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Reconciling the term structure of interest rates with the consumption-based ICAP model

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Abstract

This paper attempts to explain some of the time series features of the low end of the term structure of US interest rates using a representative-agent cash-in-advance consumption-based ICAP model, modified to allow for time variation in the conditional variances of the exogenous processes. The ability of the model to reproduce features of the actual data is evaluated using a Monte Carlo simulation technique. The statistical properties of simulated yields and spreads are shown to replicate several properties of the observed term structure of U.S. T-bills over the sample 1964–1988.

Key words: Yields; Spreads; Heteroskedasticity; Cash-in-advance; Simulation
JEL classification: C15; G12

Tus triunfos, pobres triunfos pasajeros. (Mano a mano, Tango Argentino)

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1. Introduction

One of the most widely examined theories in financial economics is the expectations theory of the term structure of interest rates, which relates the yields on long-term bonds to expected future yields on short-term bonds. The theory has been tested in several ways. Campbell (1986) and Campbell and Shiller (1987, 1992), among others, have used regression tests to examine whether the slope of the yield curve (the spread) has predictive content for the holding premium return. Fama (1984), Stambaugh (1988), and others have tested the theory using the predictive content of forward rates for realized future spot interest rates. To provide a more direct test of the predictive content of forward rates Froot (1989) uses survey data on expectations of future spot interest rates. Much of the empirical evidence presented so far indicates that the theory has severe shortcomings. The yield spread has information in predicting holding premium returns and the forward rate does not accurately predict future spot interest rates. Standard explanations for these failures, which include the presence of time-varying risk and liquidity premia or irrational behavior, have yet to provide a convincing account of the empirical features of the term structure of interest rates.

One important recent line of research examines whether the empirical evidence is in line with the implications of the Cox, Ingersoll, and Ross (1985) model. Encouraging results in matching features of conditional and unconditional second moments have been reported in Pennacchi (1991), Boudoukh (1993), and Engle and Ng (1993).

Another recent line of research has examined whether this empirical evidence is consistent with the consumption-based theory of risk premia developed by Lucas (1980) and Breeden (1979) and implicit in the one-sector growth model (see, e.g., Backus, Gregory, and Zin, 1989; Donaldson, Johnsen, and Mehra, 1990). This research has attempted to determine whether numerical versions of the theory can account for variation in risk premia which are implicit in the failure of the expectations hypothesis. Some success has been reported in matching the variability of yields (see Den Haan, 1990), the behavior of real and nominal yields over the business cycle (Donaldson, Johnsen, and Mehra, 1990; Labadie, 1991), and the predictive power of the nominal yield curve for real output (Cooley and Ohanian, 1990).

This paper contributes to this growing body of literature in several ways. As in the studies along the Lucas–Breeden tradition, we are interested in analyzing whether modifications of a standard consumption-based intertemporal asset pricing model can generate a term structure of interest rates which is consistent with US empirical evidence. We differ from the others in two ways. First, we allow the conditional variability of the exogenous forces of the economy to vary over time. Breeden (1986) and Stambaugh (1988) have argued that time variations in the conditional variability of the driving forces of the model are qualitatively important in determining the properties of interest rates. We want to determine whether changes over time in the uncertainty surrounding economic fundamentals can help to *quantitatively* explain the behavior of the yield curve. Second, we use a Monte Carlo simulation technique to assess the ability of the model to reproduce features of the actual data.

In comparing actual and simulated data we focus on a broad set of features of the actual yield curve. They are: (i) the yield curve is upward-sloping on average, (ii) the volatility of yields decreases with maturity, (iii) yields at all maturities are highly autocorrelated and highly heteroskedastic, (iv) yields on longer-term bills have significant correlation with yields on shorter-term bills, (v) the yield spread is a better predictor for future changes in short-term yields than the forward premium. The opposite is true for future changes in longer-term yields.

To attempt to replicate these features quantitatively we simulate a monetary model with time-separable preferences and exogenous endowments, fiscal policies, and monetary policies. Under general conditions we show that the equilibrium interest rates depend, among other things, on the conditional variability of the exogenous forces of the economy. In a related paper (Canova and Marrinan, 1993) we demonstrated that, by allowing for time variation in the conditional distributions, a two-country version of the model we use here is able to quantitatively replicate the variability, serial correlation properties, and heteroskedastic structure of profits from forward speculation in foreign exchange markets. Work by Ferson and Harvey (1991) also indicates that changes in the uncertainty surrounding economic variables may be important in characterizing properties of US stock returns. Therefore, it is of interest to examine whether a unified explanation for the apparent failure of the expectations theory in financial markets is possible.

To assess the quantitative properties of the theoretical economy we use a Monte Carlo methodology. The approach is advantageous in several respects. It includes both estimation by simulation and calibration techniques as special cases. It also allows us to incorporate existing evidence on the parameters of the model in a realistic way, make probability statements on the range of possible outcomes that the model can generate or on particular events we are interested in replicating, and, as a by-product, provides a global sensitivity analysis for some crucial parameters. We take the economic model to be, at best, an approximation to reality, and we view ourselves as trying to determine how good an approximation it is. This is done by taking the actual realization of the statistic of interest as a critical value and computing the probability (over replications) that the model generates that critical value. Gregory and Smith (1991) have proposed a similar approach to formalize inference in simulated macroeconomic models. Contrary to their approach, we explicitly take parameter uncertainty into consideration and randomize over *both* exogenous processes *and* parameters (see also Kwan, 1990). The parameters are drawn from the frequency distribution of the estimates compiled from the existing literature.

The results indicate that the model reproduces with high probability several interesting aspects of the term structure, namely, the upward-sloping average yield curve, the fact that volatility of yields decreases with maturity, and most of the second-order properties of the spreads. The model falls somewhat short in quantitatively accounting for other features of the term structure, including the serial correlation properties of yields and their heteroskedastic structure and, partially, the cross-correlations of long and short yields, of the spreads, and forward premia with changes in yields.

We find that the presence of heteroskedasticity in the exogenous processes is fundamental in making simulated yields comparable to actual yields. With homoschedastic exogenous processes the second-order properties of simulated yields are with probability 1 at odds with actual data. We also find that allowing for a structural break in the stochastic process of the money supply enhances the ability of the model to match the serial correlation and the heteroskedastic properties of the data.

The paper is organized in seven sections. The next section presents evidence on the short end of the term structure of US interest rates and establishes some stylized 'facts'. Section 3 describes the model economy and derives equilibrium pricing formulas for nominally risk-free interest rates. Section 4 presents our simulation technique and discusses the relationship with existing simulation approaches. Section 5 presents the results. Section 6 discusses the sensitivity of the results to several modifications of the basic model. Section 7 concludes.

2. Evidence from the term structure of US T-bill rates

This section presents some 'facts' concerning the short end of the term structure of US interest rates which emerge from the available data set. We concentrate only on the short end of the term structure because the effects of time variation in the second moments of economic fundamentals are likely to show up primarily at these maturities. Unless the conditional distributions of future fundamentals display very strong persistence, at longer horizons (say, 5 or 10 years) agents' forecasts of the conditional variances are likely to coincide with the unconditional variances, therefore reducing the importance of expected variation in second moments for the pricing of nominal bonds. In addition, by limiting the scope of the research to the short end of the term structure, we avoid having to deal with possible 'preferred habitat' considerations which may require more complicated theoretical settings.

We focus on yields as opposed to holding premium returns to maintain comparability with the recent work by Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990), and Den Haan (1990). Although this choice prevents us from examining the information content of forward rates for holding premium returns (as, e.g., Stambaugh, 1988), we can still address issues of predictability by relating forward premiums to the changes in yields.

We present simple summary statistics instead of regression coefficients because they are more robust to interpolation and measurement errors and to small sample biases.

The data we employ is obtained from the Center for Research in Security Prices (CRSP) tapes, augmented with Fama's term structure files on T-bills (Fama, 1984). Monthly data for the average of the bid and ask for spot and forward prices of 1-, 3-, 6-, and 12-month T-bills are taken from Fama's 12-month bill files. Yields are continuously compounded over a month and converted to percentages per year by multiplying the figures by 1200 to express the data in more familiar units. For each month Fama's data set chooses the bill with maturity closest to 12 months. This bill is then followed to maturity, providing in subsequent months the yields for maturities of 11 months, 10 months, etc. Since data for 12-month T-bills is available from 1963.7, data for a 1-month bill is available from 1964.6 and in our study we use data up to 1987.11 for a total of 281 observations.[1] Figs. 1 and 2 present the time plots for the yields and the spreads and their estimated univariate MA representations. Tables 1 and 2 report selected summary statistics and Table 3 some relevant cross-moments among the variables. The presence of overlapping intervals for yields on bonds with maturity greater than one month may induce spurious serial correlation and cross-correlation. However, this does not constitute an important problem in describing the properties of actual yields because simulated yields will be constructed in the same way as actual ones.

Since the yield plots display a break around 1979, we also compute statistics for two subsamples 1964.6–1979.9 and 1979.10–1987.11. Although the magnitudes of the first two moments of yields change across subsamples, neither the serial correlation properties nor any of the qualitative aspects of summary statistics presented are altered across subsamples. Therefore, we present only evidence concerning the entire sample.[2]

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[1] Since the data set on 12-month prices contains several missing values, we reconstruct missing values either by linear interpolation or by compounding the prices of bills of various maturities. The statistics we report are insensitive to which of the two procedures are used and are practically identical to those obtained by simply dropping missing values from the sample. For the sake of robustness we also examined the properties of the data set employed by McCulloch and Shiller (1987) where missing observations are interpolated with a cubic spline and the term structure data contained in the Citibase Tape, where there are no missing observations but data is reported as average over the month. We find that, apart from very minor numerical differences, the qualitative features of the statistics reported are robust. Regression coefficients, however, do differ substantially depending on the data set used.

[2] Results for subsamples are contained in an appendix available from the authors on request. Note that the spreads appear to be stationary throughout the sample.

Table 1
Monthly statistics: T-bill yields, 1964.6–1987.11

	1-month T-bill	3-month T-bill	6-month T-bill	12-month T-bill
Mean	6.74	7.09	7.34	7.53
P-value	0.000	0.000	0.000	0.000
Std. error	2.78	2.88	2.87	2.73
Skewness	− 0.73	− 0.78	− 0.57	0.39
P-value	0.000	0.000	0.000	0.000
Kurtosis	8.86	7.48	7.81	3.26
P-value	0.000	0.000	0.000	0.000
AR(1)	0.948	0.965	0.966	0.974
AR(2)	0.904	0.922	0.924	0.944
AR(4)	0.823	0.826	0.857	0.879
AR(12)	0.650	0.676	0.687	0.700
AR(24)	0.398	0.392	0.414	0.443
ARCH(13)	37.20	43.17	40.32	47.22
P-value	0.000	0.000	0.000	0.000
BP(13)	75.44	56.91	56.19	54.80
P-value	0.000	0.000	0.000	0.000
White(26)	93.01	71.86	76.95	78.79
P-value	0.000	0.000	0.000	0.000
Q(49)	55.90	60.49	6.76	33.99
P-value	0.231	0.125	1.000	0.949
KS	0.784	0.862	0.561	0.910

BP refers to Breusch–Pagan test, Q to the Ljung–Box test, KS refers to the Kolmogorov–Smirnov statistics. The numbers in parentheses after ARCH, BP, White, and Q refer to the number of degrees of freedom of the χ^2 statistics.

The evidence contained in the tables and figures can be summarized as follows:

- The arithmetic mean of nominal yields on all T-bills for the 1964–1987 period is close to 7%. The average term structure is slightly upward-sloping but average yields do not increase proportionally with the gap in maturities.
- The standard error of yields averages about 2.8, but over the term structure is slightly hump-shaped, with the hump occurring for the 3-month maturity. Volatility, defined as the standard error of the series divided by the absolute value of the mean, is slightly decreasing with maturity.

Table 2
Monthly statistics: T-bill spreads, 1964.6–1987.11

	1–3 months	1–6 months	1–12 months	3–6 months	3–12 months	6–12 months
Mean	0.35	0.60	0.79	0.25	0.44	0.18
P-value	0.000	0.000	0.000	0.000	0.036	0.000
Std. error	0.44	0.53	0.78	0.24	0.67	0.51
Skewness	1.23	0.84	0.21	0.37	0.33	− 0.63
P-value	0.000	0.000	0.039	0.004	0.008	0.000
Kurtosis	8.13	5.11	7.33	2.42	15.39	24.38
P-value	0.000	0.000	0.000	0.000	0.000	0.000
AR(1)	0.221	0.338	0.491	0.587	0.672	0.611
AR(2)	0.198	0.330	0.309	0.449	0.421	0.296
AR(4)	0.039	0.068	− 0.010	0.135	0.176	0.104
AR(12)	0.361	0.209	0.173	0.098	0.154	0.131
AR(24)	0.265	0.241	0.132	0.051	− 0.063	− 0.017
ARCH(13)	34.01	57.18	92.14	47.36	62.69	89.42
P-value	0.000	0.000	0.000	0.000	0.000	0.000
BP(13)	55.82	59.20	55.68	22.84	87.77	105.99
P-value	0.000	0.000	0.000	0.000	0.000	0.000
White(26)	75.26	128.57	139.86	72.24	162.01	141.62
P-value	0.000	0.000	0.000	0.000	0.000	0.000
Q(49)	55.84	26.84	2.68	67.48	69.12	43.29
P-value	0.233	0.995	1.00	0.041	0.030	0.702
KS	0.251	0.973	1.030	0.624	0.817	0.555

BP refers to Breusch–Pagan test, Q to the Ljung–Box test, KS refers to the Kolmogorov–Smirnov statistics. The numbers in parentheses after ARCH, BP, Q, and White refer to the number of degrees of freedom of the χ^2 statistics.

- Except for the 12-month bills, yields are skewed to the left (lower-than-average yields occur more frequently than higher-than-average yields) and highly leptokurtic. The Kendall and Stuart (1958) two-tailed test rejects the null hypothesis that their conditional distribution is normal for all maturities. The spreads between yields of different maturities also display marked non-normalities. This behavior persists when the gap between maturities increases. Therefore, contrary to what has been found in other financial markets (see, e.g., Fama, 1976; Diebold, 1988), time aggregation does not reduce nonnormalities.

Table 3
Cross-moments: 1964.4–1987.11

	− 2	− 1	0	1	2
Δ long–Δ short yields					
1–6 months	− 0.11	0.18	0.70	0.13	− 0.03
1–12 months	− 0.01	0.13	0.46	0.21	− 0.02
3–6 months	− 0.08	− 0.14	0.95	0.13	− 0.10
3–12 months	0.06	0.12	0.76	0.20	− 0.01
6–12 months	− 0.09	0.11	0.70	0.19	− 0.04
FP–Δ short yields					
1–3 months	− 0.08	0.24	− 0.29	− 0.12	− 0.08
1–6 months	− 0.05	0.16	− 0.25	− 0.14	− 0.13
1–12 months	0.07	0.25	− 0.23	− 0.08	− 0.13
3–6 months	− 0.07	0.06	− 0.14	− 0.12	− 0.04
3–12 months	0.15	0.19	− 0.12	− 0.11	− 0.06
6–12 months	0.18	0.18	− 0.07	− 0.11	− 0.02
SP–Δ short yields					
1–3 months	− 0.06	0.32	− 0.36	− 0.06	− 0.12
1–6 months	− 0.07	0.28	− 0.39	− 0.08	− 0.17
1–12 months	0.15	0.34	− 0.42	− 0.18	− 0.22
3–6 months	− 0.02	0.06	− 0.18	− 0.11	− 0.11
3–12 months	0.27	0.24	− 0.26	− 0.21	− 0.13
6–12 months	0.34	0.28	− 0.23	− 0.21	− 0.10

FP stands for forward premium, SP for spread.

- Yields at all maturities are highly serially correlated and their univariate moving average representation decays slowly. A 1% shock still generates a 0.20% displacement in the level of yields at the 48-month horizon. The degree of persistence in the autocovariance function increases with maturity. This is consistent with the idea that yields on longer-term instruments reflect events in the future which are unaffected by current business cycle conditions (see, e.g., Donaldson, Johnsen, and Mehra, 1990). Changes in T-bill yields of any maturity have a very small and insignificant first-order autocorrelation coefficient. However, except for 1-month bills and contrary to Fama (1984), we reject the hypothesis that yield changes are white noise because of significant seasonalities present in the data (see the surge in the MA representation of 3-, 6-, and 12-month yields at the 12-month horizon).
- The conditional distribution of the yields and spreads display time variation and marked nonlinearities. The conditional variances of yields of all maturities are highly volatile and display significant conditional heteroskedasticity.

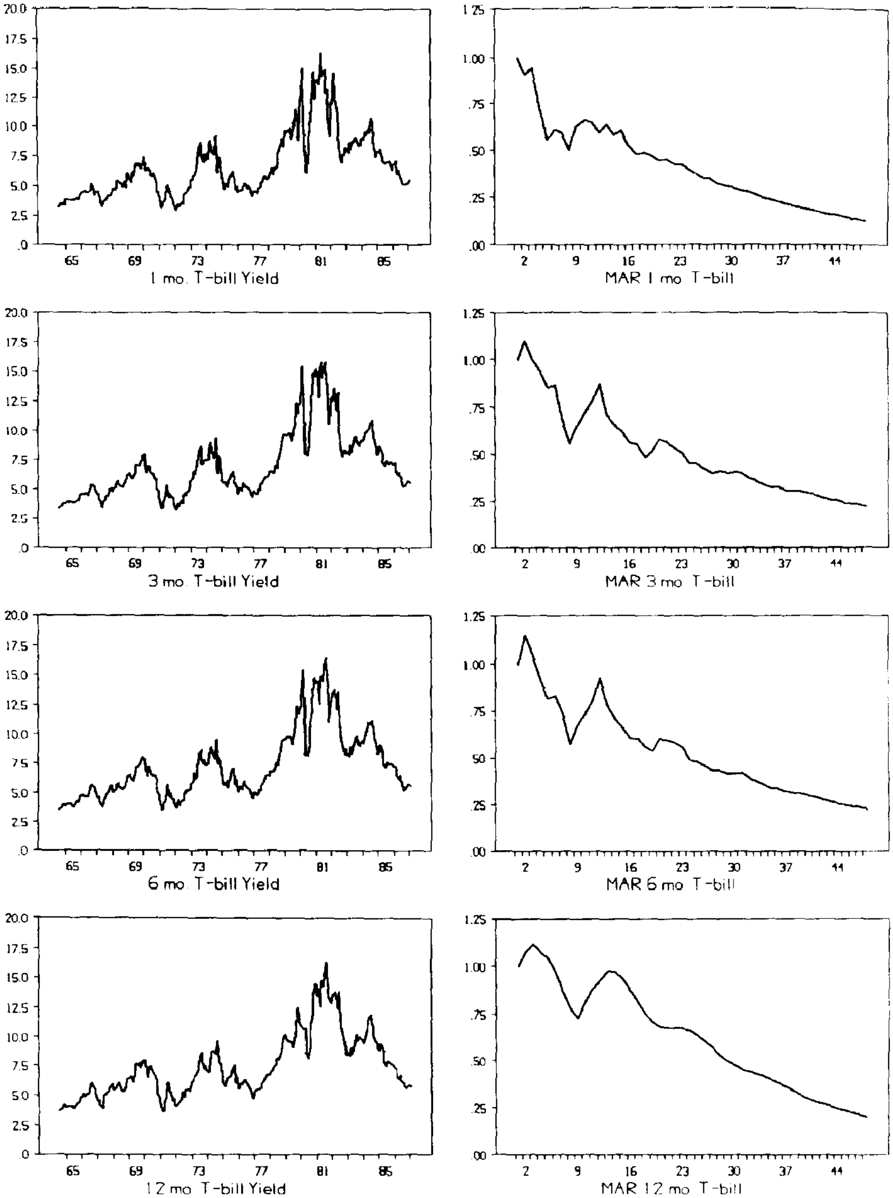
1 mo. T-bill Yield
MAR 1 mo T-bill
3 mo. T-bill Yield
MAR 3 mo T-bill
6 mo. T-bill Yield
MAR 6 mo T-bill
12 mo T-bill Yield
MAR 12 mo T-bill

Fig. 1

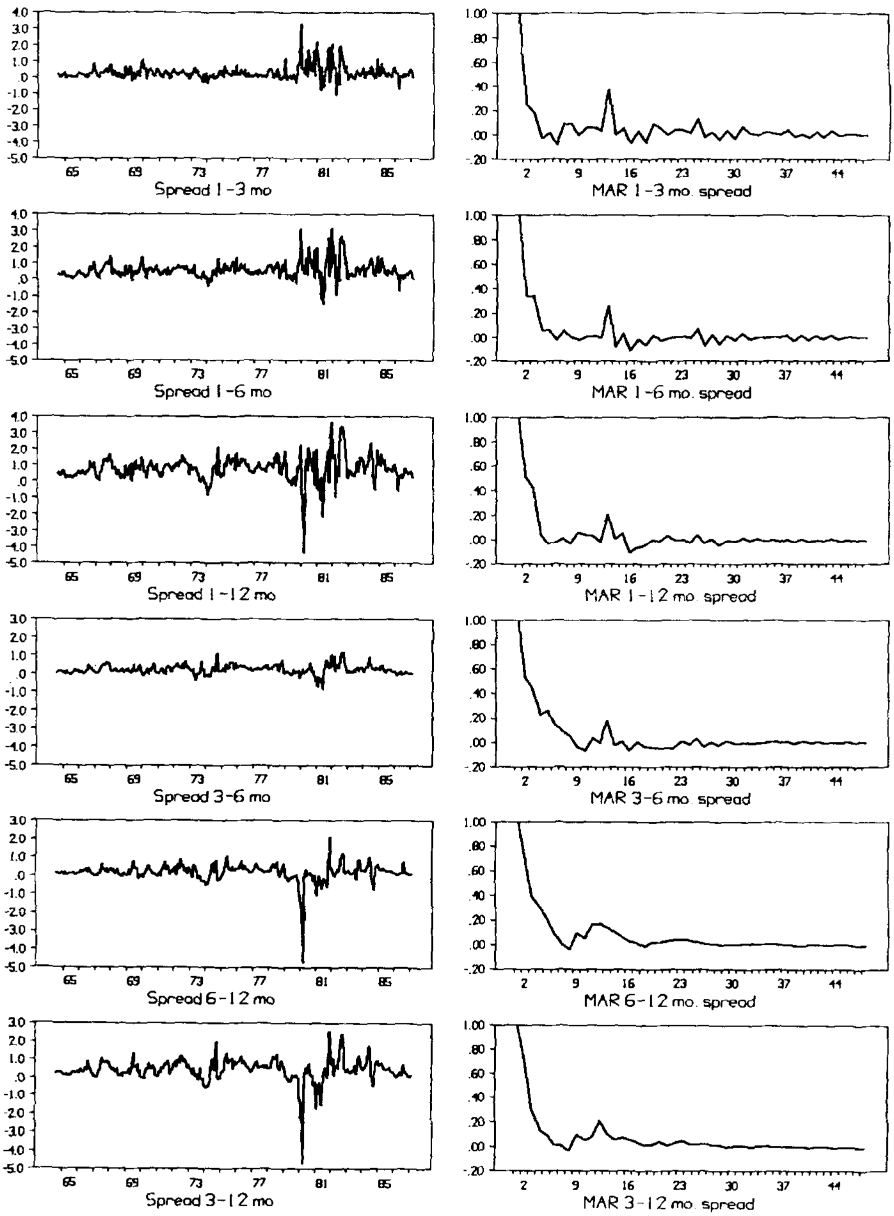
Spread 1-3 mo
MAR 1-3 mo. spread
Spread 1-6 mo
MAR 1-6 mo. spread
Spread 1-12 mo
MAR 1-12 mo. spread
Spread 3-6 mo
MAR 3-6 mo. spread
Spread 6-12 mo
MAR 6-12 mo. spread
Spread 3-12 mo
MAR 3-12 mo. spread

Fig. 2

The nature of the heteroskedasticity in yields varies across maturities since spreads still display marked heteroskedastic patterns.

- There is a negative contemporaneous correlation between the forward premia (the difference between the forward and the spot rate) and interest rate changes (on average, -0.15), which is inconsistent with the predictions of the liquidity theory of the yield curve (see, e.g., Kessel, 1965). The spread appears to have higher predictive content for future changes in short-term interest rates than the forward premium, but the difference is small. For future changes in long-term rates the opposite is true.
- The contemporaneous correlation between changes in long-term yields and changes in short-term yields averages around 0.70 and slightly decreases as the gap between maturities increases. This result, taken together with the flat pattern of standard errors at different maturities, is consistent with Mankiw and Summers (1984) and with the idea of 'undersensitivity' of longer rates to short rate fluctuations.

Our task is to construct and simulate a general equilibrium model of the term structure and examine whether it can both qualitatively and quantitatively reproduce these features.

Model-based empirical work of the term structure has generally been concerned with the estimation of the risk aversion parameter and of the discount factor of the representative consumer (see, e.g., Hansen and Singleton, 1983; Brown and Gibbons, 1985; Dunn and Singleton, 1986; Lee, 1989). It is only more recently, with the work of Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990), Cooley and Ohanian (1990), Den Haan (1990), and Labadie (1991), that the emphasis has been shifted to try to ascertain whether a consumption-based ICAP model can reproduce those features of the US yield curve which are puzzling from the point of view of the expectations theory. Our work is a direct extension of their efforts.

3. A general equilibrium model of the term structure

The theoretical framework we employ is a version of the cash-in-advance monetary model developed by Lucas (1980), modified to allow for time variation in the conditional variance of the exogenous processes. It is similar to the one used in a previous paper of ours (Canova and Marrinan, 1993) in which we study the behavior of profit from forward speculation in foreign exchange markets. We employ a similar theoretical structure because we are interested in assessing whether such a model can quantitatively account for a wide variety of features of US financial markets. Since the model is well-known in the literature, we only briefly describe its features and proceed directly to the computation of the equilibrium values of the variables of interest.

Every period the economy is endowed with Y_t units of a nonstorable consumption good. There is a government which consumes G_t units of the good. To finance these consumption requirements the government issues money, M_t, collects real lump sum taxes, T_t, and issues debt to finance any purchases in excess of money creation and tax collections. This debt is in the form of state-contingent nominal bills of maturity k, $k = 1, 2, \ldots, K$. Endowments, government consumption requirements, and money supplies are exogenous and follow a first-order Markov process with stationary and ergodic transition function.

The economy is populated by a representative household maximizing a time-separable utility function defined over consumption bundles C_t. The household is subject to both a wealth constraint and a liquidity constraint which compels it to purchase goods with cash. The timing of the model follows Lucas with asset markets open first and goods markets following. At the beginning of each period the consumer enters the asset market and decides how to allocate her wealth among the productive assets, currency, and the state-contingent nominal bonds. After the asset market closes, the consumer enters the goods market and makes her consumption purchases with previously accumulated currency.

Equilibrium requires that households optimize and all markets clear. Since capital markets are complete, this permits an unconstrained Pareto-optimal allocation of the time–state-contingent nominal bonds. Let $e^{-r_{t,k}(v)}$ denote the discount price of a bill paying one unit of currency at time $t + k$, if event v occurs and $r_{t,k}(v)$ denote the associated continuously compounded interest rate. By integrating the equilibrium pricing formulas over all possible v we can determine the price at t of a nominally riskless k-period discount bill, $e^{-r_{t,k}}$.

In equilibrium nominal interest rates reflect optimal consumption–saving decisions by equating bond prices to individuals' expected marginal rate of substitution of future nominal expenditure for current nominal expenditure,

$$e^{-r_{t,k}} = \beta^k \, \mathrm{E}_t \frac{P_t U_{t+k}(C_{t+k})}{P_{t+k} U_t(C_t)}. \tag{1}$$

Because of the timing of the model, all uncertainty is resolved prior to the household's money holding decisions so they hold just enough currency to finance their current consumption purchases. This implies that $P_t = M_t / Y_t$, and the discount price of a bill of maturity k is

$$e^{-r_{t,k}} = \frac{\beta^k \, \mathrm{E}_t M_{t+k}^{-1} Y_{t+k} U_{t+k}(C_{t+k})}{Y_t M_t^{-1} U_t(C_t)}. \tag{2}$$

From (2) it is immediate to compute forward prices for maturity q, $f_{t,q}$, as

$$e^{-f_{t,q}} = \frac{e^{-r_{t,k+q}}}{e^{-r_{t,k}}} \quad \forall k, q. \tag{3}$$

Yield and forward rates can be obtained from (2) and (3) by simple logarithmic transformations.

An expression for the slope of the yield curve (the spread) between k- and h-period nominally riskless pure discount bills with $k > h \geq 1$ is obtained from (2) as

$$SP_t^{k,h} = h^{-1} \ln\left[\frac{E_t \beta^h Y_{t+h}(M_{t+h})^{-1} U_{t+h}}{Y_t(M_t)^{-1} U_t}\right] - k^{-1} \ln\left[\frac{E_t \beta^h Y_{t+k}(M_{t+k})^{-1} U_{t+k}}{Y_t(M_t)^{-1} U_t}\right]. \quad (4)$$

Finally, an expression for the forward premium, defined as $FP_t^{q,h} = -\ln(e^{-f_{t,q}}) + \ln(e^{-r_{t,h}})$, is

$$FP_t^{q,h} = (k+q)^{-1} \ln\left[\frac{E_t \beta^{k+q} Y_{t+k+q}(M_{t+k+q})^{-1} U_{t+k+q}}{Y_t(M_t)^{-1} U_t}\right] - k^{-1} \ln\left[\frac{E_t \beta^k Y_{t+k}(M_{t+k})^{-1} U_{t+k}}{Y_t(M_t)^{-1} U_t}\right] - h^{-1} \ln\left[\frac{E_t \beta^h Y_{t+h}(M_{t+h})^{-1} U_{t+h}}{Y_t(M_t)^{-1} U_t}\right]. \quad (5)$$

Yields, forward rates, spreads, and forward premia depend on expectations about future output, future money supply, and future consumption growth. Since in equilibrium expectations about future consumption growth depend on expectations about future government purchases of goods, both supply and demand factors affect the position and the slope of the term structure. Also, uncertainty about regime shifts or regime persistence can influence the expectation formation and therefore the properties of forward and spot rates.

To obtain closed form expressions for yields, forward rates, spreads, and forward premia the instantaneous utility function is specialized to be of a constant relative risk aversion type as

$$U(C_t) = \frac{C_t^{(1-\gamma)}}{1-\gamma}, \qquad 0 \leq \gamma \leq \infty, \quad (6)$$

where γ is the parameter of risk aversion. Let Φ_t be the proportion of government consumption in total output at time t. In equilibrium $C_t = Y_t - G_t = Y_t(1 - \Phi_t)$. Evaluating the marginal utilities in (2)–(5) at these equilibrium consumption levels gives expressions for yields and forward rates entirely in terms of the distributions of the exogenous variables. The complete solution requires substituting in specific processes governing the exogenous variables.

Let $z_t = [\Delta \log(Y_t),\ \Delta \log(M_t),\ \Phi_t]$. We assume that z_t has a stationary unconditional distribution. In addition, we assume that all three processes follow a first-order autoregression,

$$z_{jt} = A_{0j} + A_{1j} z_{jt-1} + \varepsilon_{jt}, \qquad j = 1, \ldots, 3, \tag{7}$$

and that their conditional variances are time-varying and follow a GARCH(1, 1) process:

$$\sigma_{jt}^2 = a_{0j} + a_{1j}\sigma_{jt-1}^2 + a_{2j}\varepsilon_{jt-1}^2, \qquad j = 1, \ldots, 3. \tag{8}$$

We selected univariate specifications for the exogenous process to keep the dimension of the parameters space manageable and the problem tractable. In Canova and Marrinan (1993) and (1994) we show that the extent of the interaction across moments of the variables is small and can be neglected as a first approximation.

If, as in Breeden (1986), we take a second-order Taylor expansion of (2)–(5) around zero, it is immediate to show that yields, forward rates, spreads, and forward premia will all depend on the conditional means, variances, and covariances of the exogenous processes. Since there is evidence that the conditional covariances are small (see, e.g., Hansen and Hodrick, 1983; Lee, 1989) we will include them along with the higher-order terms in the approximation error and neglect them in the simulations. This allows us to focus on the contribution of time-varying conditional variances to the properties of the term structure. Expressions for the four variables of interest appear in the Appendix.[3]

Straightforward calculations indicate:

- The unconditional means and variances of the exogenous processes influence the average size of all four variables.
- Deviations of the conditional moments relative to the unconditional moments of the exogenous processes affect the unconditional autocovariance functions of all four variables.
- The discount factor β affects only the mean of yields.
- The risk aversion parameter, γ, affects both the unconditional means and the unconditional autocovariances of all four variables.

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[3] Adrian Pagan has pointed out to us that because we compute a Taylor expansion using certainty equivalence, we neglect a term due to Jensen's inequality which may be important for the second-order properties of simulated data. Therefore, our results should be interpreted as providing a lower bound to the effect that time variations in the second moments may have on the term structure of interest rates.

Since (2)–(5) hold for each k, it is possible to express long-term rates, spreads, and forward premia as a function of the distributional characteristics of short-term rates, using the approach suggested by Longstaff and Schwartz (1992). In particular, the term structure of yields for maturities greater than 1 will depend on the level of 1-month yield and on its conditional autocovariance function. Longstaff and Schwartz demonstrated that the predictive ability of a regression model for yields of long maturities improves when the conditional variance of short-term yields appears as regressor in addition to the level of short-term yields. Our model implies that, along with these two factors, time variations in the autocovariance function of short-term yields is important in explaining movements in the long end of the term structure.[4]

4. Stimulating the model

To generate time series for the variables of interest, it is necessary to select values for the 17×1 vector of parameters $\theta = (\gamma, \beta, A_{01}, A_{11}, a_{01}, a_{11}, a_{21}, A_{02}, A_{12}, a_{02}, a_{12}, a_{22}, A_{03}, A_{13}, a_{03}, a_{13}, a_{23})$. To provide discipline in the siimulation one could, as in 'calibration' exercises, select them to be consistent with existing micro studies. Alternatively, one could estimate θ by simulation. That is, one could choose θ to formally match statistics of the simulated and of the actual data in the least squares metric (see Lee and Ingram, 1991; Duffie and Singleton, 1993) or in the VAR metric (see Smith, 1993).

The approach we employ here incorporates ideas of Monte Carlo testing and has several appealing features (see Canova, 1994, for a complete description of the methodology). It allows us to summarize existing econometric evidence on the parameters in a realistic way, automatically provides a global sensitivity analysis for reasonable perturbations of the parameters and permits a more formal evaluation of the properties of the model.

Our task is to generate probability statements for statistics of the simulated data. For example, we would like to know what is the probability that the model can generate, on average, an upward-sloping yield curve. Available information on the parameters is summarized by means of a joint density $\pi(\theta \mid \mathscr{F})$, where $\mathscr{F}$ is the information set available and $\theta \in \Theta \subset R$. Let $G(x_t(z_t) \mid \theta, m)$ be the density for the $q \times 1$ vector of endogenous time series x_t, conditional on the parameter vector θ and the particular economic model m we have chosen. Here x_t includes four yields, six spreads, and six forward premia. $G(x_t(z_t) \mid \theta, m)$

[4] Using the instrumental variable procedure suggested by Pagan and Ullah (1988) we find that in the data the first two conditional autocovariance terms of 1-month yields enter significantly in a regression of longer-term yields on the level of 1-month yields, on their conditional variance and on four conditional autocovariances.

describes the likelihood of obtaining an x_t path from our model once a particular θ vector is chosen. For given θ, randomness in x_t is due to the randomness in the exogenous processes z_t.

Let $J(x_t(z_t),\ \theta \,|\, m,\ \mathscr{F})$ be the joint distribution of x_t and θ given the model specification and the information set. In the analysis we focus on statistics of the simulated data which are functions $h(\theta, z_t)$ of the parameters θ and of the exogenous processes z_t. In our case $h(\theta, z_t)$ includes the first and second conditional moments, the first four unconditional moments, terms of the autocovariance function of x_t, and some cross-correlations among its elements. Model-based probabilities for $h(\theta, z_t)$ can be obtained for any $\mathscr{A} \subset \Theta$ as a byproduct of the evaluation of integrals of the form:

$$\mathrm{E}(h(\theta, z_t) \,|\, m, \mathscr{F}, \mathscr{A}) = \int_{\mathscr{A}} h(\theta, z_t)\, J(x_t(z_t),\ \theta \,|\, m,\ \mathscr{F})\, \mathrm{d}\theta\, \mathrm{d}z_t. \tag{9}$$

Although theoretically straightforward, expressions like (9) are generally impossible to compute analytically or using simple numerical (spherical or quadrature) rules when Θ is high-dimensional. Our approach is to use a Monte Carlo methodology. The main idea is simple. Let θ_i be a $(k \times 1)$-dimensional i.i.d. vector of parameters and $\{z_{it}\}_{t=1}^{T}$ be a path for z_t where the subscript i refers to the draw. If the probability function from which the θ's and the z's are drawn is proportional to $J(x(z_t),\ \theta \,|\, m,\ \mathscr{F}_t)$, then, by the law of large numbers, $n^{-1} \sum_{i=1}^{n} h(\theta_i, z_{it})$ converges almost surely to $\mathrm{E}(h(\theta, z_t))$, where n is the number of replications. Therefore, by drawing a large number of replications for θ and z from $J(x_t(z_t),\ \theta \,|\, m,\ \mathscr{F}_t)$, we can approximate arbitrarily well $\mathrm{E}[h(\theta, z_t)]$.[5]

This Monte Carlo approach to simulation explicitly accounts for the uncertainty faced by a simulator in choosing parameter values and encompasses both calibration and estimation by simulation as special cases. Calibration is obtained when $\pi(\theta \,|\, \mathscr{F})$ has a point mass at a given $\bar{\theta}$ (usually chosen on the basis of micro-studies) and when a single draw from $G(x_t(z_t) \,|\, \theta,\ m)$ is made. Some authors report results when outcomes are averaged over a small number of simulations (see, e.g., Backus, Gregory, and Zin, 1989). In this case, $\pi(\theta \,|\, \mathscr{F})$ still has a point mass at $\bar{\theta}$ but repeated draws from $G(x_t(z_t) \,|\, \theta,\ m)$ are made.

The simulated method of moments (SMM) of Lee and Ingram (1991) or Duffie and Singleton (1993) and the GMM procedure of Burnside, Eichenbaum, and

[5] When $G(x_t(z_t) | \theta, m, \mathscr{F})$ is unknown, and numerical procedures are needed to solve the model, one could follow Geweke (1989) and draw from an *Importance Sampling* density for θ and z_t. Under mild regularity conditions the laws of large numbers still apply, i.e.: $\sum_{j=1}^{n} h(\theta_i, z_{ti})\, w_i / \sum_{i=1}^{n} w_i \equiv h_n \rightarrow \mathrm{E}(h(\theta, z_t))$ and $\sqrt{n}[h_n - \mathrm{E}(h(\theta, z_t))] \Rightarrow \mathscr{N}(0, \sigma_h^2)$ where $w_i = J(x_t(z_{t,i}), \theta_i | m, \mathscr{F}_t) / I(\theta_i, x_t(z_{t,i}))$, $\sigma_h^2 = \mathrm{var}(h(\theta))$, and $I(\theta, x_t(z_t))$ is the Importance Sampling density. Geweke (1989) describes how in practice one would select $I(\theta, x(z_t))$.

Rebelo (1993) are also special cases of this framework. In both cases $\pi(\theta \mid \mathscr{F})$ is a density with a point mass at θ^*, where θ^* is either a vector of parameters which minimizes a measure of distance between simulated and actual data or sets some orthogonality conditions equal to zero. Simulations are performed by drawing one or more realizations from $G(x_t(z_t) \mid \theta^*, m)$. Similarly, the simulated quasi-maximum likelihood technique (SQML) of Smith (1993) obtains when $\pi(\theta \mid \mathscr{F})$ has a point mass at $\hat{\theta}$, the SQML estimator of θ, and simulations are performed by drawing one or more realizations from $G(x_t(z_t) \mid \hat{\theta}, m)$.

4.1. Model evaluation

Probability statements and quantiles for the statistics of interest are easily obtained as a by-product of the Monte Carlo procedure. For example, to evaluate $P(h(\theta, z_t) \in A)$, where A is a bounded set we can choose the dth component of the function h to be $h_d(\theta, z_t) = \chi(\theta, z\colon h(\theta, z_t) \in A)$, where χ is the indicator function, i.e., $\chi(h(\theta, z_t) \geq A) = 1$ if $h(\theta, z_t) \in A$ and zero otherwise. Similarly, for any given α or H, we can compute $P[h(\theta, z_t) \leq H] = \alpha$ by appropriately selecting the indicator function. Once quantiles and probability statements are available, we can evaluate whether the model can, in a probabilistic sense, reproduce features of the actual data.

Suppose, we have a vector of statistics H from the actual data and we are interested in the probability that H could be generated by the chosen parametrization of the model. One way to evaluate the model is to take the actual realization of the statistics as a critical value and compute the probability that $h(\theta, z_t)$ is less than or equal to H, i.e., evaluate the model's likelihood of realizing the vector of statistics we observe in the data.

Another way to evaluate the model is to choose an α and, using the quantiles of the simulated distribution, compute a critical value $\tilde{H}$ satisfying $P[h(\theta, z_t) \leq \tilde{H}] \leq \alpha$. Comparing H and $\tilde{H}$ would then give a one-sided procedure to evaluate for the hypothesis that H has been generated by the model at a $\alpha\%$ level.

4.2. Sensitivity analysis

When one employs a Monte Carlo approach to compute integrals like (9) an automatic global sensitivity analysis on the support of the parameter space is performed as a by-product of the simulations. Sensitivity analyses can, however, take other more specific forms. For example, one might be interested in evaluating the probability of an x_t path associated with a specific estimate of θ (say, e.g., the simulated method of moments estimator of θ) or, perhaps, in assessing what is the maximal variation in x_t or $h(\theta, z_t)$ which is consistent, say, with θ being within a two standard error band of a particular estimated value. To perform this type of analysis simply slice the joint density for θ and z_t in the appropriate dimensions, draw a time path for z_t, and construct paths for x_t for one or more draws of θ in the particular range.

The approach to model evaluation we propose shares features with the procedure proposed by Gregory and Smith (1991). In their framework, however, parameters are calibrated. Since no allowance is made for parameter uncertainty, sensitivity analysis is roughly performed by replicating the experiment for different calibrated values of the parameters. Our approach also shares features with Kwan (1990). Similar to us he allows for parameter uncertainty in his simulation scheme but performs model evaluation by calculating the pairwise posterior odds ratio for alternative model specifications. In other words, while we evaluate the model in an absolute sense, Kwan's procedure generates probability statements relative to other possible specifications.

4.3. Selecting $\pi(\theta \mid \mathscr{F})$

The selection of $\pi(\theta \mid \mathscr{F})$ is a crucial ingredient in our simulation procedure. One could choose it to be the asymptotic distribution of the SMM estimator of θ as in Canova and Marrinan (1993) or of the GMM estimator of θ as in Burnside, Eichenbaum, and Rebelo (1993). Alternatively, one could choose it to be a 'subjective' Bayesian prior as in Kwan (1990) or an 'objective' one, as in Phillips (1991). Here we select $\pi(\theta \mid \mathscr{F})$ to reflect the cross-study variation in existing econometric evidence and to be consistent with standard simulation practices. To be as uncontroversial as possible and to maintain the approach closest to the logic of calibration, we choose $\pi(\theta \mid \mathscr{F})$ to be the frequency distribution of estimates of θ available in the literature, weighting estimates from all studies we are aware of equally.[6] If no econometric evidence is available and economic theory does not provide a range for a subset of the parameters, we assume a uniform density on a support chosen on the basis of our own calculations. In addition, since existing information about the components of θ is, in most cases, uncorrelated, $\pi(\theta \mid \mathscr{F})$ is factored into the product of lower-dimensional marginal densities.

The parameters of the model can be divided into two groups: one includes those which have an economic interpretation (β, γ) and for which a rich set of estimates exists in the literature. We use this empirical evidence to construct frequency distributions of estimates in these dimensions. A second group includes all remaining parameters characterizing the distribution of the exogenous processes. For this second group the econometric evidence is scant or nonexistent,

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[6] We neglect the fact that since some studies use the same data sets, estimates for certain parameters are not independent. As long as the resulting estimates reflect sampling variability due to different estimation techniques or different sample sizes, dependence of the estimates does not create a problem here.

and we express our ignorance by choosing a reasonable range for the support and imposing uniform densities in these dimensions.

For monthly data the discount factor β is typically estimated to be in the neighborhood of 0.996 with a small standard error (see, e.g., Hansen and Singleton, 1983; Eichenbaum, Hansen, and Singleton, 1988). The estimates vary from a minimum of 0.990 (see, e.g., Hansen and Singleton, 1983) to a maximum of 1.0022 (see, e.g., Dunn and Singleton, 1986). In general, estimates of the β are not independent of estimates of the risk aversion parameter γ. For the studies we analyzed, the rank correlation coefficient between estimates of γ and β is 0.12. When γ and β are jointly estimated the range of estimates of γ lies between 0.5–1.5 when consumption of nondurables and services are used (see, e.g., Hansen and Singleton, 1983; Brown and Gibbons, 1985; Heaton, 1993) to 2.5–3.5 when consumption of both durables and nondurables are used (see Dunn and Singleton, 1986).[7] In a study where the discount factor did not appear, Canova and Marrinan (1993) found that a value of γ close to zero best fits the data. In other studies where the discount factor is fixed, the estimated value of γ is larger (see, e.g., Burnside, Eichenbaum, and Rebelo, 1993).

In simulation studies, β is typically chosen to produce a steady-state real risk-free rate of 1–5% on an annual basis (see, e.g., Mehra and Prescott, 1985; Weil, 1989; Giovannini and Labadie, 1991; or Backus, Gregory, and Zin, 1989). This implies that on a monthly basis a reasonable range for β is [0.9951, 0.9992]. On the other hand, the range for γ is much larger and varies from 0.5 to 55 (see, e.g., Cooley and Ohanian, 1990; Giovannini and Labadie, 1991; Labadie, 1989; Donaldson, Johnsen, and Mehra, 1990; Backus, Gregory, and Zin, 1989; Kandel and Stambaugh, 1990).

We capture this information by choosing the marginal density for the β to be truncated normal centered around 0.997, with range [0.990, 1.0022] and the marginal density for γ to be $\chi^2(4)$ with range [0, 55]. Since the rank correlation between estimates of γ and β is only 0.12, we assume that the joint density of these two parameters is the product of the two marginals.

A few features of the two densities should be noted. The density of β is skewed to the left to conform to the idea that an annual real rate of 2–3% is more likely than a value in excess of 5%. The density for γ has mode at 2, which is the value most typically found in micro-econometric studies and often used for benchmark simulations. In addition, it puts very low weights on high values of γ. The 95% range of a $\chi^2(4)$ is, in fact, [0.7, 10] and less than 1.0% of the mass of the density is in the region where γ exceeds 13.

The next ten parameters (A_{01}, A_{11}, a_{01}, a_{11}, a_{21}, A_{02}, A_{12}, a_{02}, a_{12}, a_{21}) describe the conditional means and variances of output growth and money

[7] Kocherlakota (1990) shows that because of small-sample biases estimates for γ of the order of 2 are consistent with a 'true' value of about 13.

supply growth. Several studies document that the processes for output and for the monetary base in the US appear to contain at least one unit root (see, e.g., Stock and Watson, 1989). Using Citibase tape data we computed the first-order autocorrelation for the growth rate of industrial production and of the monetary base to be, respectively, 0.53 and 0.01 with standard errors equal to 0.07.[8] We use this information by selecting a density for A_{11} to be uniform on [0.46, 0.60] and for A_{12} to have 50% of the mass uniformly distributed in the interval [− 0.06, − 0.00001] and 50% of the mass at 0. This implies that we give a fifty–fifty chance to the unit root hypothesis for the process (see Sims, 1988, for the rationale for this representation). When output and the base have a unit root, A_{01}, A_{02} represent the average drift of the processes. Output in the U.S. for the period 1964–1987 grew at an average rate of 0.2% per month with a standard deviation of 0.9%. The average growth rate of the base in the U.S. has been 0.6% per month with a standard error of 0.3%. Therefore, we take A_{01}, A_{02} to be uniformly distributed over the intervals [− 0.007, 0.011] and [0.003, 0.009].

Little information about the parameters of the variances of output and the base is available. Hodrick (1989) and Canova and Marrinan (1995) estimate the conditional variances of these processes using GARCH specifications. We incorporate the information contained in these two studies by selecting a uniform prior for all parameters: a_{11} and a_{12} have densities with support on [− 0.002, 0.002], a_{21} has support on [0.14, 0.38], and a_{22} on [0.06, 0.36].[9] Finally, we select a_{01} and a_{02} so that, given the values for a_{11}, a_{12}, a_{21}, a_{22}, the unconditional variance of the two simulated processes lies within one standard error band around the point estimate of the unconditional variance over the 1964–88 sample ([0.000001, 0.00001] and [0.00008, 0.0001], respectively).

For the remaining five parameters characterizing the behavior of government expenditure (A_{03}, A_{13}, a_{03}, a_{13}, a_{23}) no econometric evidence exists because data on the size of government expenditure shares in total output is not available at a monthly frequency. We collect an estimate of the unconditional mean and variance for government expenditure share in total output at a quarterly frequency. We find that this mean share is stable across time at around 0.14, and its standard error is of the order of 0.08. We impose these restrictions on our simulated share of government expenditure by choosing A_{03} to be uniform on [0.03, 0.05] and a_{03} to be uniform on [0.02, 0.03]. Since there is no information for setting (A_{13}, a_{13}, a_{23}), we chose them to be uniform in [0, 0.5],

[8] Because of the high powered nature of the money supply in this model, we use the monetary base as opposed to broader measures of monetary aggregates.

[9] Our point estimates of the GARCH coefficients for the three processes are the median values of the assumed bands. As an alternative, and since estimates of these parameters are conditionally normal, one could also draw from a joint normal distribution. We prefer uniform distribution because the GARCH parameters for these processes are rather imprecisely estimated.

but eliminate any draw which induces a time path for the government expenditure share that, once it is aggregated at a quarterly frequency, is inconsistent with the reported quarterly evidence.

The final ingredient required is the choice of initial conditions for the exogenous processes. To make the simulations comparable with the actual data we chose as initial conditions the realized values for the exogenous processes in 1964.6.

5. The results

Tables 4–6 report statistics for yields at 1-, 3-, 6-, and 12-month maturities, for six spreads and for three cross-correlations when 10000 simulations were performed.[10] For each statistic we report a simulated 90% band and the probability that the model generates a value less than or equal to the value observed in the data.

The tables indicate that the model reproduces several qualitative features of the US term structure: the average yield curve slopes upward (the probability that the term structure is upward-sloping is 0.95), the volatility of yields decreases with maturity, yields of 3-, 6-, and 12-month maturities exhibit a high degree of serial correlation and changes in short-term yields are positively correlated with changes in long-term yields.

Quantitatively, the model matches the variability of yields and the higher moments but has three shortcomings. First, the 90% bands for the mean of yields is slightly too low. Contrary to Backus, Gregory, and Zin (1989), a high variability of yields is obtained here only at the cost of producing very low (or even negative) values for their mean. Second, the bands for the statistics testing for heteroskedasticity are also slightly too low. Third, and more importantly, the model fails to produce enough serial correlation in the simulated data. On average, simulated yields have first-order serial correlation coefficients which are 28% lower than what we observe in the US data.

The model is relatively more successful in accounting for the quantitative properties of the spreads. Only the mean of the spreads at the lowest end of the term structure and the level of heteroskedasticity are at odds with the actual data. While the former failure is significant, the latter one is minor. Because the model generates, approximately, the same amount of heteroskedasticity in all

[10] All the simulations reported are performed on a IBM Risk6000 workstation using RATS programs, which are available on request from the authors. Codes drawing random numbers from the chosen densities are also available on request. We produced our own pseudo-random numbers because we found that the periodicity of the algorithms creating random numbers in standard statistical software is too short to avoid repetitions. Our algorithm, which is based on Press et al. (1989), passes the twelve tests for randomness of Knuth (1981) and has a periodicity of 714025.

Table 4
Simulated data, 90% bands: T-bill yields

	1-month T-bill	3-month T-bill	6-month T-bill	12-month T-bill
Mean	[− 1.12, 7.09]	[3.92, 7.27]	[5.44, 7.49]	[6.63, 7.58]
$P(h(\theta, z_t) < H)$	0.90	0.92	0.91	0.90
Std. error	[0.43, 36.18]	[0.21, 15.36]	[0.11, 8.07]	[0.06, 4.02]
$P(h(\theta, z_t) < H)$	0.46	0.69	0.81	0.89
Skewness	[− 1.39, − 0.51]	[− 1.60, − 0.31]	[− 1.51, − 0.18]	[− 1.22, − 0.22]
$P(h(\theta, z_t) < H)$	0.62	0.62	0.69	1.00
Kurtosis	[1.59, 21.99]	[1.28, 21.04]	[0.99, 20.68]	[0.81, 20.41]
$P(h(\theta, z_t) < H)$	0.51	0.48	0.52	0.24
AR(1)	[− 0.24, 0.62]	[− 0.21, 0.70]	[− 0.23, 0.71]	[− 0.20, 0.74]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(2)	[− 0.02, 0.45]	[− 0.05, 0.40]	[− 0.04, 0.41]	[− 0.04,. 0.49]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(4)	[− 0.06, 0.16]	[− 0.06, 0.14]	[− 0.06, 0.19]	[− 0.06, 0.19]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(12)	[− 0.07, 0.07]	[− 0.08, 0.07]	[− 0.09, 0.08]	[− 0.09, 0.08]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(24)	[− 0.08, 0.08]	[− 0.09, 0.07]	[− 0.09, 0.07]	[− 0.10, 0.07]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
ARCH(13)	[0.11, 42.67]	[0.09, 38.03]	[0.04, 33.66]	[0.05, 34.01]
$P(h(\theta, z_t) < H)$	0.94	0.96	0.97	0.97
BP(13)	[0.27, 35.09]	[0.31, 38.88]	[0.33, 47.92]	[0.31, 48.04]
$P(h(\theta, z_t) < H)$	0.99	0.98	0.98	0.98
White(26)	[2.52, 59.99]	[2.96, 60.87]	[3.01, 65.52]	[3.07, 67.76]
$P(h(\theta, z_t) < H)$	1.00	0.98	0.97	0.98
Q(49)	[5.57, 88.81]	[5.10, 90.11]	[2.27, 92.50]	[4.40, 94.61]
$P(h(\theta, z_t) < H)$	0.61	0.74	0.08	0.34
KS	[0.82, 5.12]	[0.77, 3.67]	[1.02, 3.87]	[0.99, 3.92]
$P(h(\theta, z_t) < H)$	0.38	0.32	0.08	0.26

BP refers to Breusch–Pagan test, Q to the Ljung–Box test, KS refers to the Kolmogorov–Smirnov statistics. The numbers in parentheses after ARCH, BP, White, and Q refer to the number of degrees of freedom of the χ^2 statistics.

the yields, the spreads fail to be as heteroskedastic as we observe in the actual data.

Finally, the simulated correlations between changes in long-term yields and changes in short-term yields and correlations between forward premia and future changes in yields and between spreads and future changes in yields are too high to be consistent with the data.

To intuitively understand how the model can reproduce important qualitative features of the US term structure previously unexplained, consider its simplest version where there is no government and no money ($G_t = M_t = 0$).

The pricing formula at t for an asset delivering one unit of the consumption good at $t + k$ with certainty is

$$V_{t,k} = \beta^k \, \mathrm{E}_t \frac{U'(C_{t+k})}{U'(C_t)} = \beta^k \, \mathrm{E}_t \left(\frac{Y_{t+k}}{Y_t}\right)^{-\gamma}. \tag{10}$$

The yield on this asset at time t is

$$r_{t,k} = -\frac{\ln V_{t,k}}{k} \approx -\ln \beta + \frac{\gamma}{k} \mathrm{E}_t\left(\ln\left[\frac{Y_{t+k}}{Y_t}\right]\right) - \frac{\gamma^2}{2k} \mathrm{var}_t\left(\ln\left[\frac{Y_{t+k}}{Y_t}\right]\right), \tag{11}$$

where $\mathrm{E}_t(.)$ and $\mathrm{var}_t(.)$ refer to the conditional mean and variance of the quantity in parenthesis and the approximation comes from the truncation in the Taylor expansion.

From (11) it is clear that time variation in the conditional variance of the exogenous processes of the economy is a potentially important determinant of the cyclical behavior of interest rates. For example, if the process for output is a random walk, time variation in the variance of output growth entirely accounts for variation over time in yields. In addition, when future economic uncertainty is large, a riskless bill is more highly valued and, consequently, its yield may be very low (even negative). The very low average value of the risk-free rate observed in the US has been considered by many troublesome (see, e.g., Weil, 1989). A large amount of average variability in the exogenous processes may account for this behavior (see Huggett, 1993, for an alternative explanation). Note also that as $k \to \infty$, $\mathrm{var}_t(\ln[Y_{t+k}/Y_t]) \to \mathrm{var}(\ln[Y_{t+k}/Y_t])$, unless the conditional variance of output growth is very persistent. Therefore, variation in the uncertainty surrounding the driving processes far in the future will have no effect on current yields. This implies that heteroskedasticity in the exogenous forces of the economy is likely to impact primarily on the shorter end of the term structure.

The unconditional autocovariance function of yields (and spreads) also depends on the presence of conditional heteroskedasticity in a nontrivial way. For example, assuming that the conditional moments of output growth are uncorrelated as in Kandel and Stambaugh (1991), the unconditional variance of the

Table 5
Simulated data, 90% bands: T-bill spreads

	1–3 months	1–6 months	1–12 months	3–6 months	3–12 months	6–12 months
Mean	[0.06, 3.25]	[0.13, 4.23]	[0.44, 6.06]	[0.04, 1.23]	[0.28, 2.51]	[0.15, 1.14]
$P(h(\theta, z_t) < H)$	0.00	0.00	0.00	0.06	0.05	0.05
Std. error	[0.14, 20.97]	[0.21, 26.66]	[0.29, 30.05]	[0.08, 6.97]	[0.11, 9.81]	[0.04, 3.97]
$P(h(\theta, z_t) < H)$	0.18	0.17	0.21	0.27	0.39	0.61
Skewness	[0.31, 1.38]	[0.31, 1.39]	[0.30, 1.48]	[0.27, 1.50]	[0.26, 1.44]	[0.15, 1.35]
$P(h(\theta, z_t) < H)$	0.18	0.11	0.03	0.07	0.07	0.00
Kurtosis	[1.82, 23.01]	[1.71, 22.88]	[1.54, 22.68]	[1.50, 22.40]	[1.48, 22.50]	[0.79, 24.42]
$P(h(\theta, z_t) < H)$	0.50	0.37	0.40	0.13	0.72	0.96
AR(1)	[− 0.18, 0.52]	[− 0.19, 0.55]	[− 0.21, 0.56]	[− 0.22, 0.67]	[− 0.13, 0.70]	[− 0.19, 0.71]
$P(h(\theta, z_t) < H)$	0.82	0.80	0.89	0.82	0.92	0.85
AR(2)	[− 0.04, 0.42]	[− 0.04, 0.43]	[− 0.05, 0.45]	[− 0.05, 0.47]	[− 0.06, 0.45]	[− 0.05, 0.45]
$P(h(\theta, z_t) < H)$	0.73	0.89	0.87	0.93	0.89	0.75
AR(4)	[− 0.05, 0.15]	[− 0.07, 0.15]	[− 0.05, 0.15]	[− 0.06, 0.16]	[− 0.06, 0.16]	[− 0.09, 0.17]
$P(h(\theta, z_t) < H)$	0.74	0.80	0.33	0.89	0.95	0.82

AR(12)	[− 0.06, 0.06]	[− 0.08, 0.07]	[− 0.07, 0.07]	[− 0.08, 0.07]	[− 0.08, 0.07]	[− 0.09, 0.08]
$P(h(\theta, z_t) < H)$	1.00	0.98	0.98	0.96	0.97	0.97
AR(24)	[− 0.08, 0.05]	[− 0.08, 0.07]	[− 0.08, 0.08]	[− 0.09, 0.08]	[− 0.09, 0.07]	[− 0.09, 0.07]
$P(h(\theta, z_t) < H)$	1.00	0.99	0.98	0.91	0.14	0.42
ARCH(13)	[0.06, 44.41]	[0.06, 45.84]	[0.06, 44.06]	[0.02, 44.61]	[0.04, 43.52]	[0.05, 35.50]
$P(h(\theta, z_t) < H)$	0.93	0.98	0.99	0.95	0.96	0.98
BP(13)	[0.23, 39.98]	[0.24, 37.70]	[0.22, 37.54]	[0.32, 35.67]	[0.30, 38.11]	[0.33, 44.56]
$P(h(\theta, z_t) < H)$	0.99	1.00	0.98	0.88	1.00	1.00
White(26)	[1.92, 64.88]	[2.20, 65.55]	[2.18, 66.65]	[2.79, 68.11]	[3.12, 62.75]	[3.18, 63.84]
$P(h(\theta, z_t) < H)$	0.98	1.00	1.00	0.97	1.00	0.99
Q(49)	[0.86, 92.78]	[4.04, 89.31]	[0.99, 86.91]	[7.13, 93.68]	[1.74, 80.45]	[2.38, 95.98]
$P(h(\theta, z_t) < H)$	0.67	0.27	0.03	0.76	0.78	0.45
KS	[0.05, 1.52]	[0.61, 2.04]	[0.56, 2.71]	[0.48, 1.50]	[0.52, 1.66]	[0.11, 0.98]
$P(h(\theta, z_t) < H)$	0.40	0.49	0.53	0.33	0.37	0.63

BP refers to Breusch–Pagan test, Q to the Ljung–Box test, KS refers to the Kolmogorov–Smirnov statistics. The numbers in parentheses after ARCH, BP, Q, and White refer to the number of degrees of freedom of the χ^2 statistics.

Table 6
Simulated data, 90% bands: Cross-moments

	−2	−1	0	1	2
Δ long–Δ short yields					
1–6 months	[−0.17, 0.27]	[−0.63, −0.08]	[0.92, 0.99]	[−0.34, 0.11]	[−0.19, 0.26]
$P(h(\theta, z_t) < H)$	0.09	1.00	0.00	0.96	0.21
1–12 months	[−0.16, 0.30]	[−0.64, −0.02]	[0.88, 0.99]	[−0.65, 0.06]	[−0.18, 0.29]
$P(h(\theta, z_t) < H)$	0.25	1.00	0.00	0.97	0.22
3–6 months	[−0.21, 0.26]	[−0.61, 0.09]	[0.994, 0.999]	[−0.62, 0.14]	[−0.20, 0.24]
$P(h(\theta, z_t) < H)$	0.16	0.98	0.00	0.96	0.15
3–12 months	[−0.18, 0.27]	[−0.61, 0.01]	[0.97, 0.99]	[−0.59, 0.11]	[−0.19, 0.25]
$P(h(\theta, z_t) < H)$	0.62	0.98	0.00	0.98	0.27
6–12 months	[−0.21, 0.26]	[−0.62, 0.08]	[0.98, 0.99]	[−0.60, 0.10]	[−0.20, 0.25]
$P(h(\theta, z_t) < H)$	0.18	0.98	0.00	0.98	0.24
FP–Δ short yields					
1–3 months	[−0.22, 0.29]	[0.41, 0.75]	[−0.70, −0.43]	[−0.34, 0.22]	[−0.27, 0.01]
$P(h(\theta, z_t) < H)$	0.35	0.00	1.00	0.17	0.33
1–6 months	[−0.22, 0.31]	[0.40, 0.73]	[−0.73, −0.45]	[−0.31, 0.21]	[−0.25, 0.01]
$P(h(\theta, z_t) < H)$	0.42	0.00	1.00	0.16	0.25
1–12 months	[−0.22, 0.30]	[0.39, 0.74]	[−0.75, −0.44]	[−0.30, 0.25]	[−0.25, 0.04]
$P(h(\theta, z_t) < H)$	0.76	0.00	1.00	0.23	0.26

3–6 months	[− 0.21, 0.44]	[0.36, 0.78]	[− 0.78, − 0.40]	[− 0.32, 0.26]	[− 0.23, 0.05]
$P(h(\theta,z_t) < H)$	0.37	0.00	1.00	0.25	0.60
3–12 months	[− 0.23, 0.40]	[0.37, 0.79]	[− 0.76, − 0.39]	[− 0.34, 0.24]	[− 0.25, 0.02]
$P(h(\theta,z_t) < H)$	0.78	0.00	1.00	0.25	0.48
6–12 months	[− 0.20, 0.38]	[0.33, 0.80]	[− 0.77, − 0.44]	[− 0.37, 0.19]	[− 0.26, 0.03]
$P(h(\theta,z_t) < H)$	0.81	0.00	1.00	0.25	0.74
SP–Δ short yields					
1–3 months	[− 0.20, 0.28]	[0.42, 0.70]	[− 0.70, − 0.48]	[− 0.29, 0.18]	[− 0.23, 0.01]
$P(h(\theta,z_t) < H)$	0.39	0.03	1.00	0.31	0.26
1–6 months	[− 0.22, 0.33]	[0.42, 0.78]	[− 0.70, − 0.45]	[− 0.28, 0.19]	[− 0.24, 0.03]
$P(h(\theta,z_t) < H)$	0.37	0.00	0.99	0.21	0.21
1–12 months	[− 0.22, 0.30]	[0.43, 0.77]	[− 0.70, − 0.40]	[− 0.29, 0.22]	[− 0.20, 0.01]
$P(h(\theta,z_t) < H)$	0.85	0.03	0.96	0.10	0.08
3–6 months	[− 0.20, 0.40]	[0.38, 0.80]	[− 0.71, − 0.38]	[− 0.38, 0.19]	[− 0.27, 0.02]
$P(h(\theta,z_t) < H)$	0.53	0.00	1.00	0.26	0.31
3–12 months	[− 0.20, 0.40]	[0.39, 0.79]	[− 0.72, − 0.39]	[− 0.39, 0.20]	[− 0.28, 0.02]
$P(h(\theta,z_t) < H)$	0.87	0.00	1.00	0.19	0.29
6–12 months	[− 0.21, 0.40]	[0.36, 0.79]	[− 0.79, − 0.35]	[− 0.40, 0.21]	[− 0.29, 0.02]
$P(h(\theta,z_t) < H)$	0.88	0.00	0.99	0.21	0.40

SP stands for spread and FP for forward premium.

yield on a bill of maturity k is

$$\begin{aligned}\operatorname{var}(r_{t,k}) &= \mathrm{E}\Bigg[\frac{\gamma}{k}\Bigg(\mathrm{E}_t\Bigg(\ln\Bigg[\frac{Y_{t+k}}{Y_t}\Bigg]\Bigg) - \mathrm{E}\Bigg(\ln\Bigg[\frac{Y_{t+k}}{Y_t}\Bigg]\Bigg)\Bigg) \\ &\quad - \frac{\gamma^2}{2k}\Bigg(\operatorname{var}_t\Bigg(\ln\Bigg[\frac{Y_{t+k}}{Y_t}\Bigg]\Bigg) - \operatorname{var}\Bigg(\ln\Bigg[\frac{Y_{t+k}}{Y_t}\Bigg]\Bigg)\Bigg)\Bigg]^2 \\ &= \Bigg[\frac{\gamma^2}{k^2}\mathrm{E}\Bigg[\sum_{j=0}^{k} A_{11}^{j} z_{1t}\Bigg] + A_0 \sum_{l=1}^{k}\sum_{l=0}^{k} A_{11}^{l} - \frac{kA_{01}}{1-A_{11}}\Bigg]^2 \\ &\quad + \frac{\gamma^4}{4k^2}\mathrm{E}\Bigg[\sum_{l=1}^{k}\Bigg(\sum_{j=0}^{l-1} A_{11}^{2j}\Bigg)(a_{11}+a_{21})^{k-l} * (\sigma_{t+1}^2 - \mathrm{E}(\sigma_{t+1}^2))\Bigg]^2,\end{aligned} \tag{12}$$

where we have used the results of Baillie and Bollerslev (1992) to compute the expected value at t of the conditional variance of output growth at $t + k$. If output is a random walk, the first term in (12) drops out and, if there was no heteroskedasticity, the variance of interest rates would be identically equal to zero. When conditional heteroskedasticity is present, the variance of interest rates depends on the signs and relative magnitudes of the GARCH parameters, the maturity of the bill, and the size of the deviations of the conditional from unconditional variability of output. Similarly, using the fact that the autocorrelation function of yields can be computed as $\operatorname{corr}(r_{t,k}, r_{t-k,k}) = (\mathrm{E}(1 + R_{t,2k}) - [\mathrm{E}(1 + R_{t,k})]^2)/\operatorname{var}(r_{t,k})$ (see, e.g., Kandel and Stambaugh, 1990), it is immediate to note that heteroskedasticity in output growth will have an impact on the entire second-order properties of yields. Therefore, time variation in the conditional second moments of the driving processes may be crucial in matching the variability and the correlation structure of yields at the short end of the term structure, especially when the driving processes are nearly integrated.

To confirm the intuition provided with the above simple analytical example, we conduct a numerical experiment using conditionally homoskedastic exogenous processes. Tables 7–9 present the results and display several interesting features. First, the standard errors of yields are very small and the bands are narrow. Second, the third and fourth moments of yields are much smaller in absolute value than in the heteroskedastic case, the bands are shifted toward positive values and the median of the band is always around zero. Third, the behavior of the bands for the autocorrelations depends on the maturity of the bill: for the low end of the term structure, the bands are smaller in size and shifted toward zero. For 12-month yields, the upper tail of the distribution almost completely disappears. The behavior of the spreads tracks very closely

Table 7
Simulated data, 90% bands: T-bill yields, no heteroskedasticity in the exogenous processes

	1-month T-bill	3-month T-bill	6-month T-bill	12-month T-bill
Mean	[− 0.96, 9.75]	[3.82, 8.04]	[5.80, 8.03]	[6.67, 8.41]
$P(h(\theta, z_t) < H)$	0.76	0.77	0.80	0.79
Std. error	[0.0005, 0.01]	[0.002, 0.006]	[0.002, 0.003]	[0.006, 0.006]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
Skewness	[− 0.26, 0.22]	[− 0.22, 0.13]	[− 0.11, 0.19]	[0.05, 0.11]
$P(h(\theta, z_t) < H)$	0.00	0.00	0.00	0.00
Kurtosis	[− 0.97, 0.41]	[− 0.77, 0.06]	[− 0.84, 0.28]	[− 0.64, − 0.53]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(1)	[0.02, 0.59]	[0.02, 0.49]	[− 0.03, 0.50]	[− 0.05, 0.002]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(2)	[0.01, 0.36]	[0.06, 0.30]	[0.03, 0.28]	[0.01, 0.06]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(4)	[− 0.09, 0.17]	[− 0.04, 0.12]	[− 0.04, 0.13]	[− 0.004, 0.14]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(12)	[− 0.10, 0.09]	[0.01, 0.17]	[− 0.03, 0.10]	[− 0.004, 0.01]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
AR(24)	[− 0.12, 0.12]	[− 0.08, 0.05]	[− 0.08, 0.08]	[0.02, 0.05]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
ARCH(13)	[6.07, 21.47]	[5.65, 19.07]	[6.37, 18.53]	[9.73, 13.97]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
BP(13)	[5.14, 18.51]	[8.17, 23.69]	[5.14, 17.02]	[8.35, 12.44]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
White(26)	[14.07, 35.31]	[14.07, 35.31]	[14.21, 34.23]	[19.06, 36.06]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00
Q(49)	[6.87, 58.66]	[25.07, 46.91]	[2.96, 48.83]	[25.39, 32.70]
$P(h(\theta, z_t) < H)$	0.91	1.00	0.06	0.96
KS	[0.67, 2.01]	[0.92, 2.88]	[0.83, 2.08]	[0.67, 2.87]
$P(h(\theta, z_t) < H)$	0.36	0.18	0.09	0.33

BP refers to Breusch–Pagan test, Q to the Ljung–Box test, KS refers to the Kolmogorov–Smirnov statistics. The numbers in parentheses after ARCH, BP, White, and Q refer to the number of degrees of freedom of the χ^2 statistics.

the behavior of yields. Finally, with homoskedastic exogenous processes cross-correlations are very different from the heteroskedastic case. The median value of the contemporaneous cross-correlation of changes in short- and in long-term yields drops significantly and the lower tail of the distribution includes, in two

Table 8

Simulated data, 90% bands: T-bill spreads, no heteroskedasticity in the exogenous processes

	1–3 months	1–6 months	1–12 months	3–6 months	3–12 months	6–12 months
Mean	[− 1.60, 1.62]	[− 2.84, 2.61]	[− 3.39, 3.56]	[− 1.18, 0.99]	[− 1.68, 1.94]	[− 0.52, 0.95]
$P(h(\theta, z_t) < H)$	0.28	0.30	0.25	0.33	1.00	0.20
Std. error	[0.002, 0.004]	[0.001, 0.007]	[0.006, 0.01]	[0.001, 0.003]	[0.005, 0.007]	[0.005, 0.005]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00	1.00	1.00
Skewness	[− 0.20, 0.05]	[− 0.16, 0.20]	[− 0.06, 0.16]	[− 0.05, 0.15]	[0.003, 0.12]	[0.05, 0.11]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00	1.00	1.00
Kurtosis	[− 0.91, 0.03]	[− 0.63, 0.26]	[− 0.63, 0.04]	[− 0.74, − 0.11]	[− 0.75, − 0.27]	[− 0.63, − 0.48]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00	1.00	1.00
AR(1)	[0.0006, 0.43]	[− 0.04, 0.51]	[− 0.05, 0.32]	[− 0.03, 0.40]	[− 0.04, 0.17]	[− 0.05, 0.01]
$P(h(\theta, z_t) < H)$	0.85	0.77	1.00	1.00	1.00	1.00
AR(2)	[0.07, 0.23]	[0.01, 0.28]	[0.008, 0.18]	[0.05, 0.23]	[− 0.01, 0.10]	[0.01, 0.04]
$P(h(\theta, z_t) < H)$	0.94	0.97	1.00	1.00	1.00	1.00
AR(4)	[− 0.02, 0.10]	[− 0.03, 0.14]	[− 0.03, 0.05]	[− 0.08, 0.06]	[− 0.06, 0.00]	[− 0.02, 0.01]
$P(h(\theta, z_t) < H)$	0.88	0.83	0.21	1.00	1.00	1.00

AR(12)	[0.01, 0.17]	[− 0.06, 0.08]	[− 0.04, 0.03]	[0.03, 0.12]	[− 0.07, 0.02]	[− 0.02, − 0.001]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	0.43	1.00	1.00
AR(24)	[− 0.04, 0.06]	[− 0.07, 0.09]	[0.004, 0.07]	[− 0.04, 0.04]	[0.01, 0.07]	[0.03, 0.06]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	0.96	0.00	0.00
ARCH(13)	[6.37, 17.74]	[6.33, 18.54]	[7.21, 17.74]	[7.70, 17.06]	[10.42, 21.19]	[10.80, 14.92]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00	1.00	1.00
BP(13)	[5.58, 17.09]	[5.96, 21.14]	[6.33, 17.06]	[10.95, 27.77]	[7.27, 13.56]	[8.41, 11.51]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	0.61	1.00	1.00
White(26)	[8.12, 31.07]	[7.12, 26.88]	[8.06, 28.13]	[9.54, 29.10]	[11.05, 33.04]	[10.84, 32.28]
$P(h(\theta, z_t) < H)$	1.00	1.00	1.00	1.00	1.00	1.00
Q(49)	[22.70, 44.64]	[24.16, 48.87]	[10.60, 46.64]	[28.31, 43.08]	[8.93, 43.94]	[27.45, 37.27]
$P(h(\theta, z_t) < H)$	0.99	0.09	0.02	0.24	1.00	0.98
KS	[0.23, 1.60]	[0.80, 1.90]	[0.83, 1.67]	[0.52, 1.80]	[0.73, 1.72]	[0.44, 1.56]
$P(h(\theta, z_t) < H)$	0.28	0.43	0.49	0.36	0.29	0.32

BP refers to Breusch–Pagan test, Q to the Ljung–Box test, KS refers to the Kolmogorov–Smirnov statistics. The numbers in parentheses after ARCH, BP, Q, and White refer to the number of degrees of freedom of the χ^2 statistics.

Table 9

Simulated data, 90% bands: Cross-moments, no heteroskedasticity in the exogenous processes

Δ long–Δ short yields					
1–6 months	[− 0.18, 0.12]	[− 0.21, − 0.11]	[− 0.26, 0.90]	[− 0.20, 0.16]	[− 0.17, 0.0]
$P(h(\theta, z_t) < H)$	0.15	1.00	0.84	0.87	0.43
1–12 months	[− 0.08, 0.13]	[− 0.03, 0.26]	[− 0.56, 0.12]	[− 0.03, 0.31]	[− 0.07, 0.1]
$P(h(\theta, z_t) < H)$	0.29	0.46	1.00	0.78	0.27
3–6 months	[− 0.11, 0.09]	[− 0.43, − 0.24]	[0.73, 0.94]	[− 0.39, − 0.23]	[− 0.10, 0.0]
$P(h(\theta, z_t) < H)$	0.10	1.00	0.96	1.00	0.10
3–12 months	[− 0.01, 0.08]	[− 0.30, − 0.15]	[0.34, 0.48]	[− 0.26, − 0.11]	[− 0.06, 0.0]
$P(h(\theta, z_t) < H)$	0.45	1.00	1.00	1.00	0.85
6–12 months	[− 0.07, 0.01]	[− 0.51, − 0.23]	[− 0.51, 0.94]	[− 0.50, − 0.20]	[− 0.09, 0.0]
$P(h(\theta, z_t) < H)$	0.04	1.00	0.17	1.00	0.75
FP–Δ short yields					
1–3 months	[0.03, 0.28]	[0.09, 0.49]	[− 0.49, − 0.10]	[− 0.28, 0.03]	[− 0.18, 0.0]
$P(h(\theta, z_t) < H)$	0.00	0.25	0.73	0.68	0.50
1–6 months	[0.02, 0.28]	[0.45, 0.57]	[− 0.58, − 0.44]	[− 0.28, 0.02]	[− 0.19, 0.0]
$P(h(\theta, z_t) < H)$	0.00	0.00	1.00	0.70	0.23
1–12 months	[0.02, 0.26]	[0.33, 0.48]	[− 0.49, − 0.31]	[− 0.25, 0.03]	[− 0.17, 0.0]
$P(h(\theta, z_t) < H)$	0.39	0.01	1.00	0.53	0.14

3–6 months	[− 0.01, 0.23]	[0.42, 0.59]	[− 0.60, − 0.43]	[− 0.23, 0.04]	[− 0.17, 0.0]
$P(h(\theta,z_t) < H)$	0.00	0.00	1.00	0.21	0.89
3–12 months	[0.06, 0.22]	[− 0.03, 0.38]	[− 0.39, 0.02]	[− 0.21, 0.007]	[− 0.17, − 0.0]
$P(h(\theta,z_t) < H)$	0.82	0.82	0.25	0.19	0.93
6–12 months	[− 0.05, 0.23]	[− 0.64, 0.30]	[− 0.27, 0.63]	[− 0.23, − 0.03]	[− 0.18, 0.0]
$P(h(\theta,z_t) < H)$	0.86	0.90	0.14	0.27	1.00
SP–Δ short yields					
1–3 months	[− 0.00, 0.22]	[− 0.18, 0.41]	[− 0.41, 0.16]	[− 0.25, 0.05]	[− 0.14, 0.0]
$P(h(\theta,z_t) < H)$	0.00	0.85	0.13	0.44	0.11
1–6 months	[0.01, 0.27]	[0.36, 0.49]	[− 0.51, − 0.35]	[− 0.26, 0.03]	[− 0.18, 0.0]
$P(h(\theta,z_t) < H)$	0.00	0.01	0.79	0.70	0.07
1–12 months	[0.03, 0.20]	[0.23, 0.45]	[− 0.45, − 0.20]	[− 0.21, 0.03]	[− 0.11, 0.0]
$P(h(\theta,z_t) < H)$	0.87	0.53	0.14	0.09	0.00
3–6 months	[− 0.01, 0.19]	[0.50, 0.62]	[− 0.62, − 0.50]	[− 0.17, 0.04]	[− 0.14, − 0.0]
$P(h(\theta,z_t) < H)$	0.04	0.00	1.00	0.14	0.12
3–12 months	[0.04, 0.18]	[− 0.09, 0.24]	[− 0.23, 0.08]	[− 0.15, 0.01]	[− 0.13, − 0.0]
$P(h(\theta,z_t) < H)$	1.00	0.94	0.05	0.00	0.06
6–12 months	[0.01, 0.12]	[− 0.66, − 0.09]	[0.11, 0.66]	[− 0.10, − 0.01]	[− 0.14, − 0.0]
$P(h(\theta,z_t) < H)$	1.00	1.00	0.00	0.00	0.76

SP stands for spread and FP for forward premium.

cases, negative values. The bands for the cross-correlations of both forward premia and spreads with future changes in yields move toward zero. As expected from the above discussion, the bands for the cross-correlations of longer-term forward premia and spreads with future changes in yields are the least affected by the change.

In conclusion, the presence of heteroskedasticity in the exogenous processes appears to be important in reproducing the conditional and the unconditional variance of yields. It also helps in boosting the autocorrelation function of simulated data toward that of the actual data but does not quite do the job. The cost of introducing heteroskedastic processes in the model materializes primarily in higher values for higher moments of the yields and in the extreme values for the cross-correlations between changes in short- and long-term yields and between forward premia and spreads with future changes in yields.

6. Some sensitivity results

Some of the assumptions we made in either solving the model or in specifying the nature of the stochastic processes may be considered controversial. In this section we examine the robustness of the conclusions obtained to modifications of these assumptions. We also examine whether it is the uncertainty present in the economic parameters or in the parameters characterizing the exogenous processes which is responsible is for the large size of the bands appearing in Tables 4–6.

In deriving (2)–(5) we imposed the quantity theory. Hodrick, Kocherlakota, and Lucas (1991) show that when a version of the above model is calibrated to the U.S. economy the cash-in-advance constraints almost always bind. Therefore, there appears to be little practical gain in specifying models with more complicated nonbinding constraints. However, in principle, we can abstract from this problem entirely by simply taking the stochastic processes for consumption and prices as the primitives for our simulations. In practice, the quality of consumption data is poor. Wilcox (1992) pointed out that monthly aggregate consumption data is primarily interpolated from observations obtained at a much lower frequency. This interpolation procedure generates serially correlated measurement errors and disturbing autocorrelation properties in the data.[11] In addition Breeden, Gibbons, and Litzenberger (1989) indicated that summation biases may make the statistical properties of quarterly consumption data dubious as well.

[11] The first-order serial correlation coefficient of monthly consumption growth for the data set we use is -0.27. If we aggregate the monthly data at a quarterly level, the correlation becomes 0.13. For published quarterly data on consumption growth, the first-order correlation coefficient is 0.29.

With these caveats in mind we use (1) as our asset pricing equation. To perform simulations we have to select ten new parameters. The densities for the parameters of consumption growth and inflation processes are uniform centered around the point estimates with ranges equal to a one standard error band. Estimates of these parameters are obtained using the monthly consumption data on nondurables and services and the monthly personal consumption expenditure index.[12] Letting $z_t = [\Delta \log P_t,\ \Delta \log C_t]$, the ranges for the ten parameters characterizing the two processes are: $A_{01} \in [0.0018, 0.0022]$, $A_{11} \in [0.30, 0.38]$, $a_{01} \in [0.0000026, 0.0000034]$, $a_{11} \in [-0.0001, 0.0001]$, $a_{21} \in [0.20, 0.44]$, $A_{02} \in [0.0017, 0.0023]$, $A_{12} \in [-0.32, -0.22]$, $a_{02} \in [0.000009, 0.000011]$, $a_{12} \in [-0.0008, 0.0006]$, $a_{22} \in [0.02, 0.07]$.

Tables with the results of this and other experiments are reported in an appendix available from the authors. Here we briefly summarize the main features of the results. We find that the probability that the model generates an upward-sloping yield curve drops to about 52%. Backus, Gregory, and Zin (1989) demonstrated that, on average, an upward-sloping yield curve obtains when the growth rates of the driving processes are negatively serially correlated. In the present instance, the first-order serial correlations of consumption growth and inflation are approximately of the same order but of opposite signs. Therefore, in large samples one should expect that approximately in 50% of the simulations the average term structure will slope upward.

We also find that the variability and the amount of serial correlation and of heteroskedasticity in simulated yields and spreads decreases. However, qualitatively, none of the features reported in Tables 4–6 is dramatically altered. Therefore, the imposition of the quantity theory is not crucial in determining the outcomes of the simulations.

There is some evidence in the literature (see, e.g., Kearns and Pagan, 1992) that GARCH models fail to capture important distributional characteristics of many processes. We conducted experiments using alternative functional forms for the conditional variances (as in Schwert, 1990) or using nonparametric estimates of the conditional moments (as in Pagan and Ullah, 1988). We found numerical changes in the reported statistics, but the essence of the results is unaltered.

Lewis (1991) presents evidence that the unconditional distribution of one component of z_t is nonstationary. She argues that the uncertainty due to regime changes in monetary policy in the U.S. may have had a nonnegligible impact on the behavior of the term structure for the 1979–82 period. In addition to this, when a process is subject to structural shifts, estimates of the conditional variance obtained from GARCH (AR or nonparameteric) models are biased,

[12]This data was kindly provided by Masao Ogaki.

tend to understate the true conditional variance of the processes, and may affect the time series properties of simulated yields.

To examine the impact of a change in the unconditional distribution of the monetary base on the term structure of yields, we performed simulations drawing the parameters of the process for the base from two different densities: one which matches the properties of the base before 1979 (first subsample consisting of time periods 1 through 176) and a second one that matches its properties for the period 1979–87 (second subsample consisting of time periods 177 through 270).[13]

Estimates of the AR-GARCH parameters of the growth rate of the base are approximately identical over the two subsamples, except for the first-order serial correlation coefficient. Before 1979, A_{12} is estimated to be -0.40 with a standard error of 0.14. After 1979, A_{12} is estimated to be 0.31 with the same standard error (compare with a value of 0.01 and a standard error of 0.07 obtained for the entire sample). The densities for the five parameters characterizing the conditional moments of money growth rates are assumed to be uniform centered around the point estimate of the parameter with the following ranges: before 1979 $A_{02} \in [0.010, 0.012]$, $A_{12} \in [-0.54, -0.26]$, $a_{02} \in [0.000004, 0.000006]$, $a_{12} \in [-0.0001, 0.0001]$, $a_{22} \in [0.01, 0.35]$, and after 1979 $A_{02} \in [0.003, 0.005]$, $A_{12} \in [0.17, 0.45]$, $a_{02} \in [0.000009, 0.00012]$, $a_{12} \in [-0.00008, 0.00008]$, $a_{22} \in [0.01, 0.38]$.

The results indicate that this modification improves the performance of the model. Both the serial correlation and the heteroskedasticity present in simulated yields and spreads increases. The model now generates about 86% of the serial correlation we observe in actual yields and can account for their heteroskedastic structure. In addition, the contemporaneous cross-correlations of changes in yields are much lower than in the basic case. In three out of the five cases, the correlations observed in the data can be generated by the model with reasonable probability.

To determine whether is it the uncertainty in the estimates of the parameters of the exogenous processes or the uncertainty we face in choosing values for the 'economic' parameters which is responsible for the size of bands reported in the tables, we conduct two additional experiments. Each experiment involves slicing the joint distribution of parameters and exogenous processes in different dimensions.

In the first case, we 'calibrate' the stochastic process for the exogenous variables by selecting point estimates for the parameters of their conditional

[13] A more appropriate way to check whether Lewis's objection is relevant would be to estimate the parameters of the base recursively (with the Kalman filter) and draw parameters in the simulation from recursive densities (one for each of the 270 time periods generated in each simulation). Because of the complexity of such an approach we did not undertake this exercise.

means and variances (which are the midpoints of the ranges described in Section 4.3). This experiment should indicate how the uncertainty surrounding the parameters of the stochastic processes is reflected in the bands for the reported statistics. In the second case, we 'calibrate' the two economic parameters ($\beta = 0.997$, $\gamma = 0.0, 2.0, 10.0$) and examine the distribution of the statistics when the parameters of the exogenous processes are randomly drawn. This experiment indicates how sensitive the results are to uncertainty in the economic parameters economists care about most.

We find that the simulated statistics are somewhat sensitive to the uncertainty in the parameters of the exogenous processes. When we fix these parameters the 90% bands for the moments of all yields are tighter, the bands for the standard errors are lower and, on average, there is less serial correlation and much less heteroskedasticity in the simulated processes. Similar results emerge for the spreads.

When β and γ are fixed at some 'reasonable' value we again find that the bands for the reported statistics change substantially. In particular, the absolute level of the standard error of simulated yields and spreads drops substantially. As we increase γ from 0 up to 10, the whole band for the autocorrelations are shifted toward zero for the 1-month rate and change nonmotonically for the 6- and 12-month rates. This change is achieved at the cost of shifting the band for mean yields and introducing a large amount of skewness and kurtosis in the simulated data. As in Cooley and Ohanian (1990), we find that values of γ close to zero minimize the distortions in the second-order properties of the simulated data.

7. Conclusions

This paper attempted to reconcile the US term structure of interest rates with the predictions of a standard monetary consumption based ICAP model. We modified the basic model to allow for conditional heteroskedasticity in the exogenous processes of the economy and found the modification helpful in accounting for some puzzling features of the yield curve.

We show that the model can reproduce the average slope of the yield curve, the absolute variability of yields, and the fact that volatility decreases with maturity and comes close to (but falls short of) matching the serial correlation properties of yields. The model is more successful in accounting for features of the spreads at various maturities. For almost all statistics examined the actual statistic observed in U.S. data falls within the 90% bands and, in a large number of cases, the actual statistics fall near the medians of the simulated distributions. The model produces with high-probability contemporaneous cross-correlations for changes in long- and short-term yields which are, in general, too large and cross-correlations at leads which are too small to be consistent with US evidence. The same is true for forward premia and future changes in yields. When a break in the unconditional distribution of the monetary base is allowed, this shortcoming is partially eliminated. This is not the case for correlations between the spreads and future changes in yields. This failure is important and deserves further study.

Although the representative agent paradigm is ill-suited to understand the complexity of financial markets, we believe that further experimentations with this model are necessary to discover what features of the real world are consistent with the approach before proceeding to more complex multi-agent specifications (as, e.g., in Marcet and Singleton, 1991; Heaton and Lucas, 1992). Extensions of this single-agent model to include capital and variable labor as in DenHaan (1990) or some form of liquidity constraint as in Lucas (1991) or Huggett (1993) are likely to be fruitful in eliminating some of the problems reported here. We do not believe, however, that the introduction of habit persistence, along the lines of Costantinides (1990), is the key to solve the problems we have highlighted. Habit persistence helps to increase the variability of yields at the cost of shifting the mean of the entire yield curve toward unreasonably low or negative values. However, it is not clear it will help to break the tight links between forward premia and spreads with future changes in yields that the current model generates.

Appendix

The closed-form solution for the interest rate on a bill of maturity k, is given $\forall k$ by

$$
\begin{aligned}
r_t^k \approx & -\ln \beta + \frac{1}{k}\Bigg\{\mathcal{G}_t(k) + \gamma \log(1 - z_{3t}) + \left[\sum_{j=1}^{k} A_{12}^j z_{2t} + A_{02} \sum_{l=1}^{k} \sum_{j=0}^{l-1} A_{12}^j\right] \\
& - 0.5 * \left[\sum_{l=1}^{k} \sum_{j=0}^{l-1} A_{12}^{2j} \mathcal{H}_{2t}(k,l)\right] \\
& + (\gamma - 1) * \left[\sum_{j=1}^{k} (A_{11}^j z_{1t} + A_{01} \sum_{l=1}^{k} \sum_{j=0}^{l=1} A_{11}^j\right] \\
& - 0.5 * (\gamma - 1)^2 * \left[\sum_{l=1}^{k} \sum_{j=0}^{l-1} A_{11}^{2j} \mathcal{H}_{1t}(k,l)\right]\Bigg\},
\end{aligned}
\tag{13}
$$

where $\mathscr{H}_{it}(k,l) = \sigma_i^2 + (a_{1i} + a_{2i})^{k-l} \; (\sigma_{it+1}^2 - \sigma_i^2)$, $i = 1,2$ (see Baillie and Bollerslev, 1992), σ_i^2 is the unconditional variance of z_{it}, and $\mathscr{G}_t(k)$ involves the parameters of the process for government expenditure share and γ and is given by

$$\mathscr{G}_t(k) = -\ln\Bigg[-\left(1 - A_{13}^k z_{3t} - A_{03}\sum_{j=0}^{k-1} A_{13}^j - \sqrt{3 * \mathscr{H}_{3t}(k)}\right)^{(1-\gamma)} + \left(1 - A_{13}^k z_{3t} - A_{03}\sum_{j=1}^{k-1} A_{13}^j + \sqrt{3 * \mathscr{H}_{3t}(k)}\right)^{(1-\gamma)} + \ln\left[2 * (1-\gamma) * \sqrt{3\mathscr{H}_{3t}(k)}\right], \qquad (14)$$

where $\mathscr{H}_{3t}(k) = \sigma_3^2 + (a_{13} + a_{23})^{k-1} \; (\sigma_{3t+1}^2 - \sigma_3^2)$.

The forward rate, the spread, and the forward premium between interest rates of maturity k and h can then be computed directly from (13) and (14) using (3), (4), and (5).

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