and then run a regression like (15) on the simulated data to estimate a_2 and b_2.[7] We report two sets of results. One obtained using the simulated method-of-moment (SMM) estimates of the free parameters for each of the two maturities. Another obtained by randomizing over the free parameters using their asymptotic distribution. That is, for each draw q and each maturity k we simulate $\{s_t(\beta_q)\}_{t=1}^{T}$, $\{f_{t,k}(\beta_q)\}_{t=1}^{T}$, where $\beta_q \sim \mathrm{N}(\beta_{\mathrm{SMM}}, \mathrm{var}(\beta_{\mathrm{SMM}}))$ and where $\mathrm{var}(\beta_{\mathrm{SMM}})$ is the asymptotic covariance matrix of β_{SMM}, the SMM estimator of β. In this case we report the 90% range of the simulated distribution for a_2 and b_2 and the median value of the distribution when $q = 10{,}000$.

The results indicate that the biasedness observed in the actual data also emerges in our simulated data. For the three-month realized profits, the b_2 coefficient is negative in 58% of the simulations and the largest obtained value is 0.72 (the unbiasedness hypothesis would suggest a value of 1). For one-month profits, the 90% range for both regression parameters includes all but one pair obtained in the actual data.

[7]To simulate a time series for the forward rate we need to select four extra parameters regulating the conditional means of the two money supply processes. We use those reported in table 4.

Table 6

Regression results, (a) 1975.1–1991.9 and (b) 1974.7–1986.10.[a]

Market	a_2	b_2	R^2
	(a) $\Delta^3 s_t = a_2 + b_2 FP_{t,3} + u_t$		
DM/$	− 5.98 (− 1.69)	− 1.23 (− 1.37)	0.01
SF/$	− 10.11 (− 2.79)	− 1.42 (− 2.17)	0.02
FF/$	2.01 (0.90)	0.07 (0.13)	0.001
£/$	7.64 (3.54)	− 2.15 (− 4.11)	0.07
Y/$	− 22.05 (− 5.43)	− 4.03 (− 4.84)	0.19
Portfolio	− 4.20 (− 2.50)	− 2.31 (− 3.29)	0.06
Simulation	− 0.02 (− 0.60)	− 0.13 (− 2.58)	0.017
	[− 0.24, 1.78]	[− 0.97, 0.41]	[0.001, 0.13]
	0.48	− 0.06	
	(b) $\Delta s_t = a_2 + b_2 FP_{t,1} + u_t$		
DM/$	0.003 (0.70)	− 0.21 (− 0.16)	0.0001
Can$/$	− 0.002 (− 2.05)	− 0.24 (− 0.35)	0.008
FF/$	− 0.002 (− 0.87)	− 0.49 (− 0.81)	0.003
£/$	− 0.003 (− 0.97)	− 0.42 (− 0.38)	0.001
Y/$	0.009 (3.30)	− 1.57 (− 2.16)	0.02
Portfolio	0.0002 (0.14)	− 1.04 (− 1.24)	0.008
Simulation	0.017 (4.01)	− 0.16 (− 7.90)	0.048
	[− 0.04, 0.06]	[− 0.88, 0.56]	[0.001, 0.086]
	0.005	− 0.08	

[a] t-statistics are in parentheses. The first row of Simulation reports the regression results obtained when the data is generated with the optimal values of the parameters; the second row reports the 90% range of the simulated distribution for the regression coefficients obtained by drawing 10,000 values for the parameters from their asymptotic distribution; the third row reports the median value of the simulated distribution for regression coefficients.

One way to understand these results is to look at eqs. (3) and (5). While changes in the conditional variances of the exogenous processes affect the forward rate and the expected spot rate, they do not appear in the formula for realized spot rate. Therefore fluctuations over time in the conditional second moments affect the forward premia and the realized change in the spot rate differently, leading to a forecast error in predicting changes in the spot rate which are not serially uncorrelated, homoskedastic, and exogenous. Running a regression like (15) therefore misses the underlying dynamics of the data. Contrary to the arguments in Domowitz and Hakkio (1985), the presence of heteroskedasticity in the processes generating expected profits does not affect only the estimate of the intercept in the regression. The entire regression line is shifted from what would be expected under the simple expectational hypothesis.

7. Conclusions

This paper attempts to replicate the statistical properties of nominal profits from forward speculation on the dollar using a general equilibrium monetary model where agents are rational and fundamentals drive exchange rate behavior. It explores the influence of time variation in the conditional variability of the exogenous processes on the time series properties of nominal expected profits. We find that although the presence of conditional heteroskedasticity can generate time series which are volatile and exhibit autocorrelation, the magnitudes deviate from those of the predictable component of cross-sectional realized average nominal profits on the dollar. We also find that simulated forward rates are biased predictors of simulated future spot rates and that the risk component of expected profits does not necessarily account for many interesting properties of the data.

Two conclusions emerge from our study. First an ICCAP model appropriately formulated may help in accounting for some of the puzzling time series features of nominal profits on the dollar. This class of models therefore has the potential to explain other anomalies (the equity premium, the holding and forward premium in the term structure of interest rates) recently discovered in financial markets. Second, the identification of the expected component of nominal profits with a risk premium may lead to fallacious conclusions about the nature and sources of risk in foreign exchange markets.

Appendix A

The expressions for $\mathscr{G}_{1t}$ and $\mathscr{G}_{2t}$ used in section 3 are given by

$$\mathscr{G}_{1t} = -\frac{(1-h_{5t,k})}{h_{5t,k}}\log(1-h_{5t,k}) - \log[1-(1-h_{5t,k})^{(1+\delta)(1-\gamma)}]$$

$$+\log[1-(1-h_{5t,k})^{\delta(1-\gamma)}], \tag{16}$$

$$\mathscr{G}_{2t} = -\frac{(1-h_{6t,k})}{h_{6t,k}}\log(1-h_{6t,k}) - \log[1-(1-h_{6t,k})^{(1-\delta)(1-\gamma)}]$$

$$+\log[1-(1-h_{6t,k})^{(1+(1-\delta)(1-\gamma))}], \tag{17}$$

where $h_{it,k} = \sqrt{(12\sigma_{it,k}^2)}$, $i = 1,2$, and $\sigma_{it,k}^2$ is the variance of the process at $t+k$ conditional on information available at time t.

Appendix B

The minimization routine we use to compute SMM estimates of the parameters is numerical because the function Q is not well-behaved and a standard hill-climbing routine produces values for the gradient which are too small to move away from initial conditions. The procedure we employ is as follows. First, we evaluate Q at five different points in each of the five dimensions and use the Bayesmth interpolation procedure [see Sims (1986)] to reconstruct the shape of Q and to obtain a guess for the gradient and for the most likely direction where the minimum is located. Second, we grid the space around this first minimum using the guessed gradient to select the ranges in the five dimensions, and then repeat the function evaluation and the interpolation procedure to obtain a new guess for the minimum of Q and for the gradient. We repeat this procedure five times, and we report the minimum of Q and the values of β obtained at the last iteration. To confirm that the value of Q obtained in the fifth iteration is really the minimum we perform sensitivity analysis in two ways: first we arbitrarily perturb one parameter at a time a neighborhood of its optimal value to see if another minimum is achieved. Second, we restart the minimization procedure from different initial conditions to check if the algorithm converges to a new minimum. Because the function is ill-behaved, this second step of the sensitivity routine is often crucial to avoid getting stuck in a local minimum.

Since each grid requires $5^5 = 3125$ evaluations of Q and because we start the procedure three times from different initial conditions, the total number of function evaluations is 9,375. On a 25 MHz 486 machine using the RATS random number generator and the seed command set equal to two, the total computation time for the grid search was about 80 minutes. Given simulation results contained in Gourieroux and Monfort (1991), we set $n = 10$ in estimating the free parameters.

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