APPENDICES

A Long-run labour productivity in an open economy

As discussed in the text, there is some controversy on how the price of investment and GDP should be deflated so as to make equation (1) hold. In this note we investigate on the relative price that determines labor productivity in the long run. The question is relevant just in an open economy since in a closed economy the consumer price index and the output deflator should be the same (except possibly because of the wedge introduced by indirect taxes). We start considering a simple static model. This is just intended to characterize the steady state of an economy with intertemporal maximization. The intuition of the results are probably easier to grasp in this simple set-up. We then consider a intertemporal version of the same model with perfect capital mobility. This is done just to get fully reassured that the results also hold in a more conventional set-up.

The static model In the economy there are four goods: two consumption goods and two investment goods. The 'H'ome economy is the only producer of one consumption good and one investment good. The other two goods are produced by the 'F'oreign economy. We start considering as a numeraire the domestic consumption good. This will be equivalent to deflating nominal quantities with the output deflator. In the economy there is a representative consumer who maximize his period by period utility (i.e. his discount factor is zero) given by

$$U = a \ln C^H + (1-a) \ln \frac{C^F}{P_c^F} \tag{11}$$

where C^H and C^F denotes the consumption expenditures in the good produced by the H and F economy, respectively. P_c^F is the price (in domestic consumption units) of the consumption good produced abroad. Hereafter we use the convention that the superscript always indicates where the good is produced ('H'ome or 'F'oreign), while the subscript refers to the type of good ('c'onsumption or 'i'nvestment).

The problem is subject to the resource constraint:

$$Y = I^H + C^H + X \tag{12}$$

where I^H, C^H, and X are investment expenditures in domestic goods, in the consumption of domestic goods, and exports (in either consumption or investment goods). Domestic output is produced according to the constant-return to scale Cobb-Douglas production function:

$$Y = Z \left(K^H\right)^\alpha \left(K^F\right)^\beta \tag{13}$$

where, without loss of generality, the work force is normalized to one ($L^{1-\alpha-\beta} = 1$). Thus Y also denotes labor productivity. K^H and K^F are the stock of capital of the Home economy produced at home and abroad, respectively. The law of evolution of capital is

$$K^j = \frac{I^j}{P_i^j}, \qquad j = H, F \tag{14}$$

where, for simplicity we assume that capital fully depreciates after use (i.e. capital in the previous period does not influence capital in this period). This simplifies the analysis and it is without loss of generality given that we are interested in the long run properties of the model. Notice that we are assuming that newly purchased capital can be used to produce in this period. This assumption is particularly convenient given the static nature of the model. Finally notice that the production function (13) implies that foreign and domestic capital are separate factors of production. If instead they were perfect substitutes, all capital would be produced just by the economy with the lowest capital price. In this sense the model where the two types of capital are perfect substitutes corresponds to the particular case of our economy when either α or β are exactly equal to zero (so that just one type of capital is used in production). One can easily check that results remain unchanged when considering this limit case.

To close the model we impose the condition that the trade balance has to be zero. This is consistent with the existence of an intertemporal budget constraint that usually states that the present discounted value of future trade surpluses has to be equal to the current value of foreign debt. Thus the following condition generally holds on average:

$$I^F + C^F = X. \tag{15}$$

This says that the value of imports is equal to exports, i.e. the trade balance is zero.

Maximization The problem of the representative household of the H economy can then be written as follows:

$$\begin{aligned}
&\max_{I^F, I^H, X} a \ln C^H + (1-a) \ln C^F - (1-a) \ln P_c^F \\
&s.t. \\
&C^H = Z \left(K^H\right)^\alpha \left(K^F\right)^\beta - I^H - X \\
&C^F = X - I^F
\end{aligned}$$

and where K^H and K^F are given by (14). By maximizing with respect to I^H we obtain

$$\alpha Y = I^H, \tag{16}$$

while by maximizing with respect to X yields

$$\frac{a}{C^H} = \frac{1-a}{C^F}. \tag{17}$$

Finally, by maximizing with respect to I^F we obtain:

$$\frac{a}{C^H} \cdot \frac{\beta Y}{K^F P_i^F} = \frac{1-a}{C^F},$$

which after using (17) yields

$$\beta Y = I^F. \tag{18}$$

Our decomposition By using (16) and (18) to substitute for K^H and K^F, we have that Y satisfies

$$Y = Z\left(\frac{\alpha Y}{P_i^H}\right)^{\alpha}\left(\frac{\beta Y}{P_i^F}\right)^{\beta}.$$

Now we take logs and we denote with small letters the log of the corresponding quantity in capital letters. After solving for y (which corresponds to the steady state value of the intertemporal model) we obtain:

$$y = cte + \frac{1}{1-\alpha-\beta}z - \frac{\alpha+\beta}{1-\alpha-\beta}\left[\frac{\alpha}{\alpha+\beta}p_i^H + \frac{\beta}{\alpha+\beta}p_i^F\right] \tag{19}$$

where $cte = \alpha\ln\alpha + \beta\ln\beta$ is a constant. Now notice that the term in square brackets is a weighted average of the relative price of equipment goods produced at home and abroad. The weights are the total value of capital as a share of domestic GDP. This should approximately be the index calculated by Gordon (1990) and extended by Cummins and Violante (2002), once this is deflated by using the GDP deflator rather than the CPI index. To be more formal one can note that the exact index for the price of investment good that would permit perfect aggregation in the model (see next section for more on this) would be

$$P_i = (\alpha+\beta)\left(\frac{P_i^H}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{P_i^F}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \tag{20}$$

so we can think that the Gordon's index for the investment specific technology in logs is

$$q = cte - \left[\frac{\alpha}{\alpha+\beta}p_i^H + \frac{\beta}{\alpha+\beta}p_i^F\right]$$

where cte is an appropriately defined constant. Thus by using the GDP deflator, we obtain that, in the long run, labor productivity is just explained by the evolution of q and z and it is equal to

$$y = cte + \frac{1}{1-\alpha-\beta}z - \frac{\alpha+\beta}{1-\alpha-\beta}q.$$

This justifies using the long run identifying restrictions imposed in the paper and our choice of the numeraire. The neutral technology shock that we identify is a shock that has permanent effects on z in the long-run.

The Fischer decomposition What would it have happened if we had deflated everything by the CPI index? Now notice that the exact index for the price of consumption good that would permit perfect aggregation in the model is

$$P_c = \left(\frac{P_c^H}{a}\right)^{a}\left(\frac{P_c^F}{1-a}\right)^{1-a} \tag{21}$$

(see next section for more on this). Thus it is reasonable to think of the log of the Consumer Price Index as equal to

$$p_c = cte + ap_c^H + (1-a)\,p_c^F$$

where again cte is an appropriately defined constant. Then we can define labor productivity deflated by the CPI index as equal to

$$y^c \equiv y + p_c^H - p_c = y + (1-a)\left(p_c^H - p_c^F\right).$$

By adding $(1-a)\left(p_c^H - p_c^F\right)$ to both sides of equation (19) we obtain that

$$\begin{aligned} y^c &= cte + \frac{1}{1-\alpha-\beta}z - \frac{\alpha+\beta}{1-\alpha-\beta}\left[\frac{\alpha}{\alpha+\beta}p_i^H + \frac{\beta}{\alpha+\beta}p_i^F + (1-a)\left(p_c^H - p_c^F\right)\right] \\ &\quad + \frac{1}{1-\alpha-\beta}(1-a)\left(p_c^H - p_c^F\right). \end{aligned} \tag{22}$$

Now notice that the term in square brackets is the price of investment goods relative to aggregate consumption (i.e. deflated by the CPI index). That is

$$q^c = cte - \left\{\frac{\alpha}{\alpha+\beta}\left[p_i^H + (1-a)\left(p_c^H - p_c^F\right)\right] + \frac{\beta}{\alpha+\beta}\left[p_i^F + (1-a)\left(p_c^H - p_c^F\right)\right]\right\}$$

is the original index produced by Gordon (1990) and extended by Cummins. and Violante (2002). Thus when both output and the relative price of investment are deflated by the CPI index we have that

$$y^c = cte + \frac{1}{1-\alpha-\beta}z + \frac{\alpha+\beta}{1-\alpha-\beta}q^c + \frac{1}{1-\alpha-\beta}(1-a)\left(p_c^H - p_c^F\right) \tag{23}$$

It is obvious from the last term in the expression that with this choice of the numeraire a permanent change in the price of domestic consumption relative to foreign consumption affects the long run level of labor productivity measured in CPI units—i.e. a change in $p_c^H - p_c^F$ affects y^c in the long run. This means that z and q^c are not the only long run determinants of labor productivity measured in consumption units. When we consider a VAR with the first difference of y^c and q^c, a neutral technology shock is a shock that has permanent effects on either z or the relative price of consumption goods. Thus permanent changes in the relative price of consumption goods will be identified as "neutral" technology shocks in a VAR with y^c and q^c.

The Altig et al. decomposition Altig et al. (2005) measure the price of investment relative to consumption and output using the GDP deflator. Then, after adding and subtracting $(1-a)\left(p_c^H - p_c^F\right)$ inside the square brackets of (19), we obtain that

$$y = cte + \frac{1}{1-\alpha-\beta}z + \frac{\alpha+\beta}{1-\alpha-\beta}q^c + \frac{\alpha+\beta}{1-\alpha-\beta}(1-a)\left(p_c^H - p_c^F\right) \tag{24}$$

Again a permanent change in $p_c^H - p_c^F$ has long run effects on y. This means that z and q^c are not the only long run determinants of labor productivity measured by using the output deflator. When we consider a VAR with the first difference of y and q^c, a neutral technology shock is a shock that has permanent effects on either z or the relative price of consumption goods. Thus permanent changes in the relative price of consumption goods will be identified as "neutral" technology shocks in a VAR with y and q^c.

In the light of this result I would say that using the GDP deflator is the appropriate choice.

What determines the relative price of consumption goods? Of course the relative price of domestic and foreign consumption goods is endogenous. So it may be moved by the two technology shocks. If these are the only long-run determinants of the relative price, there would be nothing wrong in using a VAR with (the first difference of) y^c and q^c, rather than one with y and q. We next show that when we endogenize the relative price of consumption goods by imposing market clearing in the international market for consumption goods, $p_c^H - p_c^F$ is affected by the ratio of the neutral technology of the H economy to the neutral technology of the F economy. Thus changes in the neutral technology of the F economy that are not accompanied by an equal change in the neutral technology of the H economy are identified as a neutral technology shock when considering a VAR with y^c and q^c, while this would not be the case in a VAR with y and q. Arguably the interpretation of a neutral technology shock in the two alternative VARs is somewhat different.

To see this, assume that the F economy is characterized by the same preferences, and the same technology as the H economy. That is equation (11), (13), and (14) remain valid for the F economy as well. This means that the representative consumer of the F economy will maximize

$$U = a \ln C^{*H} + (1-a) \ln \frac{C^{*F}}{P_c^F} \tag{25}$$

where C^{*H} and C^{*F} denotes the consumption expenditures of the representative consumer of the F economy in the consumption goods produced by the H economy and the F economy, respectively. P_c^F is the price (in domestic consumption units) of the consumption good produced by the F economy. Notice that we use the convention that the superscript '*' denotes the analogue for the F economy of the previously defined quantities for the H economy. Output of the F economy is produced according to the constant-return to scale Cobb-Douglas production function:

$$Y = Z^* \left(K^{*H}\right)^{\alpha} \left(K^{*F}\right)^{\beta} \tag{26}$$

where Z^* is the neutral technology of the F economy, while K^{*H} and K^{*F} are the stock of capital of the F economy produced by the H and F economy, respectively. Notice that again the work force is normalized to one ($L^{*1-\alpha-\beta} = 1$). Thus Y^* also denotes labor productivity. The law of evolution of capital is

$$K^{*j} = \frac{I^{*j}}{P_i^j}, \qquad j = H, F \tag{27}$$

The analogous of constraint (12) and (15) for the foreign economy will be

$$P_c^F Y^* = I^{*F} + C^{*F} + X^* \tag{28}$$

$$I^{*H} + C^{*H} = X^*. \tag{29}$$

The first equation says that the value of production of the F economy is equal to the value of its uses. The second that the trade balance of the F economy is equal to zero.

Maximization in the F economy The problem of the representative household of the F economy can then be written as follows:

$$
\begin{aligned}
&\max_{I^{*F}, I^{*H}, X^{*}} a \ln C^{*H} + (1-a) \ln C^{*F} - (1-a) \ln P_c^F \\
&s.t. \\
&C^{*H} = P_c^F Z^{*} \left(K^{*H}\right)^{\alpha} \left(K^{*F}\right)^{\beta} - I^{*H} - X^{*} \\
&C^{*F} = X^{*} - I^{*F}
\end{aligned}
$$

and where K^{*H} and K^{*F} are given by (27). By maximizing with respect to I^{*H} we obtain

$$\alpha P_c^F Y^{*} = I^{*H}, \tag{30}$$

while by maximizing with respect to X^{*} yields

$$\frac{a}{C^{*H}} = \frac{1-a}{C^{*F}}. \tag{31}$$

Finally, by maximizing with respect to I^{*F} we obtain that

$$\frac{a}{C^{*H}} \cdot \frac{\beta Y}{K^{*F} P_i^F} = \frac{1-a}{C^{*F}},$$

which after using (31) yields

$$\beta P_c^F Y^{*} = I^{*F}. \tag{32}$$

Market clearing in the world economy We now use (29) to substitute for X^{*} in (28). Then we use (30), (31), and (32) to substitute for I^{*H}, C^{*H}, and I^{*F}, respectively. After some algebra we obtain that

$$C^{*F} = (1-a)(1-\alpha-\beta) P_c^F Y^{*} \tag{33}$$

By proceeding analogously with the constraints (12) and (15) of the H economy and the associated first order conditions (16), (17), and (18), we also have that

$$C^{F} = (1-a)(1-\alpha-\beta) Y. \tag{34}$$

Market clearing in the market for the goods produced by the F economy implies that

$$P_c^F Y^{*} = C^{*F} + I^{*F} + C^{F} + I^{F}$$

which says that the total production of the F economy is equal to the total demand (either for consumption or investment purposes) by the F and the H economy. We can then use (33), (32), (34), and (18) to substitute for C^{*F}, I^{*F}, C^{F} and I^{F}, respectively. Manipulating the resulting expression yields

$$P_c^F = \frac{[(1-a)(1-\alpha-\beta)+\beta] Y}{[1-\beta-(1-a)(1-\alpha-\beta)] Y^{*}}. \tag{35}$$

After taking logs, using (19) and its analogous for the F economy to substitute for y and y^*,we finally obtain that

$$p_c^F - p_c^H = cte + \frac{1}{1-\alpha-\beta}(z - z^*) \tag{36}$$

where cte is an appropriately defined constant (equal to the log of the constant term in 35). In a model where L^* was not normalized to one also the log difference between L and L^* would affect the relative price of consumption goods. Equation (36) shows that the price of consumption goods produced by the F economy is greater when its demand is also greater. This tends to be the case when the H economy becomes relatively more technologically advanced and thereby richer.

The intertemporal model To get reassured about the previous results, one can consider a fully specified intertemporal model. I do not think that this part is necessary, but maybe it is useful to us. The notation, the specification of technology and preferences are exactly as in the static previously described model. Now however the representative consumer has a discount factor $\rho \in (0,1)$—so strictly greater than zero. We also replace the constraints (12) and (15) with the more traditional resource constraint, that characterize an economy with perfect capital mobility:

$$B' = (1+r)B + \left(Y - C^H - C^F - I^H - I^F\right) \tag{37}$$

where B and B' are the net holdings of foreign assets of the representative consumer in the current and future period respectively. r is the real interest rate available in international financial markets, Y is domestic GDP and I^H and I^F are investment expenditures in domestic goods and foreign goods, respectively. The law of motion of the two types of capital is given by

$$K^j = (1-\delta)K_{-1}^j + \frac{I^j}{P_i^j}, \quad j = H,\ F$$

while GDP satisfies (13). Notice that the combination of a standard No-Ponzi condition and a transversality condition imply that the problem is also subject to the standard intertemporal constraint:

$$B_t = \frac{1}{1+r}\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s NX_{t+s}$$

where $NX_{t+s} \equiv Y_t - C_t^H - C_t^F - I_t^H - I_t^F$ is the trade balance at time $t+s$.

Perfect Aggregation One can easily check that the maximization problem implies that both the relative consumption of domestic and foreign goods and the relative value of foreign and domestic capital remain constant over time. More specifically:

$$\frac{C^H}{C^F} = \frac{a}{1-a}$$

and

$$\frac{K^H P_i^H}{K^F P_i^F} = \frac{\alpha}{\beta}$$

that were also two properties of the static model. This allows to simplify the problem of the representative household as follows:

$$\max E\left(\sum \rho^s \ln C_s\right)$$
$$s.t.$$
$$B' = (1+r)\,B + (Y - P_c C - P_i I) \tag{38}$$
$$K = (1-\delta)K_{-1} + \frac{I}{P_i} \tag{39}$$

where Y is given by (13) while "aggregate" consumption, investment, and capital are defined as equal to

$$\begin{aligned} C &= \left(C^H\right)^a \left(C^F\right)^{1-a}, \\ I &= \left(I^H\right)^{\frac{\alpha}{\alpha+\beta}} \left(I^F\right)^{\frac{\beta}{\alpha+\beta}}, \end{aligned}$$

and

$$K = \left(K^H\right)^{\frac{\alpha}{\alpha+\beta}} \left(K^F\right)^{\frac{\beta}{\alpha+\beta}},$$

respectively. The price of consumption and investment are P_c and P_i which are given by (21) and (20), respectively. Notice that our choice of the numeraire imposes that $P_c^H = 1$ in (21).

One can then consider the Bellman equation associated with this problem. This would read:

$$V(B, K_{-1}) = \max_{B',K} \ln\left\{(1+r)B + ZK^{\alpha+\beta} - B' - P_i\left[K - (1-\delta)K_{-1}\right]\right\} + \beta E\left[V(B',K)\right]$$

where we have set B' and K as the relevant control variables by using the aggregate resource constraint (38) and the capital accumulation (39) to express C and I as a function just of B', B, K and K_{-1}.

The envelope conditions with respect to B and K_{-1} are:

$$\begin{aligned} V_1 &= \frac{1+r}{C} \\ V_2 &= \frac{(1+\delta)P_i}{C}. \end{aligned}$$

The first order conditions with respect to B' and K, after using the two previous envelope conditions can be expressed as

$$\begin{aligned} \frac{1}{C} &= \beta E\left(\frac{1+r}{C'}\right) \\ \frac{(\alpha+\beta)Y - P_i K}{C} &= \beta(1-\delta)KE\left(\frac{P_i'}{C'}\right) \end{aligned}$$

where a "$'$" always indicates the value of the corresponding variable in the next period. One can then use the first above condition to simplify the second and solve for K. This yields

$$K = \frac{(\alpha+\beta)Y}{P_i\left[1 + \frac{(1-\delta)E(1+g_i')}{1+r}\right]}$$

where $E(1+g_i')$ is the expected future growth rate of the relative price of investment. One can then use this expression for K to substitute into (13). After taking and solving for y, we finally obtain a representation for y analogous to (19) which reads:

$$y = cte + \frac{1}{1-\alpha-\beta} z - \frac{\alpha+\beta}{1-\alpha-\beta} p_i + v \tag{40}$$

where cte is an appropriately defined constant while v is a stationary error that arises because the conditional expected value of the rate of growth of the price of investment can fluctuate over time (say because p_i is not a random walk). One can then proceed as in the previous section to derive the analogous of (23) and (24) in the intertemporal version for the static model. This confirms the conclusions reached by using the static model.

What determines the relative price of consumption goods? One could proceed as in the static model and endogenize the relative price of consumption goods. Again one would find that the neutral technology of the F economy relative to the H economy, $z^* - z$, would be a key determinant of the long run value of the relative price of consumption goods, $p_c^H - p_c^F$.

B Derivation of equilibrium conditions

In this appendix we derive the equilibrium conditions of the model discussed in Section 7. Before proceeding note that the distribution of old jobs f_t evolves as

$$f_t(\tau) = (1\text{-}\lambda)\left[\int_{-\infty}^{\tau^*_{t\text{-}1}} g_\epsilon(i + \mu_z + \varepsilon_{z,t}\text{-}\tau) f_{t\text{-}1}(i)di + g_\epsilon(\mu_z + \varepsilon_{z,t}\text{-}\tau)n_{t\text{-}2}\right],\ \forall\tau \in \mathbb{R}$$

where g_ϵ denotes the density function of the idiosyncratic shock ϵ, which is symmetric around zero. To understand the expression, consider the sequence of events that characterize the evolution of the distribution of old jobs between time $t-1$ and t. At time $t-1$, some old jobs are destroyed while others with technological gaps less than τ^*_{t-1} remain in operation and produce. To obtain the distribution of old jobs at time t one has to take account i) of the aggregate and idiosyncratic shocks to the job neutral technology that determine the job technological gap, ii) of the probability that jobs are exogenously destroyed and iii) of the inflow of new jobs that start producing at time $t-1$, n_{t-2}, and that will belong to the pool of old jobs at time t. To understand the term in the integral consider a job, which, at time $t-1$, produces with technological gap i. Then, this job will end up with a technological gap τ at the beginning of time t, only if it is not exogenously destroyed and the realization of the idiosyncratic shock ϵ is equal to $\tau - i - \mu_z - \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is the aggregate shock to the leading neutral technology. Then the measure of jobs with technological gap τ at time t is obtained by integrating over all possible values of technological gap i, which do not lead to job destruction at time $t-1$. Now we can solve for the equilibrium conditions of the model by writing the Bellman equation for the social planner problem. Let $\tilde{K}$, f, n_{-1}, z, and q denote the current capital stock, the beginning of period distribution, the measure of jobs that start producing in this period, the leading edge neutral technology and the investment specific technology, respectively. The social planner problem of our economy can then be written as follows

$$\begin{aligned}\tilde{W}(\tilde{K}, f,\ n_{-1}, z, q) &= \max_{\tilde{C},n,\tau^*} \ln\tilde{C} - c_w(1-u) - cu^{-\eta_0}n^{\eta_1} \\ &\quad +\beta E\left[\tilde{W}(\tilde{K}', f', n,\ z', q')\right] \end{aligned} \tag{41}$$

which is subject to the following set of transition equations

$$\begin{aligned}\tilde{K}' &= (1-\delta)\tilde{K} + e^q\left(\tilde{K}^\alpha\tilde{H}^{1-\alpha} - \tilde{C}\right), \\ f'(\tau) &= (1-\lambda)\left[\int_{-\infty}^{\tau^*} g_\epsilon(i+\mu_z+\varepsilon'_z-\tau)f(i)di + g_\epsilon(\mu_z+\varepsilon'_z-\tau)n_{-1}\right] \\ q' &= \mu_q + q + \varepsilon'_q, \\ z' &= \mu_z + z + \varepsilon'_z,\end{aligned}$$

and to the two identities:

$$\tilde{H} = \int_{-\infty}^{\tau^*} e^{\frac{z-\tau}{1-\alpha}} f(\tau)d\tau + e^{\frac{z}{1-\alpha}}n_{-1}, \tag{42}$$

$$u = 1 - \int_{-\infty}^{\tau^*} f(\tau)d\tau - n_{-1}. \tag{43}$$

B.1 The Euler equation for consumption

By deriving with respect to $\tilde{C}$ in (41), after taking into account (42) and (43) we obtain:

$$\frac{1}{\tilde{C}} = \beta e^{q} E\left(\tilde{W}'_K\right) \tag{44}$$

where $\tilde{W}'_K$ denote the partial derivative of the value function of next period with respect to capital. The envelope condition with respect to capital reads as:

$$\tilde{W}_K = \beta E\left(\tilde{W}'_K\right)\left[(1-\delta) + e^{q}\alpha\left(\frac{\tilde{H}}{\tilde{K}}\right)^{1-\alpha}\right]$$

that, after using (44) to replace $\beta E\left(\tilde{W}'_1\right)$, can be expressed as

$$\tilde{W}_K = \frac{1}{\tilde{C}}\left[(1-\delta)e^{-q} + \alpha\left(\frac{\tilde{H}}{\tilde{K}}\right)^{1-\alpha}\right].$$

After evaluating this derivative in the next period we obtain

$$\tilde{W}'_K = \frac{1}{\tilde{C}'}\left[(1-\delta)e^{-q'} + \alpha\left(\frac{\tilde{H}'}{\tilde{K}'}\right)^{1-\alpha}\right] \tag{45}$$

which substituted into (44) yields

$$\frac{1}{\tilde{C}} = \beta E\left(\frac{1}{\tilde{C}'}\left[(1-\delta)e^{q-q'} + e^{q}\alpha\left(\frac{\tilde{H}'}{\tilde{K}'}\right)^{1-\alpha}\right]\right). \tag{46}$$

B.2 Destruction

To calculate the first order condition with respect to τ^* notice that by deriving with respect to τ^* in (42) and (43) we obtain:

$$\begin{aligned} \frac{\partial u}{\partial \tau^*} &= -f(\tau^*), \\ \frac{\partial H}{\partial \tau^*} &= e^{\frac{z-\tau^*}{1-\alpha}} f(\tau^*). \end{aligned}$$

After using these two results, deriving with respect to τ^* in (41) yields:

$$(1-\alpha)\left(\frac{\tilde{K}}{\tilde{H}}\right)^{\alpha} e^{\frac{z-\tau^*}{1-\alpha}}\frac{1}{\tilde{C}} - c_w - c\eta_0 u^{-\eta_0-1} n^{\eta_1} + J_t(\tau^*) = 0 \tag{47}$$

where

$$J(i) \equiv \beta (1-\lambda) E_t \left[\int_{-\infty}^{\tau^{*\prime}} V'(j) g_\epsilon(i + \mu_z + \varepsilon_z' - j) dj \right].$$

Notice that in writing the condition we made use of (44) to replace $\beta E \left(\tilde{W}_1' \right)$. If we denote by $V'(i) \equiv \tilde{W}_{f(i)}'$ the net social value of a job with technological distance i in the next period, the envelope condition allows to write

$$V(i) = (1-\alpha) \left(\frac{\tilde{K}}{\tilde{H}} \right)^{\alpha} e^{\frac{z-i}{1-\alpha}} \frac{1}{\tilde{C}} - c_w - c\eta_0 u^{-\eta_0 - 1} n^{\eta_1} + J(i)$$

With this notation (47) can simply be expressed as

$$V(\tau^*) = 0.$$

B.3 Creation

The first order condition with respect to n reads as follows:

$$c\eta_1 u^{-\eta_0} n^{\eta_1 - 1} = \beta E \left(V'(0) \right) \tag{48}$$

where $V'(0) \equiv \tilde{W}_{n_{-1}}'$ is the next period value of a job that produces with technological gap zero (i.e. a newly created job). This is equal to the partial derivative of the value function of next period with respect to the measure of newly created jobs. The envelope condition allows to write

$$V(0) = (1-\alpha) \left(\frac{\tilde{K}}{\tilde{H}} \right)^{\alpha} e^{\frac{z}{1-\alpha}} \frac{1}{\tilde{C}} - c_w - c\eta_0 u^{-\eta_0 - 1} n^{\eta_1} + J(0).$$

Equation (48) is equivalent to (10) in the paper.

B.4 Equilibrium definition

One can easily check that the economy evolves around the stochastic trend given by

$$X \equiv e^{\frac{z}{1-\alpha}} \; e^{\frac{\alpha q}{1-\alpha}}$$

To make the environment stationary we defined the following scaled quantities:

$$K_t \equiv \frac{\tilde{K}_t}{e^{\frac{z_t + q_t}{1-\alpha}}}, \;\; H_t = \frac{\tilde{H}_t}{e^{\frac{z_t}{1-\alpha}}}, \;\; \text{and} \;\; C_t \equiv \frac{\tilde{C}_t}{e^{\frac{z_t}{1-\alpha}} \; e^{\frac{\alpha q_t}{1-\alpha}}}.$$

Then an equilibrium consists of a stationary tuple

$$(K_t, u_t, H_t, f_t, C_t, V_t, n_t, \tau_t^*, \Delta z_t, \Delta q_t),$$

where f_t and V_t are functions of technological gap while the remaining quantities are scalar, that satisfies the following conditions:

1. The law of motion of capital:

$$K_t = (1-\delta)K_{t-1}e^{-\frac{\mu_z+\mu_q+\varepsilon_{z,t}+\varepsilon_{q,t}}{1-\alpha}} + \left(K_{t-1}^{\alpha}H_{t-1}^{1-\alpha} - C_{t-1}\right)e^{-\frac{\mu_z+\mu_q+\varepsilon_{z,t}+\varepsilon_{q,t}}{1-\alpha}} \tag{49}$$

2. The definition of unemployment:

$$u_t = 1 - \int_{-\infty}^{\tau_t^*} f_t(\tau)d\tau - n_{t-1} \tag{50}$$

3. The definition of efficiency units of labor:

$$H_t = \int_{-\infty}^{\tau_t^*} e^{\frac{-\tau}{1-\alpha}} f_t(\tau)d\tau + n_{t-1} \tag{51}$$

4. The law motion of the distribution of technological gaps of old jobs:

$$f_t(\tau) = (1\text{-}\lambda)\left[\int_{\text{-}\infty}^{\tau_{t\text{-}1}^*} g_\epsilon(i+\mu_z+\varepsilon_{z,t}\text{-}\tau)f_{t\text{-}1}(i)di + g_\epsilon(\mu_z+\varepsilon_{z,t}\text{-}\tau)n_{t\text{-}2}\right] \tag{52}$$

5. The Euler equation for consumption:

$$\frac{1}{C_t} = \beta E\left\{\frac{1}{C_{t+1}\, e^{\frac{\mu_z+\mu_q+\varepsilon_{z,t+1}+\varepsilon_{q,t+1}}{1-\alpha}}}\left[(1-\delta)+\alpha\left(\frac{H_{t+1}}{K_{t+1}}\right)^{1-\alpha}\right]\right\} \tag{53}$$

6. The marginal value of jobs at any given technological distance $\tau \leq \tau_t^*$:

$$V_t(\tau) = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} e^{\frac{-\tau}{1-\alpha}}\frac{1}{C_t} - c_w - c\eta_0 u_t^{-\eta_0-1}n_t^{\eta_1} + J_t(\tau) \tag{54}$$

where

$$J_t(\tau) \equiv \beta\,(1-\lambda)\,E_t\left[\int_{-\infty}^{\tau_{t+1}^*} V_{t+1}(i)g_\epsilon(\tau+\mu_z+\varepsilon_{z,t+1}-i)di\right]$$

7. The optimal job destruction decision: $V_t(\tau_t^*) = 0$ which can be also expressed as

$$(1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} e^{\frac{-\tau_t^*}{1-\alpha}}\frac{1}{C_t} - c_w - c\eta_0 u_t^{-\eta_0-1}n_t^{\eta_1} + J_t(\tau_t^*) = 0 \tag{55}$$

8. The optimal number of newly created jobs:

$$c_1 \eta_1 u_t^{-\eta_0} n_t^{\eta_1 - 1} = \beta E\left(V_{t+1}(0)\right)$$

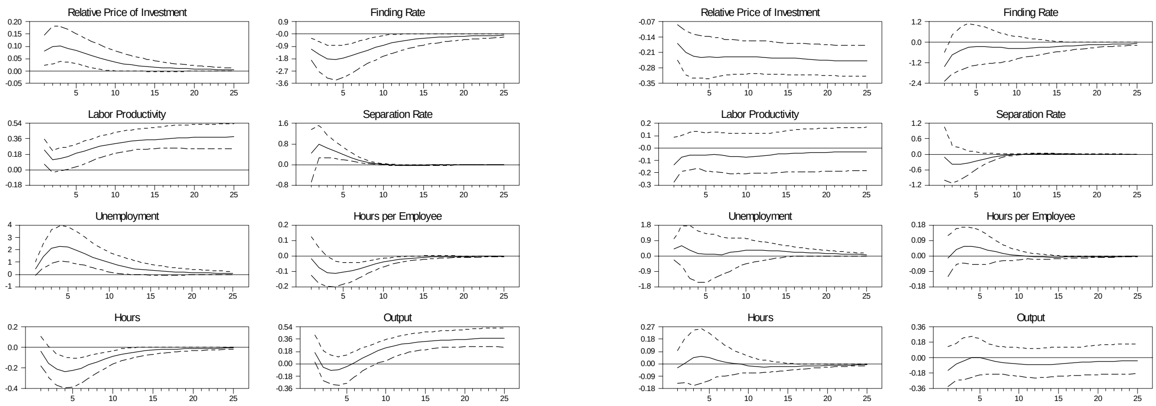
9. The laws of motion of z_t and q_t :

$$\begin{aligned} \Delta z_t &= \mu_z + \varepsilon_{z,t} \\ \Delta q_t &= \mu_q + \varepsilon_{q,t} \end{aligned}$$

where $\varepsilon_{z,t}$ and $\varepsilon_{q,t}$ are iid over time.

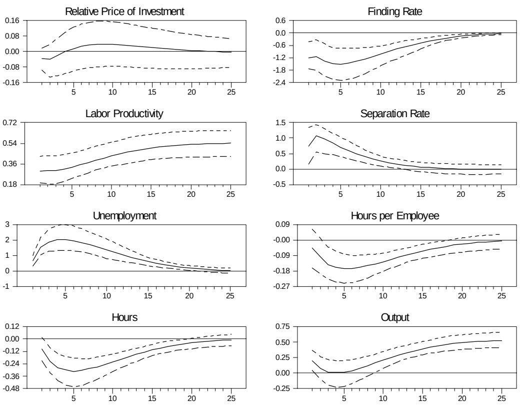
C Additional empirical results

In this appendix we report some figures that are discussed in the main text. They are reported here just for completeness.

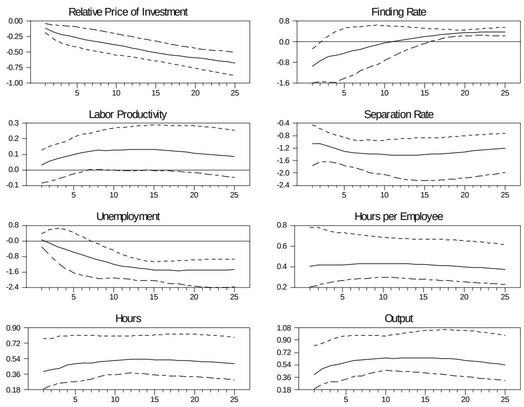


(a) Neutral technology shock (b) investment specific technology shock

Figure 11: The sample period is 1955:I-1973:I. The VAR has eight lags and contains six variables: the rate of growth of the relative price of investment, t he rate of growth of labour productivity, the (logged) job finding rate, the (logged) job separation rate, the (logged), unemployment rate (logged), and the (logged) aggregate number of hours worked per capita. Dotted lines represent the 5% and 95% quantiles of the distribution of the responses simulated by bootstrapping 500 times the residuals of the VAR. The continuous line corresponds to median estimate from bootstrap replications.

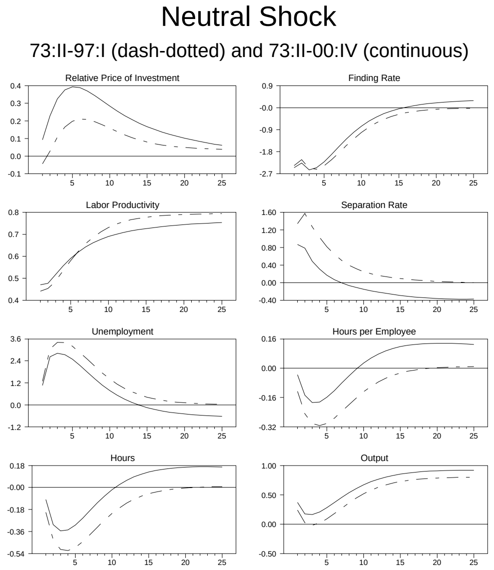


(a) Neutral technology shock

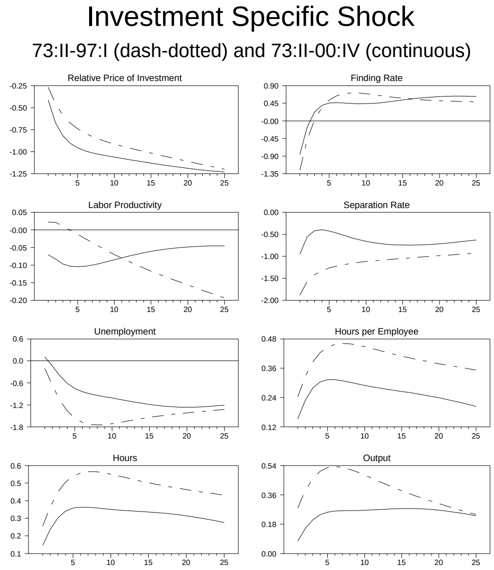


(b) Investment specific technology shock

Figure 12: The sample period is 1973:II-1997:II. The VAR has eight lags and contains six variables: the rate of growth of the relative price of investment, the rate of growth of labour productivity, the (logged) job finding rate, the (logged) job separation rate, the (logged), unemployment rate (logged), and the (logged) aggregate number of hours worked per capita. Dotted lines represent the 5% and 95% quantiles of the distribution of the responses simulated by bootstrapping 500 times the residuals of the VAR. The continuous line corresponds to median estimate from bootstrap replications.

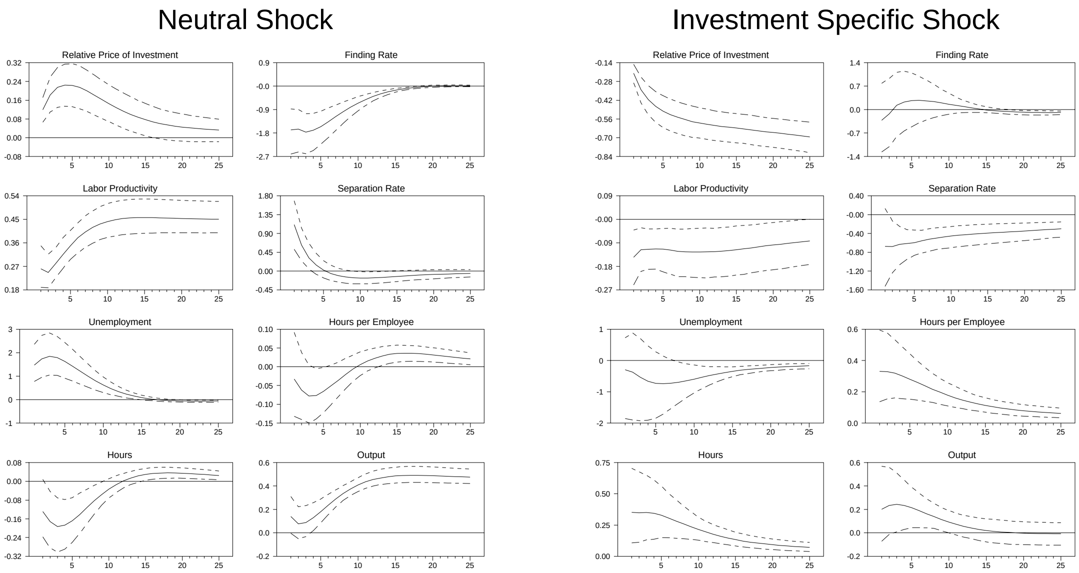


(a) Neutral technology shock



(b) Investment specific technology shock

Figure 13: Response to a neutral or an investment-specific technology shock in two different subperiods: 1973:II-1997:I, and 1973:II-2000:IV. The VAR has 8 lags and six variables: the rate of growth of the relative price of investment, the rate of growth of labour productivity, the (logged) unemployment rate, and the (logged) aggregate number of hours worked per capita, the log of separation and finding rates. The continuous line correponds to the 1973:II-2000:IV period, and the dash-dotted line to the 1973:II-1997:II period. Impulse responses correspond to point estimates.



(a) Neutral technology shock

(b) Investment specific technology shock

Figure 14: Response to a neutral or an investment-specific technology shock in a nine variables VAR with approximated rates. 1955:I-2000:IV sample with intercept deterministically broken at 1973:II and 1997:I. Dotted lines represent the 5% and 95% quantiles of the distribution of the responses simulated by bootstrapping 500 times the residuals of the VAR. The continuous line corresponds to median estimate.

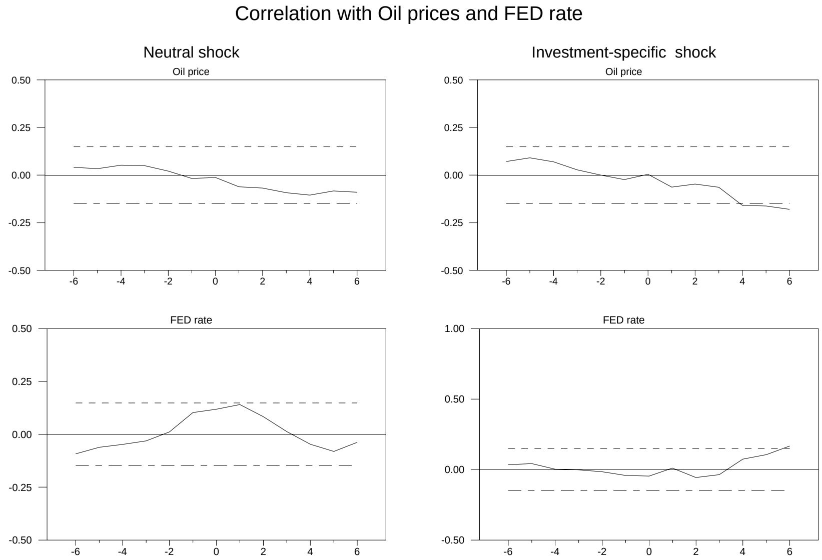
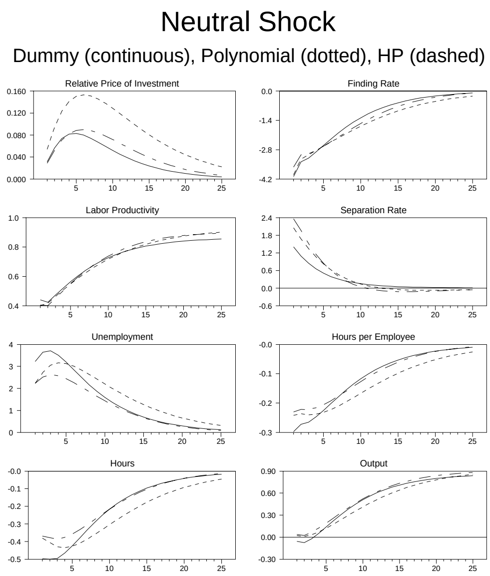
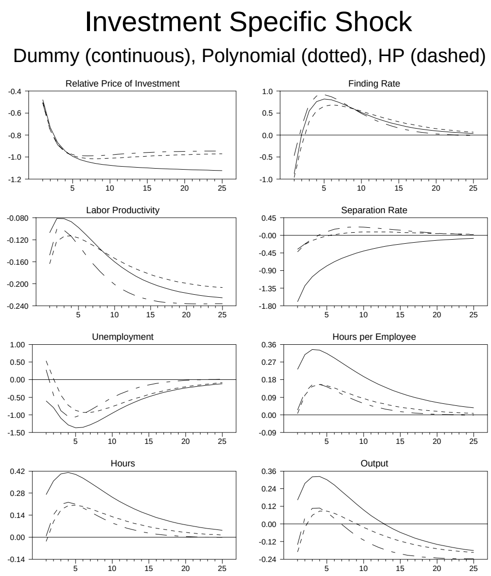


Figure 15: Left column corresponds to neutral technology shocks; right column to investment specific technology shocks. The first row plots the correlation of the corresponding technology shock with relative oil price shocks (i.e. relative to consumption). The second row with Federal fund rate shocks at different time horizons. The shocks are estimated from the nine variables VAR, approximated rates, full sample with deterministic dummies. The horizzontal lines correspond to an asymptotic 95 percent confidence interval centered around zero.

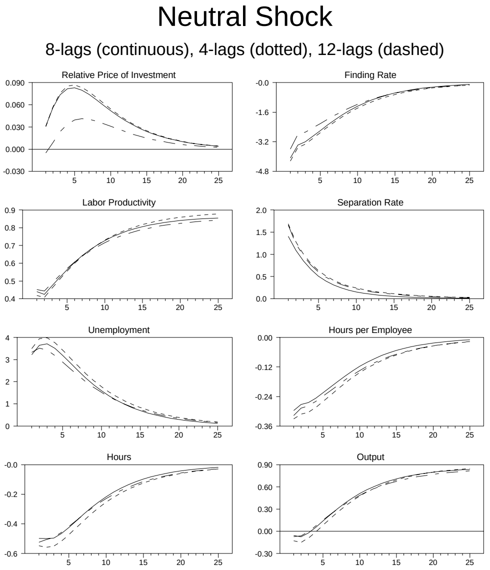


(a) Neutral technology shock

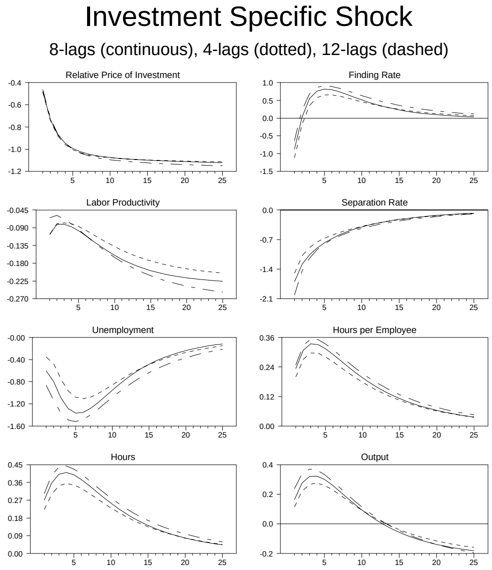


(b) Investment specific technology shock

Figure 16: The continous line corresponds to dummy specification, the dotted line to the case where the intercept is a 3rd order polynomial in time. The dashed lines are the responses after detrending the original series with an Hodrick Prescott filter with smoothing parameter $\lambda = 10000$. VAR with approximated rates, with 8 lags, and six variables. Plotted impulse responses correspond to point estimates.

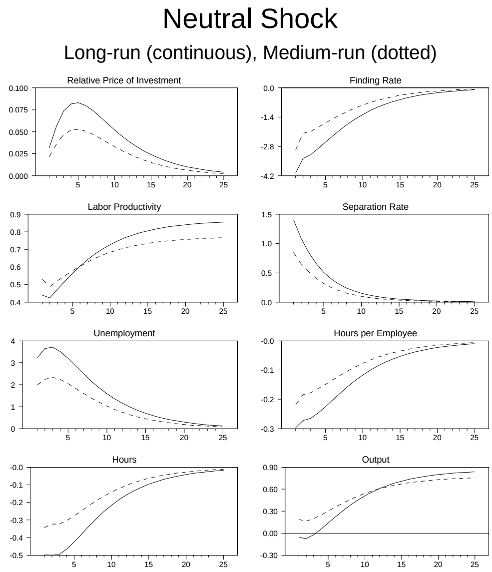


(a) Neutral technology shock

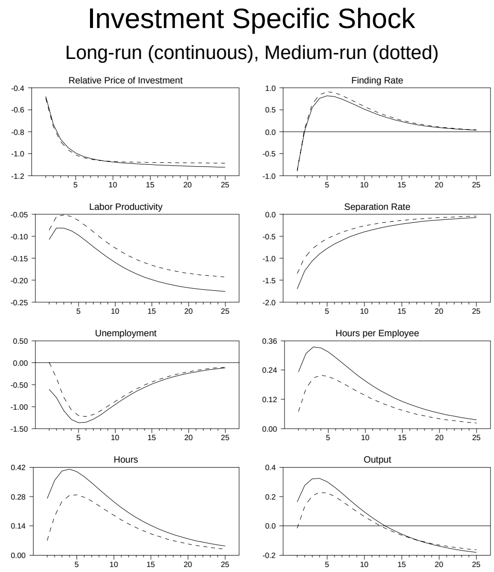


(b) Investment specific technology shock

Figure 17: Dummy specification with different lags in the VAR: continous line corresponds to 8 lags, dotted line to 4 lags, dashed line to 12 lags. VAR with approximated rates, with 8 lags, and six variables. Plotted impulse responses correspond to point estimates.

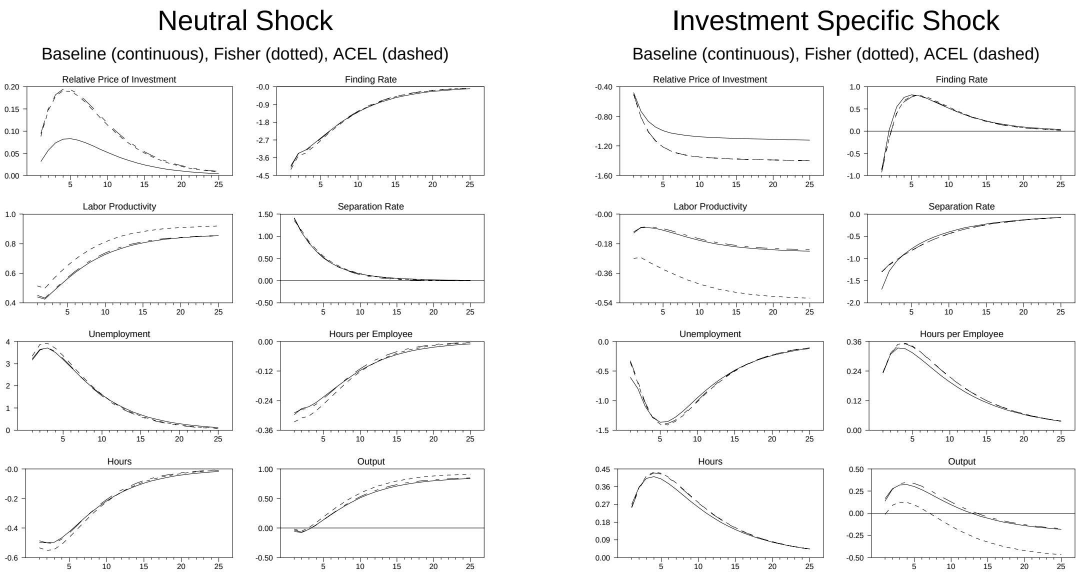


(a) Neutral technology shock



(b) Investment specific technology shock

Figure 18: Dummy specification with identifying restrictions imposed at different time horizons: continous line corresponds to long run restriction, dotted line corresponds to the specification where restrictions are imposed at an horizon of 3 years. VAR with approximated rates, with 8 lags, and six variables. Plotted impulse responses correspond to point estimates.



(a) Neutral technology shock

(b) Investment specific technology shock

Figure 19: Results from VAR in the dummy specification when the variables in VAR are deflated with a different price index: continuous line corresponds to baseline specification, dotted line corresponds to the VAR where output and price of investment are deflated by using the CPI index, the dashed line corresponds to the case where output is deflated with the output deflator and the price of investment with the CPI index. VAR with approximated rates, with 8 lags, and six variables. Plotted impulse responses correspond to point estimates.