Computer Exercises: Solution to a RBC model with capacity utilization
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Exercise 1:

Consider a simple RBC model where output is produced with three inputs, capital, labor and utilization. The representative agent problem is:

$$\max_{c_t,k_{t+1},N_t,u_t} E_0 \sum_t \beta^t (\log c_t - AN_t) \tag{1}$$

subject to

$$\begin{aligned} c_t + i_t &= y_t && (2)\\ K_{t+1} - (1-\delta_1 u_t^{\delta_2})K_t &= i_t && (3)\\ \zeta_t (K_t u_t)^\alpha N_t^{1-\alpha} &= y_t && (4) \end{aligned}$$

This formulation implies that the more capital is utilized, the faster it depreciates.

The FOC of the households are:

$$A = \frac{y_t}{c_t N_t}(1-\alpha)$$

$$\alpha \frac{Y_t}{K_t} = \delta_1 \delta_2 u_t^{\delta_2}$$

$$\frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}} r_{t+1}$$

$$r_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta_1 u_{t+1}^{\delta_2})$$

a) Calculate the steady state values of $c, y, k, i, n, r.u$ as a function of $A, \beta, \alpha, \delta_1, \delta_2$.

Solution: The steady state relationship are given by

$$\begin{aligned} c + i &= y && (5)\\ \delta_1 u^{\delta_2} K &= i && (6)\\ \zeta (Ku)^\alpha N^{1-\alpha} &= y && (7)\\ \frac{y}{cN}(1-\alpha) &= A && (8)\\ \alpha \frac{Y}{K} &= \delta_1 \delta_2 u^{\delta_2} && (9)\\ \beta r &= 1 && (10)\\ \alpha \frac{Y}{K} + (1-\delta_1 u^{\delta_2}) &= r && (11) \end{aligned}$$

The sixth equation implies $\beta = 1/r$, the fifth and the last one imply $\bar{u} = [(r-1)/(\delta_1(\delta_2 - 1))]^{1/\delta_2}$. Then the fifth implies $(\overline{Y/K}) = \delta_1\delta_2\bar{u}^{\delta_2}/\alpha$ and the second implies $(\overline{i/K}) = \delta_1\delta_2\bar{u}^{\delta_2}$. From the third we have $(\overline{N/Y}) = [(1/\bar{\zeta})*((\overline{Y/K})/\bar{u})^{\alpha}]^{1/(1-\alpha)}$ and from the fourth one $\bar{c} = ((1-\alpha)/A)*(\overline{Y/N})$.

From these we can get $((\overline{i/Y})$ using the solution for $(\overline{Y/K})$ and $(\overline{i/K})$. $(\overline{c/y})$ can be obtained from the first equation. Then from the solution for $\bar{c}$ and the above we can get $\bar{y}$ and from these $\bar{K}, \bar{i}, \bar{N}$. and this completes the computation of the steady states.

b) Log linearize the four equilibrium conditions and the three constraints around the steady state

The log-linearized conditions are:

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\begin{aligned}
\bar{y}y_t &= \bar{c}c_t + \bar{i}i_t && (12)\\
y_t &= \zeta_t + \alpha(k_t + u_t) + (1-\alpha)n_t && (13)\\
\bar{i}i_t &= \bar{k}(k_{t+1} - (1 - \delta_1\delta_2\bar{u}^{\delta_2})k_t + \delta_1\delta_2\bar{u}^{\delta_2}u_t) && (14)\\
y_t &= c_t + n_t && (15)\\
y_t &= \delta_2 u_t + k_t && (16)\\
c_t &= c_{t+1} - r_{t+1} && (17)\\
r_{t+1} &= \beta[\alpha(\overline{Y/K})(y_{t+1} - k_{t+1}) - \delta_1\delta_2\bar{u}^{\delta_2}u_{t+1}] && (18)
\end{aligned}
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c) Modify the program RBC1.m (which solves the basic RBC model using Uhlig's toolkit) to fit a model with variable utilization. Assuming $\beta = 0.99, A = 1.0, \alpha = 0.33$ and appropriately choosing δ_1, δ_2 so that in the steady state utilization is positive and close to one, trace out the responses of the variables of the system to a one standard error shock in technology. Repeat the exercise choosing a δ_1, δ_2 that imply a low level of utilization in the steady state.

see program rbc-capu-solution.m