Derivations of the conditions used in RBC1.m

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Consider a simple RBC model where output is produced with capital, labor of the form

$$\max_{c_t,k_{t+1},N_t} \sum_t \beta^t \left(\frac{c_t^{1-\eta}}{1-\eta} - AN_t\right) \tag{1}$$

subject to

$$c_t + k_{t+1} = \zeta_t {K_t}^{\rho} N_t^{1-\rho} + (1-\delta)k_t \tag{2}$$

The FOC of the households are given by:

$$c_t^{-\eta} = \lambda_t$$

$$A = c_t^{-\eta}(1-\alpha)\zeta_t(K_t)^{\rho}N_t^{-\rho} = \frac{y_t}{c_t^{\eta}N_t}(1-\rho)$$

$$c_t^{-\eta} = \beta E_t c_{t+1}^{-\eta}\left\{\rho\zeta_{t+1}{K_{t+1}}^{\rho-1}N_{t+1}^{1-\rho} + (1-\delta)\right\} = \beta E_t c_{t+1}^{-\eta}\left\{\rho\frac{y_{t+1}}{K_{t+1}} + (1-\delta)\right\}$$

$$\zeta_t K_t^{\rho} N_t^{1-\rho} = y_t$$

The steady state of the model is:

$$Ac^{\eta} = (1-\rho)y/N$$

$$1 = \beta(\rho y/K + (1-\delta))$$

$$c + \delta K = y$$

$$y = \bar{\zeta}K^{\rho}N^{1-\rho}$$

Fix $\bar{R}, \rho, \delta, \eta, \bar{\zeta}$. The steady state values are

$$\beta = 1/\bar{R}$$

$$\bar{y/K} = [\frac{1}{\beta} - (1-\delta)]/\rho = (\bar{R} - 1 + \delta)/\rho$$

$$\bar{c/y} = 1 - \delta \bar{K/y}$$

$$\bar{N/y} = [1/\bar{\zeta}(\bar{y/K})^{\rho}]^{1/(1-\rho)}$$

$$i/\bar{K} = \delta$$

$$\bar{c} = (y/\bar{N} \frac{1-\rho}{A})^{1/\eta}$$

The log linearized first order conditions can be written as

$$\begin{aligned}
0 &= -\bar{i}\ i(t) - \bar{c}\ c(t) + \bar{y}\ y(t) && (3) \\
0 &= \bar{i}\ i(t) - \bar{K}\ k(t) + (1-\delta)\ \bar{K}\ k(t-1) && (4) \\
0 &= \rho\ k(t-1) - y(t) + (1-\rho)\ N(t) + \zeta(t) && (5) \\
0 &= -\eta\ c(t) + y(t) - N(t) && (6) \\
0 &= -\rho\ \bar{y}\ k(t-1) + \rho\ \bar{y}\ y(t) - \bar{K}\ \bar{R}\ r(t) && (7) \\
0 &= E_t[-\eta\ c(t+1) + r(t+1) + \eta\ c(t)] && (8) \\
\zeta(t+1) &= \psi\ \zeta(t) + \epsilon(t+1) && (9)
\end{aligned}$$

This is a system of 7 equations in 7 variables. The exogenous shock is ζ_t the state is k_t, the control variables are c_t, i_t, y_t, N_t, r_t. There are five static equation, one expectational equation and the law of motion of the technological disturbance.

The model in Uhlig's toolkit format is in rbc1.m